ADVANCED COMPUTATIONAL TECHNIQUES FOR THE DESIGN OF DEFORMATION PROCESSES

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Objectives

A continuum framework for sensitivity analysis of large inelastic deformations is being developed with emphasis on a gradient-based optimization approach to robust design of multi-stage metal forming processes. It provides the means to select the sequence of deformation processes, design the dies/preforms for each process stage as well as the process conditions such that a product is obtained with desired shape/microstructure and with minimal material utilization. This virtual process laboratory will assist the aircraft manufacturing industry by providing the means for an accelerated materials process insertion.

1 Status of effort

A number of advances in the development of thc continuum sensitivity method (CSM) for the design of deformation processes will be reported in this summary. In particular:

- A computational framework was developed to evaluate the shape and parameter sensitivities of finite coupled thermo-inelastic deformations using the continuum sensitivity method (CSM). The effects of perturbation in the preform, die surface, or other process parameters are carefully considered in the CSM development for the computation of the die temperature sensitivity fields [1, 2, 3].
- A mathematically rigorous CSM was also introduced to evaluate the gradients of the objective function and constraints in the optimization-based design of multi-stage deformation processes. The multi-stage analysis takes a form similar to the updated Lagrangian (UL) sensitivity framework for the design of single-stage processes. It allows us to treat in a unified manner shape and parameter sensitivity analyses [4, 5].
- Finally, a CSM was recently completed for large thermoplastic deformations combined with ductile damage. The computed sensitivity fields are used for the design of metal forming processes of porous materials. In addition to die design problems, novel preform (shape) design problems are examined for near net shape manufacturing that accounts for the volume change induced during deformation [6].

The developed forming design simulator can at present address a variety of design problems for geometrically complex two-dimensional and axisymmetric multi-stage deformation processes. An updated Lagrangian framework is used that allows accurate data transfer operations during remeshing in both the direct and sensitivity analyses [7]. Multi-objective design optimization is considered and various constraints can be imposed on the process conditions or variation of the design variables.

1.1 The CSM for hot-forming design [1]

A weak thermal sensitivity problem was introduced for the workpiece. It takes the form:

$$\begin{aligned}
&\int_{B_{n+1}} \frac{\rho c}{\Delta t}\left(\overset{\circ}{\theta}_{n+1} - \overset{\circ}{\theta}_{n}\right) \vartheta \, dV + \int_{B_{n+1}} \frac{(\rho c)'}{\Delta t}\left(\theta_{n+1} - \theta_{n}\right) \overset{\circ}{\theta}_{n+1} \ \vartheta \, dV + \\
&+ \int_{B_{n+1}} K \nabla_{n+1} \overset{\circ}{\theta}_{n+1} \cdot \nabla_{n+1} \vartheta \, dV + \int_{B_{n+1}} K' \, \overset{\circ}{\theta}_{n+1} \, \nabla_{n+1} \theta_{n+1} \cdot \nabla_{n+1} \vartheta \, dV + \\
&+ \int_{\partial B_{n+1}} \left[\overset{\circ}{q}_{n+1}\right] \cdot n \vartheta dA = \int_{B_{n+1}} \overset{\circ}{W}_{mech,n+1} \ \vartheta \, dV + \\
&\int_{B_{n+1}} \nabla_{n+1} q_{n+1} \cdot L_{n+1} \ \vartheta \, dV - \int_{B_{n+1}} L^{T}_{n+1} q_{n+1} \cdot \nabla_{n+1} \vartheta \, dV
\end{aligned} \tag{1}$$

for every admissible sensitivity temperature field ϑ. The primary unknowns of Eq. (1) are the design differentials $\overset{\circ}{x} = \tilde{\overset{\circ}{x}}(X,t;\beta,\Delta\beta)$ and $\overset{\circ}{\theta} = \tilde{\overset{\circ}{\theta}}(X,t;\beta,\Delta\beta)$. The required relationships between (a) $\overset{\circ}{F}$ and $\overset{\circ}{x}$, (b) $\overset{\circ}{W}_{mech,n+1}$ and [$\overset{\circ}{x}, \overset{\circ}{\theta}$] and more details are given in [1].

Sensitivity thermal analysis in the die with respect to changes in process parameters or the workpiece initial shape are similar to the parameter sensitivity thermal analysis for the workpiece but with careful consideration of the workpiece/die interface conditions. The effect of change in the die shape resembles the shape sensitivity analysis for the workpiece. The weak form that defines the sensitivity of the temperature field in the die for perturbations in the die surface itself takes the form: For every admissible ϑ,

$$\begin{aligned}
&\int_{B_{\mathrm{d}}} \frac{\rho c}{\Delta t}\left(\overset{\circ}{\theta}_{n+1} - \overset{\circ}{\theta}_{n}\right) \vartheta \, dV + \int_{B_{\mathrm{d}}} \frac{(\rho c)'}{\Delta t}\left(\theta_{n+1} - \theta_{n}\right) \overset{\circ}{\theta}_{n+1} \ \vartheta \, dV + \\
&+ \int_{B_{\mathrm{d}}} K \nabla_{d} \overset{\circ}{\theta}_{n+1} \cdot \nabla_{d} \vartheta \, dV + \int_{B_{\mathrm{d}}} K' \, \overset{\circ}{\theta}_{n+1} \, \nabla_{d} \theta_{n+1} \cdot \nabla_{d} \ \vartheta \, dV + \int_{\partial B_{d}} \left[\overset{\circ}{q}_{n+1}\right] \cdot n \vartheta dA \\
&= \int_{B_{\mathrm{d}}} \nabla_{d} q_{n+1} \cdot L_{n+1} \ \vartheta \, dV - \int_{B_{\mathrm{d}}} L^{T}_{n+1} q_{n+1} \cdot \nabla_{d} \vartheta \, dV
\end{aligned} \tag{2}$$

where $L \equiv L_d$ is the design velocity gradient defined from perturbations in the die surface. All parts of the die surface (contact and no-contact) contribute to L_d.

1.2 Sensitivity analysis of multi-stage processes [4]

Let β^h be a set of design parameters that define the processing history (from the design-independent reference configuration B_0 to $B^h_f \equiv B^c_0$). We herein use h and c to denote the processing history and current stage, respectively. The subscripts 0 and f denote the initial and final configurations in a given stage, respectively. The deformation history leading to the intermediate preform B^c_0 for each selection of β^h is thus described as follows (see Fig. 1):

$$X \ = \ \bar{x}(Y, t^h_f; \beta^h), \quad Q^c_0 \ = \ \bar{Q}(Y, t^h_f; \beta^h), \ \forall \, Y \in B_0 \tag{3}$$

For a given β^h, the configurations B^c_i, $i = 1, \ldots, f$, of the current stage are defined from:

$$x = \tilde{x}(X,t;\beta^c) = \bar{x}(Y,t;\beta^h,\beta^c), \ \ Q = \tilde{Q}(X,t;\beta^c) = \bar{Q}(Y,t;\beta^h,\beta^c), \ \forall \, X \in B^c_0, \ t \in [t^c_0, t^c_f] \tag{4}$$

where β^c represents the design variables in the current stage.

Let us consider at first the sensitivities with respect to the design parameters β^c of field variables in the current forming stage with a fixed β^h (thus a given B_0^c and Q_0^c). Fig. 1 presents a schematic that shows the variation of the fields x^c and Q^c induced by a variation in the process parameter β^c. An UL representation is adopted here.

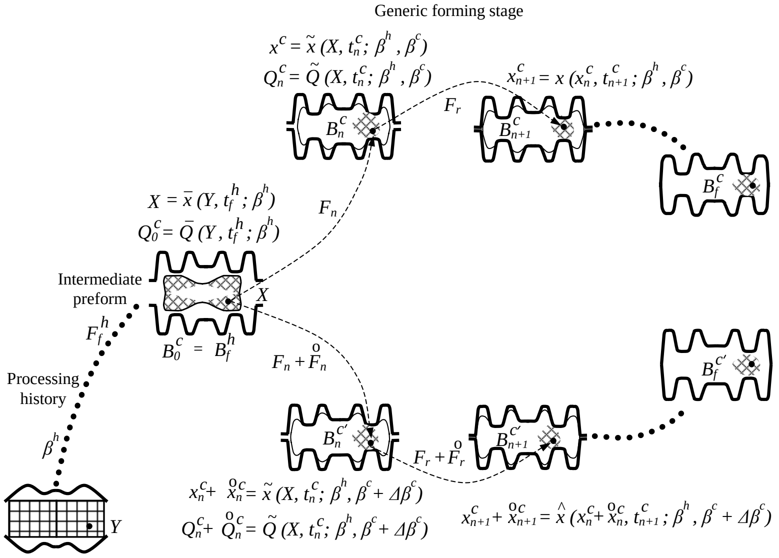

Figure 1: Schematic of the design sensitivity of the deformation in the current forming stage due to variations in the design parameters β^c of the current stage. $Q = \{F^p, s, \theta\}$ characterizes the field variables needed to define each deformation stage.

Let us consider a generic field Φ that can represent x, Q or any other material or deformation related field. The dependence of the UL field $\Phi = \hat{\Phi}(x_n, t)$ on β^c is expressed as:

$$\Phi = \hat{\Phi}(x_n^c, t; \beta^c) = \hat{\Phi}(\tilde{x}(X, t; \beta^c), t; \beta^c) = \tilde{\Phi}(X, t; \beta^c),\ t \in [t_n^c, t_{n+1}^c] \tag{5}$$

with the position x_n^c referred to the reference configuration B_n^c. The parameter sensitivity $\overset{\circ}{\Phi}$ on β^c is defined as the total Gateaux differential of $\Phi = \hat{\Phi}(x_n^c, t; \beta^c)$ in the direction $\Delta\beta^c$:

$$\hat{\mathring{\Phi}}(x_n^c, t; \beta^c, \Delta\beta^c) = \tilde{\mathring{\Phi}}(X, t; \beta^c, \Delta\beta^c) = \frac{d}{d\omega}\,\tilde{\Phi}(X, t; \beta^c + \omega\Delta\beta^c)\bigg|_{\omega=0},\ t \in [t_n^c, t_{n+1}^c] \tag{6}$$

The sensitivity $\overset{\circ}{\Phi} = \hat{\mathring{\Phi}}(x_n^c, t; \beta^h, \Delta\beta^h)$ on β^h of an UL field $\Phi = \hat{\Phi}(x_n^c, t)$ is defined as

$$\hat{\mathring{\Phi}}(x_n^c, t; \beta^h, \Delta\beta^h) = \tilde{\mathring{\Phi}}(X, t; \beta^h, \Delta\beta^h) = \bar{\mathring{\Phi}}(Y, t; \beta^h, \Delta\beta^h) = \frac{d}{d\lambda}\,\bar{\Phi}(Y, t; \beta^h + \lambda\Delta\beta^h)\bigg|_{\lambda=0} \tag{7}$$

Let us consider the dependence of $\hat{\Phi}(x_n^c, t_{n+1}^c)$ on β^h. This dependence results in the solution of the incremental deformation problem in the current forming state from the fact that the reference shape ∂B_n^c and field distribution Q_n^c depend on β^h. In particular, one can write:

$$\hat{\Phi}(x_n^c, t_{n+1}^c; \beta^h) \;=\; \hat{\Phi}(x_n^c, t_{n+1}^c; \partial B_n^c(\beta^h), Q_n^c(\beta^h)) \tag{8}$$

Using the definition of Eq. (7) and Eq. (8), one can obtain that:

$$\hat{\mathring{\Phi}}(x_n^c, t_{n+1}^c; \beta^h) = \frac{\partial\hat{\Phi}(x_n^c, t_{n+1}^c; \partial B_n^c, Q_n^c)}{\partial(\partial B_n^c)}\left[\frac{\partial(\partial B_n^c)}{\partial\beta^h}[\Delta\beta^h]\right] + \frac{\partial\hat{\Phi}(x_n^c, t_{n+1}^c; \partial B_n^c, Q_n^c)}{\partial Q_n^c}\left[\frac{\partial Q_n^c}{\partial\beta^h}[\Delta\beta^h]\right] \tag{9}$$

To evaluate $\overset{\circ}{\Phi}$ in an updated Lagrangian framework, one must thus compute the Gateaux differentials of ∂B^c_{n+1} and Q^c_{n+1} with respect to ∂B^c_n and Q^c_n, for $n = 1, 2, \ldots, f$. In this incremental calculation, the effect of the previous processing history becomes apparent and relevant only as initial conditions at $t = t^c_0$:

$$\frac{\partial(\partial B^c_0)}{\partial\beta^h}[\Delta\beta^h] = \frac{\partial(\partial B^h_f)}{\partial\beta^h}[\Delta\beta^h], \;\; \frac{\partial Q^c_0}{\partial\beta^h}[\Delta\beta^h] = \frac{\partial Q^h_f}{\partial\beta^h}[\Delta\beta^h] \tag{10}$$

The calculations of Eq. (9) and Eqs. (10) indicate that the sequential updated Lagrangian approach proposed here treats various types of sensitivity fields in a unified manner, e.g. it allows Eqs. (9) to be used regardless if the initial conditions of Eq. (10) refer to shape or process-based sensitivities in the processing history.

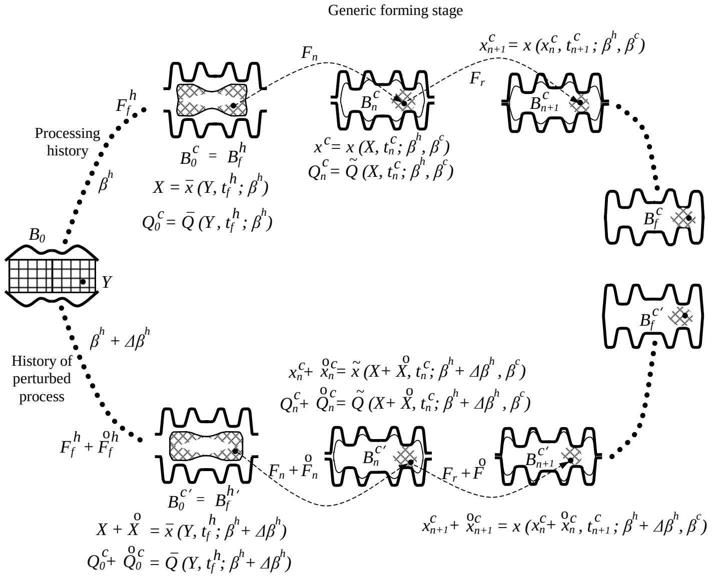

Figure 2: Schematic representation of the design sensitivity of the deformation in the current forming stage due to variations in the design parameters of previous forming stages.

1.3 CSM for materials with ductile damage [6]

A decomposition $F = F^e\, F^p\, F^\theta$ was introduced and $\det F^p = (1 - f_o)/(1 - f)$ was taken as a measure of internal damage, where f_o and f are the void volume fractions in the initial and deformed configurations, respectively. In the hyperelastic constitutive equations used, the shear modulus μ and bulk modulus κ are functions of f and temperature θ. The equivalent stress σ_m of the matrix was defined implicitly in terms of the Cauchy stress T and f:

$$\Phi = \Phi(\sigma_m, f, p, S) = 0 \tag{11}$$

where the dependence of Φ on $\bar{T}$ is restricted to its first and second invariants. Here, S, is the norm of the stress deviator, $\bar{T}'$ and p is the mean normal pressure.

Two scalar state variables are considered here, one representing the isotropic material hardening behavior, denoted by s, and the other, the volume void fraction, f. The evolution of the plastic part of the deformation gradient F^p is given by the normality rule,

$$\bar{D}^p = \mathrm{sym}\left(\dot{F}^p\, F^{p-1}\right) = \dot{\gamma}\,\partial_{\bar{T}}\Phi \tag{12}$$

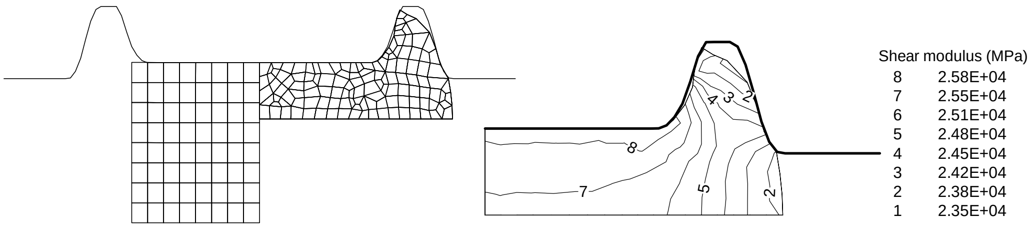

Figure 3: Initial guess preform shape, the final product using this guess and the distribution of shear modulus in the final product. For a non-porous material, this preform will fill the die as required without flash.

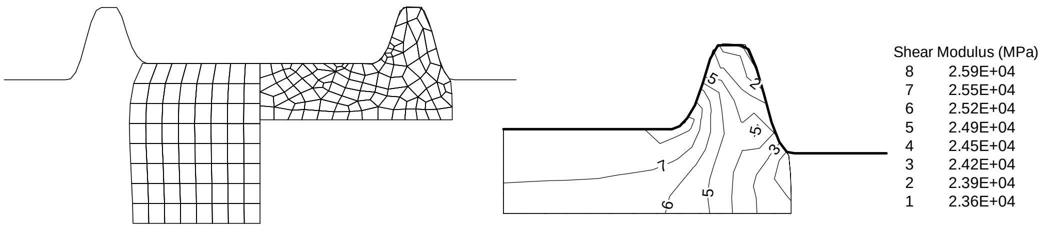

Figure 4: Optimal preform shape, the final product using this preform and the distribution of the shear modulus.

The work relation, $\bar{T}\bullet\bar{D}^p = (1-f)\sigma_m\dot{\bar{\varepsilon}}^p_m$, computes $\dot{\gamma}$. The evolution of the equivalent tensile plastic strain and of the isotropic scalar resistance s are experimentally specified as

$$\dot{\bar{\varepsilon}}^p_m = f(\sigma_m, s, \theta), \;\; \dot{s} = g(\sigma_m, s, \theta) = h(\sigma_m, s, \theta)\dot{\bar{\varepsilon}}^p_m - \dot{r}(s, \theta) \tag{13}$$

The evolution equation for the void fraction is finally computed from $\dot{f} = (1-f)tr(\bar{D}^p)$. In our analysis and design, the body is assumed to behave as a continuum despite the presence of microvoids. A mathematically rigorous extension of the CSM method to the material model described above was introduced [6]. Full design-differentiation of all material constitutive equations was performed for both shape and process parameter sensitivity analysis. It is interesting to note that the preform design analysis developed is able to account for the precise change of volume induced during the deformation process. To the best of our knowledge this is the first time of mathematically addressing preform design problems for porous materials for near-net shape optimization.

1.4 An example of a preform design for closed-die forging

The objective here is to design the volume and the free surface of a cylindrical preform of height 2.0 mm, that when compressed with a given closed forging die, fills the die completely with minimum flash, after a specified stroke of 0.65 mm. The material is taken to be $2024-T351$ Al, initially at 300 K and with variable elastic moduli [6].

Fig. 3 shows the quarter geometry of the initial guess preform (a right circular cylinder of radius 0.8 mm and height of 2 mm) and final product shape obtained using this guessed preform. Fig. 4 shows the optimal preform and the product shape achieved using this optimal preform. The variation of the objective function with iterations is shown in Fig. 5. Also shown in Figs. 3 and 4 is the variation of the shear modulus in the forged product.

2 Personnel Supported

N. Zabaras (PI), S. Ganapathysubramanian & Velamur asokan Badri Narayanan (GRAs).

3 Interactions/Transitions

- Companies interacting with us include: General Electric (Srikanth Akkaram, a former PhD student from our group is now employed with GE-CRD), Pratt and Whitney (P&W) and Alcoa.
- Our collaboration with Alcoa is continuing towards the experimental evaluation of computational designs, on the design of flat-die extrusion and multiple-pass rolling processes as well as on the design of processes for damaged materials.
- The PI has reviewed this project in various agencies and universities including at Purdue (3/29/01), Alcoa (7/17/01), AFRL (7/25/01), DARPA (8/27/01), Michigan State University (2/5/02) & GE-CRD (3/5/02).

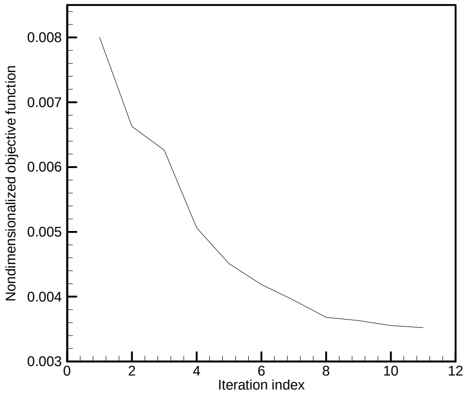

Figure 5: Variation of the nondimensionalized objective function with iteration index for a theromechanical closed die forming process.

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