

# ADVANCED COMPUTATIONAL TECHNIQUES FOR THE DESIGN OF DEFORMATION PROCESSES

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## Objectives

The objective of this work is to develop a continuum sensitivity finite element analysis for the robust design of multi-stage metal forming processes in aircraft manufacturing. The computational forming design simulator being developed is applied to industrial forming design and provides the means to select the sequence of deformation processes, design the dies and preforms for each process stage as well as the process conditions such that a product is obtained with desired shape and microstructure and with the minimal material utilization and overall cost. This virtual process laboratory will assist the aircraft manufacturing industry in reducing time for process and product development, in trimming the cost of an extensive experimental process development effort and in developing processes for tailored material properties.

## 1 Status of Effort

Substantial progress was accomplished towards the project objectives during the first year of this AFOSR award. In particular, we addressed issues of remeshing and data transfer for sensitivity analysis, issues of incompressibility and element locking for a consistent (to the direct analysis) treatment of sensitivity analysis. We also developed a 2D implementation of a continuum framework for the sensitivity analysis of non-isothermal deformation processes and a computational framework for computing the sensitivities in multi-stage processes. Finally, we computed the solution to a number of practical optimization problems in the design of forming processes.

The design simulator has been tested with a number of examples that involve complex evolving geometries and realistic material representation and contact conditions. The simulator has been shown to produce very accurate sensitivity fields in Lagrangian analysis for elasto-viscoplastic state-variable-based materials that undergo large deformations in the presence of contact and friction. A regularized treatment of the contact conditions has been shown to produce very accurate representation of the sensitivities of the contact tractions thus allowing for correct modeling of shape sensitivity (preform design) problems driven mainly by changes in the contact conditions as a result of shape changes in the initial workpiece.

The 2D forming design simulator in its current form can be used to optimize single and some preliminary multi-stage deformation processes. The simulator allows design with multi-objective functions and constraints and design variables that refer to pro-

cess parameters (die speed or stroke, operating temperature, etc.), die surfaces, and preforms (initial workpiece geometry). The design objectives can be defined in the final product (such as shape or material state control of the final product) or through out the deformation processes (such as the total work of deformation, etc.)

## 2 Accomplishments

A brief description is provided below of the technical accomplishments during FY2000.

### 2.1 Sensitivity Framework for Large Deformations

In our earlier work [1]-[3], the direct deformation problem was posed in an updated Lagrangian (UL) framework and the sensitivity problem was posed in a total Lagrangian (TL) framework. These analyses were performed without remeshing and were restrictive in the range of applications that could potentially be handled. The sensitivity deformation problem is now modeled with an UL formulation on the reference configuration  $\mathbf{B}_n$  [4]. The design sensitivity of the equilibrium equation results in a variational form posed as follows [2]: Calculate  $\overset{\circ}{\mathbf{x}} = \overset{\circ}{\hat{\mathbf{x}}}(\mathbf{x}_n, t; \boldsymbol{\beta}, \Delta\boldsymbol{\beta})$  such that

$$\int_{\mathbf{B}_n} \left\{ \overset{\circ}{\mathbf{P}}_r \cdot \nabla_n \tilde{\boldsymbol{\eta}} - P_r \nabla_n \cdot \mathbf{L}_n^T \cdot \tilde{\boldsymbol{\eta}} - P_r \mathbf{L}_n^T \cdot \nabla_n \tilde{\boldsymbol{\eta}} \right\} dV_n = \int_{\Gamma} \left\{ \overset{\circ}{\boldsymbol{\lambda}} - [\mathbf{L}_n \cdot (\mathbf{N} \otimes \mathbf{N})] \boldsymbol{\lambda} \right\} \cdot \tilde{\boldsymbol{\eta}} dA_n$$

where  $\tilde{\boldsymbol{\eta}}$  is an admissible field,  $\boldsymbol{\beta}$  is the design vector,  $\Delta\boldsymbol{\beta}$  is the perturbation of  $\boldsymbol{\beta}$ ,  $\mathbf{N}$  is the unit normal in  $\partial\mathbf{B}_n$  and  $\mathbf{L}_n = \overset{\circ}{\mathbf{F}}_n \mathbf{F}_n^{-1}$  is the design velocity gradient at  $t_n$ , and the subscript  $r$  indicates fields defined relative to  $\mathbf{B}_n$ . To solve for  $\overset{\circ}{\mathbf{x}}$ , the relationships between  $\overset{\circ}{\mathbf{F}}_r$  and  $\overset{\circ}{\mathbf{x}}$ ,  $\overset{\circ}{\mathbf{P}}_r$  and  $\overset{\circ}{\mathbf{F}}_r$ , and  $\overset{\circ}{\boldsymbol{\lambda}}$  and  $\overset{\circ}{\mathbf{x}}$  need to be developed [2]. Parameter and shape sensitivity analyses are now treated in the same manner. However, in a shape sensitivity analysis  $\mathbf{L}_o$  drives the sensitivity problem in contrast to a parameter sensitivity analysis (e.g. die design) where  $\mathbf{L}_o = \mathbf{0}$  and the sensitivity problem is driven by changes in the die shape.

### 2.2 Remeshing and Data Transfer Techniques for Sensitivity Analysis

A computational framework has been developed to evaluate the sensitivity of finite deformations when remeshing operations are performed during the FE analysis. The issue of accurate data transfer after a remeshing operation was examined for both direct and sensitivity analyses. The developed algorithms (a) maintain consistency of the constitutive equations with the computed deformation; (b) satisfy equilibrium; (c) minimize the numerical diffusion of transferred state fields; and (iv) account for the incompressible nature of plastic deformations.

The remeshing operation occurs in  $\mathbf{B}_n$  based on the element quality obtained in  $\mathbf{B}_{n+1}$ . Once the data transfer at  $t_n$  has occurred, the overall problem is resolved in  $[t_n, t_{n+1}]$  thus providing a new equilibrated configuration  $\mathbf{B}_{n+1}$  that is consistent with the computed state fields. In [4], we advocated the Gauss point to Gauss point data transfer mappings but an extension of these techniques was recently proposed based on an interpolation of Gauss point values of the old mesh using distance-based weights.

The required transfer fields for the direct analysis include  $(\mathbf{F}_n, \mathbf{F}_n^e, s)$ , whereas for the sensitivity analysis we also transfer  $(\mathbf{F}_n^p, \overset{\circ}{\mathbf{F}}_n, \overset{\circ}{\mathbf{F}}_n^e, \overset{\circ}{\mathbf{F}}_n^p, \overset{\circ}{s}_n)$ .

### 2.3 Incompressibility Issues in Sensitivity Deformation Analysis

The standard bilinear quadrilateral with full integration performs poorly in the incompressible limit. Assumed strain ( $\mathbf{B}$ - &  $\mathbf{F}$ -bar) as well as enhanced strain methods have been proposed to address this issue for the direct deformation problem. In the context of our simulator, a discrete relative deformation gradient  $\bar{\mathbf{F}}_h$  in a typical finite element is defined as  $\bar{\mathbf{F}}_h = \bar{\mathbf{F}}_h^{vol} \mathbf{F}_h^{dev}$ , where  $\bar{\mathbf{F}}_h^{vol}$  is computed with a reduced quadrature integration points. An a-priori stabilization method is used to define the assumed relative deformation gradient as  $\mathbf{F}_h^{ave} \equiv \epsilon \mathbf{F}_h + (1 - \epsilon) \bar{\mathbf{F}}_h$  ( $\epsilon \rightarrow 0$ ). In the following discussion, we limit ourselves to the  $\mathbf{F}$ -bar method. Thus  $\mathbf{P}_r \equiv \mathbf{P}_r(\mathbf{F}_h^{ave})$  is used in the finite element representation of the internal work term. Using the continuum sensitivity constitutive relationships one can compute a linear relationship  $\overset{\circ}{\mathbf{P}}_r = \overset{\circ}{\mathbf{P}}_r(\overset{\circ}{\mathbf{F}}_h^{ave})$ . The sensitivity of the assumed (discrete) relative deformation gradient is obtained by the design differentiation of the volumetric-deviatoric decomposition introduced earlier. The kinematic relationship between  $\overset{\circ}{\mathbf{F}}_h^{ave}$  and  $\overset{\circ}{\mathbf{x}}_h$  is also required. The  $\mathbf{F}$ -bar method for the sensitivity analysis finally takes the form:

$$\sum_e \left[ \int_{\Omega_e} \left[ \overset{\circ}{\mathbf{P}}_r(\overset{\circ}{\mathbf{F}}_h^{ave}) \cdot \nabla_n \tilde{\eta}_h - (\mathbf{P}_r(\mathbf{F}_h^{ave}) [\nabla_n \cdot \mathbf{L}_n^T]) \cdot \tilde{\eta}_h - [\mathbf{P}_r(\mathbf{F}_h^{ave}) \mathbf{L}_n^T] \cdot \nabla_n \tilde{\eta}_h \right] dV_n \right] = 0$$

Preliminary results using the  $\mathbf{F}$ -bar method are given in [4]. For similar developments using the  $\mathbf{B}$ -bar method see [5].

### 2.4 Implementation of Sensitivity Analysis for Hot Forming

A sensitivity analysis was developed to account for all couplings between the deformation and thermal problems such as volumetric heat generation, frictional heating, state evolution for non-isothermal conditions, change of thermal boundary conditions resulting from changes in the design, etc. [6], [7]. As an example, the weak thermal sensitivity sub-problem for a preform design analysis is defined in  $\mathbf{B}$  as:

$$\begin{aligned} & \int_{\mathbf{B}} \frac{\rho c}{\Delta t} (\overset{\circ}{\theta}_{n+1} - \overset{\circ}{\theta}_n) \vartheta dV + \int_{\mathbf{B}} \frac{(\rho c)'}{\Delta t} (\theta_{n+1} - \theta_n) \overset{\circ}{\theta}_{n+1} \vartheta dV + \int_{\mathbf{B}} K \nabla_x \overset{\circ}{\theta}_{n+1} \cdot \nabla_x \vartheta dV + \\ & \int_{\mathbf{B}} K' \overset{\circ}{\theta}_{n+1} \nabla_x \theta_{n+1} \cdot \nabla_x \vartheta dV + \int_{\partial \mathbf{B}} [\overset{\circ}{\mathbf{q}}_{n+1}] \cdot \mathbf{n} dA = \int_{\mathbf{B}} \overset{\circ}{\mathcal{W}}_{mech, n+1} \vartheta dV + \\ & \int_{\mathbf{B}} \nabla_x \mathbf{q}_{n+1} \cdot \mathbf{L}_{n+1} \vartheta dV - \int_{\mathbf{B}} \mathbf{L}_{n+1}^T \mathbf{q}_{n+1} \cdot \nabla_x \vartheta dV \end{aligned}$$

for every admissible  $\vartheta$ . This continuum sensitivity analysis framework clearly identifies all sources that contribute to the calculation of the sensitivity temperature field including the design velocity gradient  $\mathbf{L}$ , the sensitivity of the deformation induced heat  $\overset{\circ}{\mathcal{W}}_{mech, n+1}$  as well as changes in the boundary thermal conditions induced by

changes in the design. The primary unknowns are the design differentials  $\overset{\circ}{\mathbf{x}}$  and  $\overset{\circ}{\theta}$ . A similar set of equations is derived for the sensitivity of the deformation problem. In order to obtain the final form of the variational sensitivity problem, the relationships between (a)  $\overset{\circ}{\mathbf{F}}$  and  $\overset{\circ}{\mathbf{x}}$  (b)  $\overset{\circ}{\mathcal{W}}_{mech,n+1}$  and  $[\overset{\circ}{\mathbf{x}}, \overset{\circ}{\theta}]$  are used [6].

We solve the overall sensitivity problem in a two-step iterative process in order to maintain consistency with the direct analysis [6], [7]. In addition to the enormous potential of this sensitivity analysis for hot forming design, we plan to investigate the feasibility of independently controlling the thermal conditions in the die in order to further affect the microstructure development in the workpiece.

## 2.5 Framework for sensitivity analysis of multi-stage processes

We are currently developing sensitivity algorithms for multi-stage forming processes (see Fig. 1). For simplicity of the presentation non-shape parameters are used here and we consider design sensitivity with respect to  $\beta_Y$  (Fig. 1). We can write the following:  $\hat{\Phi}(\mathbf{X}, t; \beta_Y) = \tilde{\Phi}(\mathbf{Y}, t; \beta_Y) = \bar{\Phi}(\mathbf{Y}, t; \partial \mathbf{B}_o(\beta_Y), Q(\beta_Y))$ . The sensitivity of the field  $\hat{\Phi}$  with respect to  $\beta_Y$  is computed as follows:

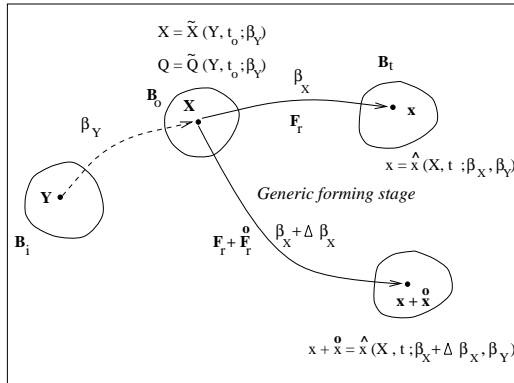


Figure 1: The design sensitivity of the deformation in the current forming stage due to variations in the (non-shape) design parameters of current forming stage.  $\beta_X$  represents design variables in the current stage and  $\beta_Y$  design variables in the previous stage.  $Q$  denotes the material state and plastic deformation field variables.

$$\overset{\circ}{\Phi} = \frac{\partial \bar{\Phi}(\mathbf{Y}, t; \partial \mathbf{B}_o, Q)}{\partial (\partial \mathbf{B}_o)} \left[ \frac{\partial (\partial \mathbf{B}_o)}{\partial \beta_Y} [\Delta \beta_Y] \right] + \sum_i \frac{\partial \bar{\Phi}(\mathbf{Y}, t; \partial \mathbf{B}_o, Q)}{\partial Q_i} \left[ \frac{\partial Q_i}{\partial \beta_Y} [\Delta \beta_Y] \right] \quad (1)$$

To evaluate  $\overset{\circ}{\Phi}$ , one first computes the differentials of  $\partial \mathbf{B}_o$  and  $Q$  with respect to  $\Delta \beta_Y$ . Thus, the sensitivities for multi-stage processes are computed in ‘a sequential manner’. These developments are presented in [8], [9].

## 2.6 Industrial forming design examples

The workpiece is 1100-Al at a temperature of 673 K [1] and an isothermal forging process is considered. Bézier curves are used for the representation of the dies and preforms [8]. When a single stage process is applied (Fig. 2), the die cavity cannot be fully filled. This problem is posed here as a two-stage design, where an open die is adopted in the performing stage and a closed die in the finishing stage. The closed die forms the desirable boundary of the final product. The objective is to design the open die shape such that the finishing die cavity can be completely filled. The BFGS algorithm is employed using the results of the sensitivity analysis.

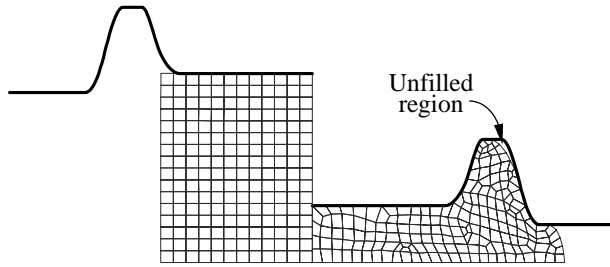


Figure 2: A single stage forging process for an axisymmetric ribbed disk. The initial billet is a cylinder of 2 mm in height and 0.8 mm in radius. Further details are given in [8].

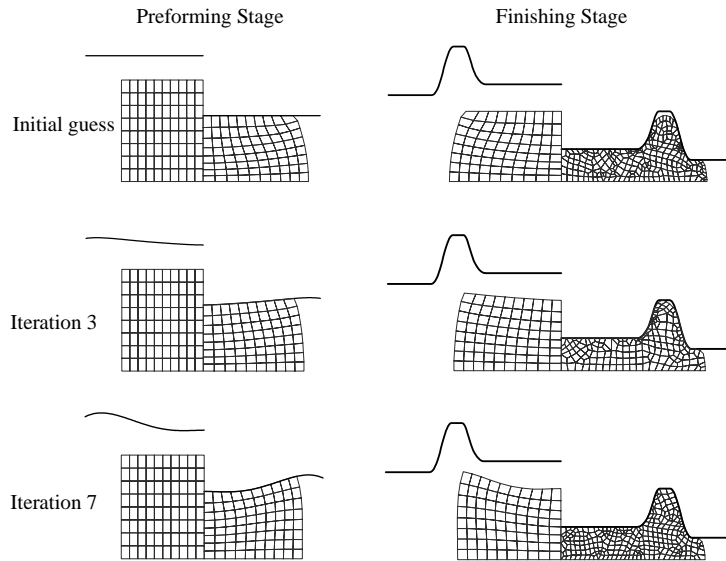


Figure 3: A two-stage forging design process for an axisymmetric ribbed disk.

Figure 3 shows the design process of the performing die. The initial guess for the shape of the performing die is a straight line. After 7 design iterations, a perfect fill is obtained. In contrary to the optimal two-stage design, the single stage process requires a higher force to accomplish the prescribed stroke, whereas the die cavity has not been fully filled (Fig. 4(a)). The convergence rate is shown in Fig. 4(b).

### 3 Personnel Supported

N. Zabararas (PI), S. Ganapathysubramanian (full time GRA), Dr. R. Sampath (GRA, Fall 2000) and H. Hou (GRA, Spring 2000). Others associated with this effort (supported by AFRL) include: Drs. S. Akkaram (former GRA), Qing Li, and Z. Hu.

### 4 Interactions/Transitions

- With the support of AFRL (Drs. S. LeClair and W. G. Frazier), we organized at Cornell (9/2000), a symposium in order to kick off this project and to allow direct interactions between our team, AFRL and aircraft manufacturers.
- Companies interacting with us include: General Electric (GE), Pratt and Whitney (P&W), Alcoa and Scientific Forming Technologies Corporation (SFTC).
- Alcoa is experimentally verifying the design simulator. Alcoa's work with AFRL on ideal-forming will be incorporated in our algorithms as 'initial design'. Dr. S. Alexandrov of Alcoa worked in our lab for 2 weeks to facilitate these transitions.

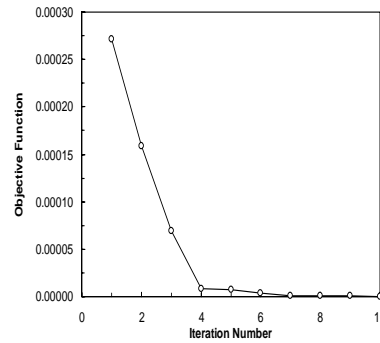
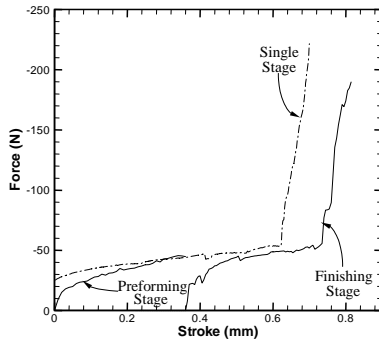


Figure 4: (a) Comparison of forces between the single-stage and two-stage processes, (b) Convergence of the optimization process.

- SFTC has agreed to work with the Cornell team in transitioning the algorithmic developments to ‘DEFORM’ (a widely used industrial forming simulator).
- The PI has reviewed this project at AFRL (‘FSI Tech Man Review’, 4/23/01 & EP’01, 7/25/01), GECD, Alcoa and academic sites (Cornell & Purdue).

Interactions with GE & P&W have been coordinated by Dr. Frazier. However, his recent resignation from AFRL has led to a delay in implementing these plans.

## 5 Honors/Awards

Professor N. Zabaras received a Presidential Young Investigator Award in 1991 from the National Science Foundation. He was promoted to Full Professor during FY2000.

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