

## **A Computational Statistics Approach to Stochastic Inverse Problems and Uncertainty Quantification in Heat Transfer**

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### **Abstract**

As most engineering systems and processes operate in an uncertain environment, it becomes increasingly important to address their analysis and inverse design in a stochastic manner using statistical data-driven prior and concurrent information on the system response. Recent advances in computational Bayesian and spatial statistics enable complete and efficient solution procedures to such problems. Herein, a novel framework based on Bayesian inference is presented for the solution of multi-dimension and multi-mode inverse heat transfer problems. The posterior probability density function (PPDF) of unknowns (modeled as random variables or stochastic processes), such as the boundary heat flux or heat sources, is computed given finite set of thermocouple temperature measurements. Markov Random Field (MRF) was adopted for prior distribution modeling. Markov Chain Monte Carlo (MCMC) algorithms are exploited to obtain estimates of statistics of random unknowns. One and two-dimensional linear boundary heat flux reconstruction problems are first discussed to demonstrate stochastic regularization through prior modeling and MCMC posterior state space exploration. A one-dimensional inverse heat conduction problem (IHCP) with consideration of uncertainty in thermo-physical properties and sensor location is also presented for complete stochastic description of the IHCP. Finally, a problem of reconstructing heat sources in three-dimensional radiation heat transfer is presented. This example will demonstrate the need for model reduction in the likelihood calculation when applying MCMC algorithms to complex continuum problems. Simulation results demonstrate the great potential of applying a Bayesian approach to stochastic estimation and design problems. Although discussed in the context of thermal systems, the methodology presented is general and applicable to design and estimation problems in diverse areas in engineering.

### **REFERENCES**

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