

A Variational Multiscale Stabilized Finite Element Method for Stochastic Advection-Diffusion and Stochastic Incompressible Flow

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Abstract

An extension of the deterministic variational multiscale (VMS) approach with algebraic subgrid scale (SGS) modeling is considered for developing stabilized finite element formulations for the linear stochastic scalar advection-diffusion equation and the incompressible stochastic Navier-Stokes equations. Different models for the intrinsic subgrid time scale will be discussed based on the level of subgrid characterization desired, phase lag and transient behavior restrictions. All these models possess similar behavior in the limits of pure advection and pure diffusion. The model chosen for τ in our numerical implementation ensures that the subgrid scale solution has finite statistical moments. The statistical behavior of τ depends on the kind of probability models chosen for the advection velocity and diffusion coefficient. Most probability models with finite support space are compatible with the proposed intrinsic time scale model. The stabilized formulations are numerically implemented using the spectral stochastic formulation of the finite element method (SSFEM). Generalized polynomial chaos and Karhunen-Loève expansion techniques are used for representation of uncertain quantities. A parallel finite element implementation using cluster specific implementation of BLAS I and BLAS II and a preconditioned parallel GMRES solver with restart capability were developed for parallel solution of the stochastic partial differential equations. Though the computational effort involved with the proposed method is an order of magnitude higher than that of a deterministic problem, it was still found to be two orders of magnitude faster than the Monte Carlo simulation for advection-diffusion problems. The proposed stabilized method is applied to various standard advection-diffusion and fluid flow examples with uncertainties in essential boundary conditions. Comparisons are drawn between the numerical solutions and Monte Carlo/analytical solutions wherever possible.

REFERENCES

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