THE STEFAN PROBLEM: A STOCHASTIC ANALYSIS USING THE EXTENDED FINITE ELEMENT METHOD

Baskar Ganapathysubramanian, Badri Velamur Asokan and Nicholas Zabaras
Materials Process Design and Control Laboratory
Sibley School of Mechanical and Aerospace Engineering
188 Frank H. T. Rhodes Hall
Cornell University
Ithaca, NY 14853-3801
baskigs@gmail.com, zabaras@cornell.edu

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The evolution of the solid-liquid interface for a pure material undergoing a phase transition can be simply represented by the Stefan equation. This equation has been extensively studied in a deterministic sense using a variety of interface capturing methodologies [1]. The presence of uncertainties in the physical properties of the material as well as fluctuations in the boundary conditions can cause significant deviations of the physical system from the deterministically predicted evolution. The uncertainties may include material property (thermal properties) uncertainty due to the presence of impurities as well random fluctuations in the boundary and initial conditions.
We will introduce methodologies using the support method [2] to quantify these uncertainties in a stochastic sense using the extended finite element method as the deterministic solution procedure. We will discuss how such uncertainties change the evolution of the interface shape. Several examples based on problems of increasing complexity will be shown to showcase the procedure.

References

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