An efficient Bayesian inference approach to inverse problems based on adaptive sparse grid collocation method

Xiang Ma and Nicholas Zabaras
Materials Process Design and Control Laboratory
Sibley School of Mechanical and Aerospace Engineering
101 Frank H. T. Rhodes Hall
Cornell University
Ithaca, NY 14853-3801

xm25@cornell.edu , njz1@cornell.edu

A new approach for modeling inverse problems using a Bayesian inference method is introduced. The Bayesian approach considers the unknown parameters as random variables and seeks the probabilistic distribution of the unknowns. By introducing the concept of the stochastic prior state space to the Bayesian formulation, we reformulate the deterministic forward problem as a stochastic one. The adaptive hierarchical sparse grid collocation method (ASGC) is used for constructing an interpolant of the solution to this stochastic forward model in this prior space which is large enough to capture all the variability/uncertainty in the posterior distribution of the unknown parameters. This solution can be considered as a stochastic function of the random unknowns in the stochastic space. This function serves as a stochastic surrogate model for the likelihood calculation. Hierarchical Bayesian formulation is used to derive the posterior probability density function (PPDF). The spatial model is represented as a convolution of a smooth kernel and Markov random field to achieve dimension reduction for facilitating the inference algorithm. Then the state space of the PPDF is explored using Markov chain Monte Carlo (MCMC) algorithms in order to obtain statistics of the unknowns. Instead of repeated evaluations of the deterministic forward model, the likelihood calculation is reduced by directly sampling the approximate stochastic solution obtained through the ASGC method. The technique is assessed on two non-linear inverse problems: source inversion and permeability estimation in flow through porous media.

Reference:

[1] X. Ma and N. Zabaras, “An efficient Bayesian inference approach to inverse problems based on adaptive sparse grid collocation method”, Inverse Problem, in press.