A nonlinear dimension reduction strategy for generating data driven stochastic input models[1]

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Computational resources have reached a point where it is now possible to analyze realistic problems in engineering in a stochastic setting. The bottleneck seems to be the construction of reliable and accurate models that describe the variability in properties of these realistic applications. These models serve as the stochastic input in the solution of the stochastic partial differential equations describing these systems. We illustrate an efficient technique for constructing a viable stochastic input model starting from a set of plausible 3D property variation data. A non-linear dimension reduction strategy is developed for generating this input stochastic model. It is shown that the space of input samples is a compact manifold, M embedded in a high dimensional input space. The methodology provides a transformation from M to a low dimensional, compact, connected set A. The algorithm ensures that M and A are locally isometric. Starting from a finite set of samples of the data (microstructures), the methodology attempts to recover the geometry of M (and hence A). Asymptotic convergence of the approximation to the actual geometry is shown. A non-parametric mapping from the low-dimensional set A to the high dimensional manifold M is constructed. The set A then serves as an accurate, low dimensional, data driven representation of the property variations. Illustrative examples using two phase microstructures are shown.

References

[1] B. Ganapathysubramanian and N. Zabaras, "Modelling diffusion in random heterogeneous media: Data-driven models, stochastic collocation and the variational multi-scale method", Journal of Computational Physics, in press

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