STOCHASTIC VARIATIONAL MULTISCALE METHOD FOR STOCHASTIC ELLIPTIC EQUATIONS WITH MULTISCALE COEFFICIENTS

V. A. Badri Narayanan and Nicholas Zabaras

Materials Process Design and Control Laboratory
Sibley School of Mechanical and Aerospace Engineering
188 Frank H. T. Rhodes Hall
Cornell University
Ithaca, NY 14853-3801
bnv2@cornell.edu, zabaras@cornell.edu

The stochastic variational multiscale (VMS) method is presented as a computational paradigm for stochastic elliptic equations with multiscale coefficients. In particular, an elliptic equation with a heterogenous stochastic diffusion coefficient possessing multiple scales is addressed. The stochastic VMS approach considers the solution as an additive sum decomposition of two scale components viz. coarse scales that can be resolved by the computational grid and subgrid scales. The variational formulations for the two scales are then derived in appropriately defined stochastic function spaces.

The subgrid variational formulation is now cast as a localized stochastic subgrid problem with appropriately defined boundary conditions. The subgrid solution is then represented as a sum of two components viz. the coarse-to-subgrid map and an inhomogeneous part. The coarse-to-subgrid component is defined based on the coarse scale solution nodal values and the inhomogeneous part does not contain a coarse scale reference. Parallel implementation of the subgrid problem and coupling with coarse scale problem will be discussed. Scalability and other issues will be presented.

Two separate stochastic modeling approaches will be considered viz. generalized polynomial chaos and a novel support-space approach. In the former, any stochastic quantity is represented as a sum of its projections on the Askey basis spanning the input probability space. In the latter, a random output is represented in a piecewise finite element representation in the input support space (regions with positive input joint probability distribution).

Issues of extension of the techniques for solving flow problems in porous media will be discussed. Finally, numerical examples validating the accuracy of the proposed algorithms will be presented.