# A STOCHASTIC VARIATIONAL MULTISCALE METHOD WITH EXPLICIT SUBGRID MODELING FOR ADVECTION-DIFFUSION SYSTEMS

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The derivation and implementation of a stochastic variational multiscale (VMS) approach for addressing multiscale stochastic advection-diffusion systems is presented. The stochastic VMS approach involves an additive sum decomposition of the stochastic solutions of the governing advection-diffusion equations into two scale components viz. coarse scales that can be resolved by a computational grid and the subgrid scales. The variational formulations for the two scales are derived in an appropriately defined stochastic function space. A generic construction of the subgrid problem with appropriate stochastic boundary conditions is then presented. Explicit stochastic subgrid algorithms based on Green's functions and partition of unity will be discussed with specifics on parallel implementation.

Comparisons will be drawn between the above approach and current deterministic subgrid modeling and upscaling techniques. Two separate stochastic modeling approaches will be considered viz. generalized polynomial chaos representation and a novel support-space approach. In the first approach, any stochastic quantity is represented as a sum of its projections onto a basis spanning the input probability space. In the latter approach, a random output is represented in a piecewise finite element representation in the input support space (regions where the input joint probability distribution is non-negative).

Issues regarding the applicability of the above stochastic modeling techniques are presented followed by a discussion on the importance of adaptive algorithms selection of multiscale regions in increasing the overall computational efficiency. Finally, numerical examples for validating the approach are provided.

## References

[1] V.A. Badri Narayanan and N. Zabaras, "Variational multiscale stabilized FEM formulations for transport equations: stochastic advection-diffusion and incompressible stochastic Navier-Stokes equations," *Journal of Computational Physics*, v. 202/1, p. 94--133, 2005.