

An adaptive hierarchical sparse grid collocation algorithm for the solution of stochastic differential equations¹

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In recent years, there has been a growing interest in analyzing and quantifying the effects of random inputs in the solution of ordinary/partial differential equations. To this end, the spectral stochastic finite element method (SSFEM) is the most popular method due to its fast convergence rate. Recently, the stochastic sparse grid collocation method has emerged as an attractive alternative to SSFEM. It approximates the solution in the stochastic space using Lagrange polynomial interpolation. The collocation method requires only repetitive calls to an existing deterministic solver, similar to the Monte Carlo method. However, both the SSFEM and current sparse grid collocation methods utilize global polynomials in the stochastic space. Thus when there are steep gradients or finite discontinuities in the stochastic space, these methods converge very slowly or even fail to converge. In this work, we develop an adaptive sparse grid collocation strategy using piecewise multi-linear hierarchical basis functions. Hierarchical surplus is used as an error indicator to automatically detect the discontinuity region in the stochastic space and adaptively refine the collocation points in this region. Numerical examples, especially for problems related to long-term integration and stochastic discontinuity, are presented. Comparisons with the multi-element generalized polynomial chaos and Monte-Carlo methods are also given to show the efficiency and accuracy of the proposed method.

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