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## Continuum sensitivity method for the design of multi-stage metal forming processes

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### Abstract

As part of this continuing DMI-funded NSF project, a novel, efficient and mathematically rigorous continuum based sensitivity method is being developed that can be used to accurately evaluate the gradients of the objective function and constraints in the design optimization of multi-stage deformation processes. The sensitivity methods developed allow for a unified treatment of both shape and parameter sensitivity analysis that are present in a typical design problem of multi-stage deformation processes. The continuum sensitivity method was recently extended to thermoplasticity combined with ductile damage at finite strains. The computed sensitivity fields are used within the developed gradient-based optimization framework for the computational design of metal forming processes for porous materials. The effectiveness of the proposed methodology is demonstrated with the solution of practical problems in the design of one-stage and two-stage metal forming processes.

## 1 Introduction

As we step into the 21st century, there is an increasing demand for high quality formed products with reduced material waste and low manufacturing costs. Finite element simulations play a vital role in materials processing and account for coupled non-linear mechanisms like microstructure evolution, varying contact and frictional conditions, large deformation plasticity, thermal effects and dissipation. In contrast to direct analysis tools, most of the methods for industrial deformation process design are currently focussed on traditional trial and error techniques which rely on handbooks and the experience of the design engineer. The high cost of manufacturing critical structural components can be greatly reduced with the development of mathematically and physically sound computational methodologies for process design and control. Metal forming design, hence, requires an accurate description of the thermo-mechanical deformation mechanisms in order to achieve design objectives as summarized in our earlier report [1].

During this project period, we extended our design analysis to multi-stage deformation processes. In addition, emphasis was given in the design of processes for porous materials. Over the years,

though there have been extensive studies in continuum damage mechanics, both numerical and experimental, very little effort has been invested towards the implementation of damage models into the design of industrial forming processes. Significant portion of our recent activities was concentrated in the development of a continuum sensitivity method to address this issue, related to the design of forming processes for porous materials.

A review of the direct deformation process simulator utilized in our design simulator is given in [2, 3]. We summarize it with Figure 1 which presents the various material configurations through which the workpiece underwent in its processing history as well as the configurations obtained in the current processing step. Let us denote with  $\mathcal{B}_0$  the initial virgin configuration of the body before any processing and with  $\mathcal{B}_f^h$  the final configuration obtained at time  $t_f^h$  as a result of the processing history. This configuration is the same as the initial configuration  $\mathcal{B}_0^c$  (time  $t_0^c \equiv t_f^h$ ) in the current processing stage.

The direct problem can be stated as follows: Compute the time history of the deformation, temperature, material state and plastic deformation of a body deforming as a result of external forces and/or deformation due to contact and friction at the workpiece-die interface. The deformation problem is sub-divided into kinematic, constitutive, contact and thermal sub-problems. An updated Lagrangian FEM formulation is used to solve the direct deformation problem in a generic forming stage in which material occupying an initial configuration (intermediate preform)  $\mathcal{B}_0^c$  ( $t = t_0^c$ ) is deformed to obtain a final configuration (final preform)  $\mathcal{B}_f^c$  ( $t = t_f^c$ ). To compute the material configuration  $\mathcal{B}_{n+1}^c$  for  $n = 0, 1, \dots, (f-1)$ , we proceed in an incremental fashion using the configuration  $\mathcal{B}_n^c$  as the reference configuration.

Let  $\mathbf{Y}$  be a material particle in  $\mathcal{B}_0$  and let  $\mathbf{X}$  be its location in the configuration  $\mathcal{B}_0^c$  as a result of the processing history. The location of the particle  $\mathbf{Y}$  in the current processing stage is defined with the mapping

$$\mathbf{x}^c = \bar{\mathbf{x}}(\mathbf{Y}, t) = \tilde{\mathbf{x}}(\mathbf{X}, t) = \hat{\mathbf{x}}(\mathbf{x}_n^c, t), \quad t \in [t_n^c, t_{n+1}^c] \quad (1)$$

where  $\mathbf{x}_n^c$  is the particle location in the reference configuration  $\mathcal{B}_n^c$ . In an appropriate kinematic framework for large deformation inelastic analysis including thermal effects, the total deformation gradient is decomposed into thermal, plastic and elastic parts as follows:

$$\mathbf{F} \equiv \mathbf{F}_{n+1}^c = \mathbf{F}^e \mathbf{F}^p \mathbf{F}^\theta \quad (2)$$

where  $\mathbf{F}^e$  is the elastic deformation gradient,  $\mathbf{F}^p$ , the plastic deformation gradient and  $\mathbf{F}^\theta$  is the thermal part of the deformation gradient in  $\mathcal{B}_{n+1}^c$ .  $\det \mathbf{F}^p$  is here taken as a measure of internal damage. Applying balance of mass for the matrix material, we obtain the following:

$$\det \mathbf{F}^p = \frac{1 - f_o}{1 - f} \quad (3)$$

where  $f_o$  and  $f$  represent the void volume fractions in the initial and deformed configurations, respectively. The equilibrium equation at  $t = t_{n+1}^c$  can be expressed in the reference configuration  $\mathcal{B}_n^c$  as,

$$\nabla_n \cdot \mathbf{P}_r + \mathbf{f} = \mathbf{0} \quad (4)$$

where  $\nabla_n \cdot$  denotes the divergence in  $\mathcal{B}_n^c$ . The Piola-Kirchhoff I stress  $\mathbf{P}_r$  is expressed per unit area of  $\mathcal{B}_n^c$  and given as follows:

$$\mathbf{P}_r = \det \mathbf{F}_r \mathbf{T} \mathbf{F}_r^{-T} \quad (5)$$

where  $\mathbf{F}_r$  is the relative deformation gradient. The solution of the deformation problem in the current processing stage proceeds incrementally in time starting from the initial configuration  $\mathcal{B}_0^c$ . In order to solve the equilibrium Eq. (4) at time  $t = t_{n+1}^c$ , the constitutive relationship between the Cauchy stress  $\mathbf{T}$  and the relative deformation gradient  $\mathbf{F}_r$  and temperature  $\theta$  should be evaluated. This is discussed in detail in [4, 5]. A hyperelastic-viscoplastic constitutive framework is adopted and the well known implicit definition of the equivalent tensile stress  $\sigma_m$  of the matrix material in terms of the Cauchy stress  $\mathbf{T}$  and the void fraction  $f$  [6, 7] is used. Two scalar state variables are considered here, one representing the isotropic material hardening behavior, denoted by  $s$ , and the other, the volume void fraction,  $f$ . The direct problem is described extensively in [3, 8] and can be easily extended to multistage processes as described in [2].

In [2], we introduce notions of the sensitivity of various physical fields with respect to small changes in the design variables of the forming sequence given in Fig. 1. Let us consider, for completeness, the sensitivities with respect to the design parameters  $\beta^c$  of field variables in the current forming stage with a fixed  $\beta^h$  (design parameters describing the processing history - thus a given  $\mathcal{B}_0^c$  and  $Q_0^c$ ). Such typical process parameters may include the ram speed history, the die surface of the current stage, and others. Fig. 2 presents a schematic that shows the variation of the fields  $\mathbf{x}^c$  and  $Q^c$  induced by a variation in the process parameter  $\beta^c$ . An updated Lagrangian representation is adopted here. Let us consider a generic field  $\Phi$  that can represent  $\mathbf{x}$ ,  $Q$  or any other material or deformation related field. The dependence of the UL field  $\Phi = \hat{\Phi}(\mathbf{x}_n, t)$  on  $\beta^c$  can be expressed as follows:

$$\Phi = \hat{\Phi}(\mathbf{x}_n^c, t; \beta^c) = \hat{\Phi}(\tilde{\mathbf{x}}(\mathbf{X}, t; \beta^c), t; \beta^c) = \tilde{\Phi}(\mathbf{X}, t; \beta^c), \quad t \in [t_n^c, t_{n+1}^c] \quad (6)$$

with the position  $\mathbf{x}_n^c$  referred to the reference configuration  $\mathcal{B}_n^c$ . The parameter sensitivity  $\hat{\Phi} = \hat{\Phi}(\mathbf{x}_n^c, t; \beta^c, \Delta\beta^c)$  is defined as the total Gateaux differential of  $\Phi = \hat{\Phi}(\mathbf{x}_n^c, t; \beta^c)$  in the direction  $\Delta\beta^c$  computed at  $\beta^c$ :

$$\begin{aligned} \hat{\Phi}(\mathbf{x}_n^c, t; \beta^c, \Delta\beta^c) &= \tilde{\tilde{\Phi}}(\mathbf{X}, t; \beta^c, \Delta\beta^c) \\ &= \left. \frac{d}{d\omega} \tilde{\Phi}(\mathbf{X}, t; \beta^c + \omega\Delta\beta^c) \right|_{\omega=0}, \quad t \in [t_n^c, t_{n+1}^c] \end{aligned} \quad (7)$$

Further developments relating to shape and parameter sensitivities to the current design variables as well as the dependence of a field on the history through  $\beta^h$  are described extensively in [2].

## 2 The continuum sensitivity method for a generic forming stage

The developed scheme adopts the ‘design-differentiate then discretize’ approach instead of the ‘discretize then design-differentiate’ approach. The governing equations of the various subproblems in the direct analysis (e.g. the kinematic, constitutive, contact and thermal analysis) are first design-differentiated and then appropriate weak forms, time integration and discretization are introduced. The resulting linear sensitivity subproblems can be coupled together in each forming stage to produce a linear problem for computing the sensitivity of the deformation, plastic deformation gradient and material state. A unified weak form is developed here to compute sensitivities with respect to any design parameter  $\beta$  in any stage. Appropriate features can then be identified which contrast the sensitivity analysis performed with respect to  $\beta^h$  from  $\beta^c$ . In the following analysis,

hot forming processes with ductile damage is discussed. The analysis for non-porous materials is for example presented in [9].

A weak form for the linear sensitivity analysis of a generic forming stage is identified by considering the sensitivity of the equilibrium equations and boundary conditions at the continuum level. The sensitivity deformation problem is developed on the reference preform  $\mathcal{B}_n^c$ . The design sensitivity of the equilibrium equation (4) at  $t = t_{n+1}^c$  results in:

$$\overline{\nabla_n \bullet \mathbf{P}_r} + \mathring{\mathbf{f}} = \mathbf{0}, \quad \forall \quad \mathbf{x}_n^c \in \mathcal{B}_n^c \quad (8)$$

A variational form for the above sensitivity equilibrium equation can be posed as follows [10]:

$$\begin{aligned} \int_{\mathcal{B}_n^c} \mathring{\mathbf{P}}_r \bullet \nabla_n \tilde{\boldsymbol{\eta}} dV_n^c &- \int_{\mathcal{B}_n^c} (\mathbf{P}_r [\nabla_n \bullet \mathbf{L}_n^{cT}]) \bullet \tilde{\boldsymbol{\eta}} dV_n^c - \int_{\mathcal{B}_n^c} (\mathbf{P}_r \mathbf{L}_n^{cT}) \bullet \nabla_n \tilde{\boldsymbol{\eta}} dV_n^c \\ &= \int_{\Gamma_n^c} \left\{ \mathring{\boldsymbol{\lambda}} - [\mathbf{L}_n^c \bullet (\mathbf{N} \otimes \mathbf{N})] \boldsymbol{\lambda} \right\} \bullet \tilde{\boldsymbol{\eta}} dA_n^c \end{aligned} \quad (9)$$

where  $\tilde{\boldsymbol{\eta}}$  is a kinematically admissible sensitivity deformation field expressed over the reference configuration  $\mathcal{B}_n^c$ ,  $\mathbf{N}$  is the unit normal vector to  $\Gamma_n^c$  and the (known) design velocity gradient  $\mathbf{L}_n^c$  at  $t_n^c$  is defined as follows:

$$\mathbf{L}_n^c \equiv \nabla_n \hat{\mathbf{x}}(\mathbf{x}_{n-1}^c, t_n^c; \boldsymbol{\beta}, \Delta \boldsymbol{\beta}) = \mathring{\mathbf{F}}_n^c \mathbf{F}_n^{c-1} \quad (10)$$

The primary unknown of equation (9) is the design differential  $\mathring{\mathbf{x}}_{n+1}^c = \hat{\mathbf{x}}(\mathbf{x}_n^c, t_{n+1}^c; \boldsymbol{\beta}, \Delta \boldsymbol{\beta})$ . In order to obtain the final form of the variational sensitivity problem, the relationships between (a)  $\mathring{\mathbf{F}}_r$  and  $\mathring{\mathbf{x}}_{n+1}^c$  (b)  $\mathring{\mathbf{P}}_r$  and  $[\mathring{\mathbf{x}}_{n+1}^c, \mathring{\theta}_{n+1}^c]$  (c)  $\mathring{\boldsymbol{\lambda}}$  and  $\mathring{\mathbf{x}}_{n+1}^c$  need to be developed. The relationship between  $\mathring{\mathbf{F}}_r$  and  $\mathring{\mathbf{x}}_{n+1}^c$  is purely kinematic and is given as follows:

$$\mathring{\mathbf{F}}_r = \overline{\nabla_n \mathring{\mathbf{x}}_{n+1}^c} = \nabla_n \mathring{\mathbf{x}}_{n+1}^c - \mathbf{F}_r \mathbf{L}_n^c \quad (11)$$

The relationship between  $\mathring{\mathbf{P}}_r$  and  $[\mathring{\mathbf{x}}_{n+1}^c, \mathring{\theta}_{n+1}^c]$  is obtained from the sensitivity constitutive problem and takes the form:

$$\mathring{\mathbf{P}}_r = \mathcal{A} [\mathring{\mathbf{F}}_r] + B \mathring{\theta}_{n+1}^c + C \quad (12)$$

where  $\mathcal{A}$  is a fourth order tensor and  $B$ ,  $C$  are second order tensors. These tensors are constants defined from known direct fields at the current time step and sensitivity fields at the previous time step [3]. The relationship between  $\mathring{\boldsymbol{\lambda}}$  and  $\mathring{\mathbf{x}}_{n+1}^c$  is obtained from the sensitivity contact problem as:

$$\mathring{\boldsymbol{\lambda}} = D [\mathring{\mathbf{x}}_{n+1}^c] + d \quad (13)$$

where  $D$  is a second order tensor and  $d$  a vector. The derivation of these tensors resulting by design-differentiation of a regularized contact problem can be found in [11]. The most notable

feature in this analysis is that the elastic moduli are assumed to be functions of the void fraction and temperature.

The thermal sensitivity problem has been solved in [3]. We note, again, that  $\rho c$  and  $K$  are functions of the void fraction,  $f$ , and temperature,  $\theta$ . The sensitivity thermal evolution equation is obtained by the design differentiation of the energy equation:

$$\rho c \frac{\overset{\circ}{\partial} \overset{\circ}{\theta}}{\overset{\circ}{\partial} t} + (\rho c)_\theta \overset{\circ}{\theta} \frac{\overset{\circ}{\partial} \theta}{\overset{\circ}{\partial} t} + (\rho c)_f \overset{\circ}{f} \frac{\overset{\circ}{\partial} \theta}{\overset{\circ}{\partial} t} = \overset{\circ}{\mathcal{W}}_{mech} - \overset{\circ}{\nabla}_{n+1} \cdot \mathbf{q} \quad (14)$$

where  $\overset{\circ}{\theta}$  and  $\overset{\circ}{f}$  refer to the current configuration,  $\mathcal{B}_{n+1}^c$  and the equation for the heat flux is given by the Fourier's law. After careful design differentiation followed by a temporal integration scheme (consistent with that used in the direct analysis), one obtains the weak-form of the time integrated thermal sensitivity equation posed on the deformed configuration. Let  $\vartheta$  represent an admissible sensitivity temperature field expressed over  $\mathcal{B}_{n+1}^c$ . Therefore a variational form of the thermal sensitivity equation is posed as:

$$\begin{aligned} & \int_{\mathcal{B}_{n+1}^c} \frac{\rho c}{\Delta t} (\overset{\circ}{\theta}_{n+1} - \overset{\circ}{\theta}_n) \vartheta dV + \int_{\mathcal{B}_{n+1}^c} \frac{(\rho c)_\theta}{\Delta t} (\theta_{n+1} - \theta_n) \overset{\circ}{\theta}_{n+1} \vartheta dV \\ & + \int_{\mathcal{B}_{n+1}^c} K \nabla_{n+1} \overset{\circ}{\theta}_{n+1} \cdot \nabla_{n+1} \vartheta dV + \int_{\mathcal{B}_{n+1}^c} K_\theta \overset{\circ}{\theta}_{n+1} \nabla_{n+1} \theta_{n+1} \cdot \nabla_{n+1} \vartheta dV + \\ & \int_{\partial \mathcal{B}_{n+1}^c} [\overset{\circ}{\mathbf{q}}_{n+1}] \cdot \mathbf{n} dA = \int_{\mathcal{B}_{n+1}^c} \overset{\circ}{\mathcal{W}}_{mech,n+1} \vartheta dV + \int_{\mathcal{B}_{n+1}^c} \nabla_{n+1} \mathbf{q}_{n+1} \cdot \mathbf{L}_{n+1} \vartheta dV \\ & - \int_{\mathcal{B}_{n+1}^c} \mathbf{L}_{n+1}^T \mathbf{q}_{n+1} \cdot \nabla_{n+1} \vartheta dV - \int_{\mathcal{B}_{n+1}^c} K_f \overset{\circ}{f}_{n+1} \nabla_{n+1} \theta_{n+1} \cdot \nabla_{n+1} \vartheta dV \\ & - \int_{\mathcal{B}_{n+1}^c} \frac{(\rho c)_f}{\Delta t} (\theta_{n+1} - \theta_n) \overset{\circ}{f}_{n+1} \vartheta dV \end{aligned} \quad (15)$$

In summary, the thermo-mechanical sensitivity analysis highlighted here is novel, mathematically rigorous and capable of providing very accurate sensitivities. It has been incorporated within the multi-stage forming design simulator and some design examples for one- and two-stage metal forming processes will be presented later in this report.

## 2.1 Evaluation of gradients from the computed sensitivities

Let us concentrate on the evaluation of the gradients using the sensitivity fields (directional derivatives) developed earlier. Let us assume that the sensitivities of a continuum Lagrangian field  $\Omega(\mathbf{x}^c, t; \boldsymbol{\beta}) = \Omega(\mathbf{x}^c, t; \beta_1, \beta_2, \dots, \beta_n)$  have been evaluated through the CSM problem where  $\boldsymbol{\beta}$  is the vector denoting the design parameters. We can then write the following:

$$\overset{\circ}{\Omega}(\mathbf{x}^c, t, \beta_1, \beta_2, \dots, \beta_n, \Delta\beta_1, \Delta\beta_2, \dots, \Delta\beta_n) \equiv \sum_{i=1}^n \frac{\partial \Omega}{\partial \beta_i} \Delta\beta_i \quad (16)$$

Thus one can compute the gradient  $\nabla\Omega = \left( \frac{\partial\Omega}{\partial\beta_1}, \frac{\partial\Omega}{\partial\beta_2}, \dots, \frac{\partial\Omega}{\partial\beta_n} \right)$  using the continuum based sensitivity fields as follows:

$$\frac{\partial\Omega}{\partial\beta_i} = \frac{\overset{\circ}{\Omega}(\mathbf{x}^c, t, \beta_1, \beta_2, \dots, \beta_n, 0, \dots, 0, \Delta\beta_i, 0, \dots, 0)}{\Delta\beta_i} \quad (17)$$

The evaluation of the gradient thus requires  $n$  sensitivity problems where the  $i^{\text{th}}$  sensitivity problem is driven by  $\Delta\beta_i$  with  $\Delta\beta_j = 0$  for  $j \neq i$ . Thus we need to solve a total of  $n + 1$  problems, one nonlinear direct problem and  $n$  linear sensitivity problems.

### 3 Design examples

#### 3.1 Preform design for closed-die forging - fill die cavity and achieve uniform state distribution (Example 1)

In this example, the design of the preforming die shape (in a two-stage forming process) is addressed so as to achieve a distribution of the state variable in the final product obtained after the finishing stage that is as uniform as possible. The initial workpiece in the preforming stage is fixed and is a right circular cylinder of radius  $r = 0.85$  mm and height 2.4 mm. The height reduction at  $r = 0$ , in the first stage is 1.2 mm and in the second stage is 0.6 mm. The friction coefficient for both stages is taken as 0.2, workpiece is assumed to be made of 1100-Al at a temperature of 673 K.

The preforming die (contact) surface  $R_{\beta}(\alpha)$  is represented using a degree 6 Bézier curve with 5 independent variables.

$$r(\alpha) = 1.5\alpha, \quad z_{\beta}(\alpha) = \sum_{i=1}^7 \beta_i \phi_i(\alpha) \quad (18)$$

The die height at  $r = 0$  was specified and the die shape is assumed to have a zero slope at  $r = 0$ . The following basis functions

$$\begin{aligned} \phi_1 &= (1 - \alpha)^6, \quad \phi_2 = 6(1 - \alpha)^5\alpha, \quad \phi_3 = 15(1 - \alpha)^4\alpha^2, \quad \phi_4 = 20(1 - \alpha)^3\alpha^3, \\ \phi_5 &= 15(1 - \alpha)^2\alpha^4, \quad \phi_6 = 6(1 - \alpha)\alpha^5, \quad \phi_7 = \alpha^7 \end{aligned} \quad (19)$$

are assumed. We herein choose  $\beta_6$  and  $\beta_7 = 1.3$  mm, in order to fix the height and slope at  $r = 0$ . The finishing die surface is defined as follows ( $\eta \in [0, 1]$ ):

$$\text{shape}(\eta) = \begin{cases} r(\eta) = 1.3 * (1 - \eta) \\ z(\eta) = \begin{cases} 1.3 + 10.58 * \eta - 62.43 * \eta^2 + 122.8 * \eta^3 & \eta \in [0.00, 0.17] \\ 1.9 & \eta \in [0.17, 0.37] \\ 3.1 - 11.9 * \eta + 44.38 * \eta^2 - 76.8 * \eta^3 \\ + 60.5 * \eta^{4.0} - 17.7 * \eta^5 & \eta \in [0.37, 1.0] \end{cases} \end{cases}$$

The finite dimensional optimization problem is now posed as follows:

$$\min_{\beta} f(\beta) = \frac{\omega_1}{M} \sum_{i=1}^M (s_i(\beta) - \bar{s}(\beta))^2 \quad (20)$$

where  $M$  is here defined as the total number of sampling points in the volume and  $\bar{s}$  is the resulting average value of the state variable. The factor  $\omega_1 = 6.25 \times 10^{-4}$  MPa<sup>-2</sup> is introduced to non-dimensionalize the cost functional and is obtained from the solution of a reference single-stage

problem for which the mean state variable obtained is 40 MPa. The steepest descent method with optimal step size selection is used in this example. The initial preforming die shape is chosen to be flat. The products obtained using the initial preforming die shape and the final (i.e. optimal) solution are shown in Fig. 3. The average value and the deviation of the material state in the initial product were computed as 50.2 MPa and 3.73 MPa, respectively. The corresponding values in the optimum product are 52.3 MPa and 1.88 MPa, respectively. Fig. 4 gives the variation of the objective function with iterations. The change in the preforming die shape with iterations is presented in Fig. 5.

### 3.2 Preform design for closed-die forging - Thermo-mechanical process with ductile damage - Varying elastic properties (Example 2)

The objective here is to design the volume and the free surface of a cylindrical preform of height 2.0 mm, that when compressed with a closed forging die, fills the die completely with minimum or no flash, after a specified stroke of 0.65 mm. The workpiece material is taken to be 2024 – T351 Al at an initial temperature of 300K. The material parameters correspond to those described in [3]. The friction coefficient between the die and workpiece is taken as 0.1. The desired final radius around the flash is taken as 1.2 mm and the desired height at  $r = 0$  as 0.7 mm. Since the problem is complex in nature, it involves multiple remeshing operations. We incorporate remeshing's depending on the size of the elements, amount of distortion and spacing. The free surface of the billet  $R_{\boldsymbol{\beta}}(\alpha)$  is represented with a degree 6 Bézier curve (with 7 Bernstein basis functions). Using the restriction  $R'_{\boldsymbol{\beta}}(0) = 0$ , the representation of  $R_{\boldsymbol{\beta}}$  can be defined with 6 independent design variables  $\beta_i$ ,  $i = [1 \dots 6]$  as follows:

$$\begin{aligned} R_{\boldsymbol{\beta}}(\alpha) &= \sum_{i=1}^6 \beta_i \phi_i(\alpha) \\ Z &= 1.5\alpha \quad 0 \leq \alpha \leq 1 \end{aligned} \quad (21)$$

where  $Z$  represents the axial coordinate,  $\alpha$  represents the normalized radial coordinate and the basis functions are given as

$$\begin{aligned} \phi_1 &= (1.0 - \alpha)^5 (1.0 + 5.0\alpha) \\ \phi_2 &= 15.0 (1.0 - \alpha)^4 \alpha^2 \\ \phi_3 &= 20.0 (1.0 - \alpha)^3 \alpha^3 \\ \phi_4 &= 15.0 (1.0 - \alpha)^2 \alpha^4 \\ \phi_5 &= 6.0 (1.0 - \alpha) \alpha^5 \\ \phi_6 &= \alpha^6 \end{aligned} \quad (22)$$

The finite dimensional optimization problem is then posed as follows:

$$\min_{\boldsymbol{\beta}} \mathcal{F}(\boldsymbol{\beta}) = \frac{1}{N} \sum_{i=1}^N (x_1^i(\boldsymbol{\beta}) - x_1^{desired})^2 + (x_2^i(\boldsymbol{\beta}) - x_2^{desired})^2 \quad (23)$$

where  $\boldsymbol{\beta} = \{\beta_1, \dots, \beta_6\}$ ,  $(x_1^{desired}, x_2^{desired})$  defines the desired boundary and  $N$  refers to the number of nodes on the free surface and regions in contact of the final product. Fig. 6 shows the quarter

geometry of the initial guess preform (a right circular cylinder of radius 0.8 mm and height of 2 mm) and final product shape obtained using this guessed preform. Fig. 7 shows the optimal preform and the final product shape achieved using this optimal preform. The variation of the objective function with the iteration index is shown in Fig. 8. The volume of the optimum preform was evaluated as  $2.5446 \text{ mm}^3$  whereas the volume enclosed by the die cavity is  $2.5353 \text{ mm}^3$ . Also shown in Figs. 6 and 7 is the variation of the shear modulus in the forged product. More about varying elastic properties is described in [3].

## 4 Summary and research activities in progress

Our completed activities in deformation process design in the first year of this DMI grant included the development of a general purpose continuum sensitivity analysis that can be used to design industrial multi-stage deformation processes. We have also completed the development of CSM techniques towards the design of processes for materials with ductile damage.

During thermo-mechanical deformation processes, materials experience significant microstructural changes which strongly affect their mechanical properties. The changes in microstructure, under thermo-mechanical processing, involve strain hardening, recrystallization and grain growth. We have recently enhanced our direct simulator to implicitly incorporate the various microstructural length scales that describe the evolving microstructure. A continuum sensitivity analysis has been implemented to allow us to design deformation processes for control of microstructure in the presence of recrystallization and grain growth mechanisms. This work is near completion [12].

Several additional research activities have been initiated motivated by the work presented here. They include the following:

- Microstructure-sensitive multi-length scale computational design of deformation processes. In particular, we are developing a multi-length scale direct and sensitivity analysis for polycrystals with emphasis on control of microstructural properties (e.g. properties controlled by texture) through macroscopic design parameters.
- We have finally initiated an effort to allow to design processes while accounting for uncertainty (robust design). In particular, given uncertainty in the process conditions, can we design a process with desired robustness limits in our design objectives? In our work in [13], we introduce a novel spectral method for the analysis and design of thermal systems where the boundary heat flux is designed that leads to desired temperature statistics at points within the domain. Both desired temperatures and computed heat flux are considered as stochastic processes. The material properties are also considered to be given stochastic processes. We plan to extend this work to continuum mechanics problems before an eventual application to stochastic robust design of deformation processes.

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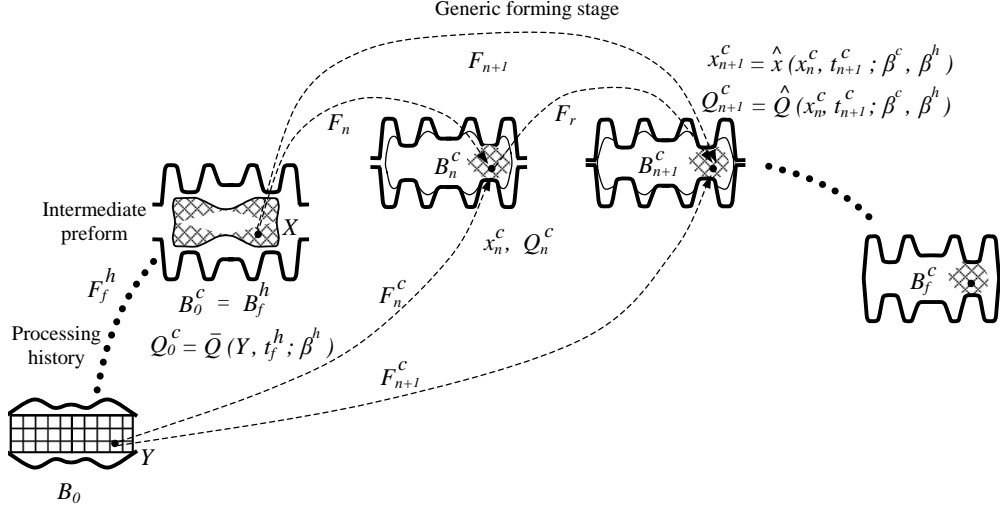


Figure 1: Schematic of an updated Lagrangian framework for a multi-stage deformation process. The superscript  $c$  refers to the current stage, whereas the superscript  $h$  refers to the deformation processing history. The configuration  $B_n^c$  is used as the reference configuration in computing the configuration  $B_{n+1}^c$  and the set  $Q_{n+1}^c$ , where  $Q = \{F^p, s, f, \theta\}$ .

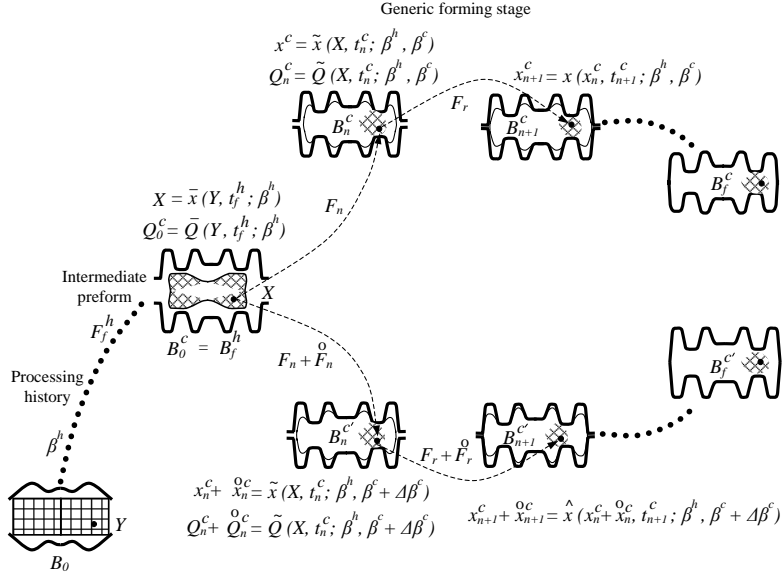


Figure 2: Schematic representation of the design sensitivity of the deformation in the current forming stage due to variations in the (non-shape) design parameters  $\beta^c$  of the current forming stage.

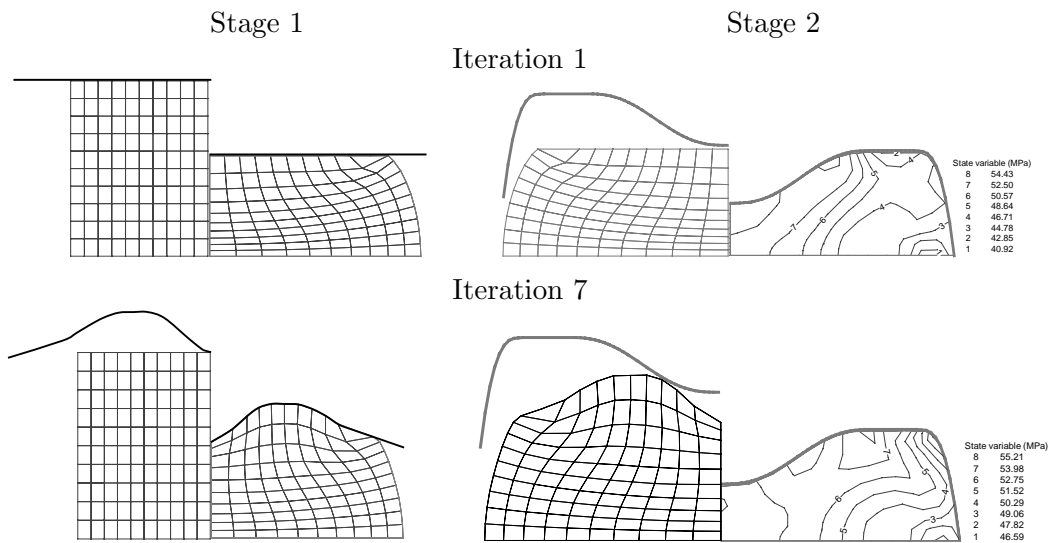


Figure 3: Comparison of the final product obtained using the initial and optimal solution for the thermo-mechanical preform forging design problem (Example 1).

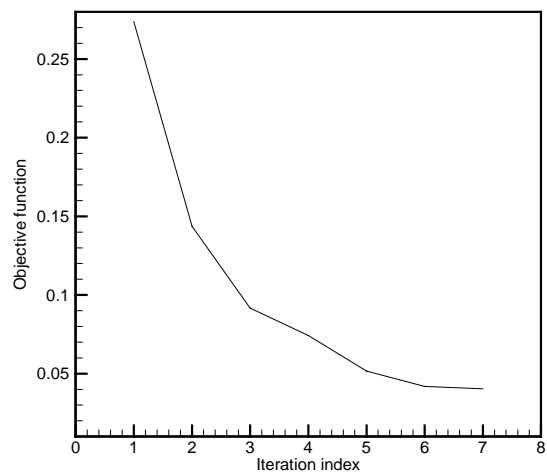


Figure 4: Variation of the objective function with optimization iterations (Example 1).

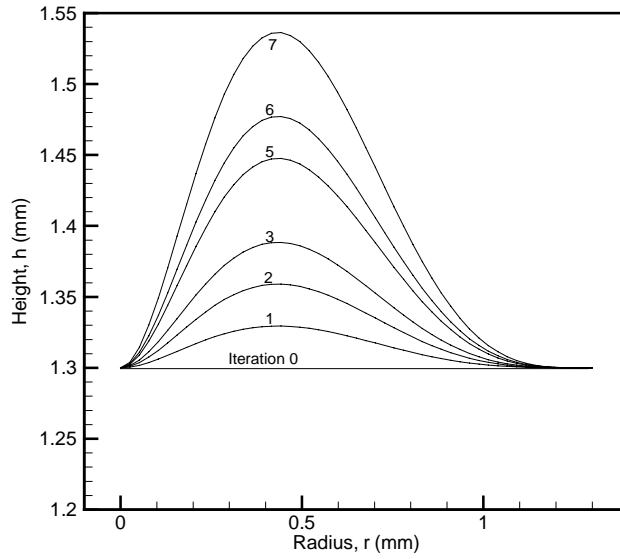


Figure 5: Variation of the preforming die shape with optimization iterations (Example 1).

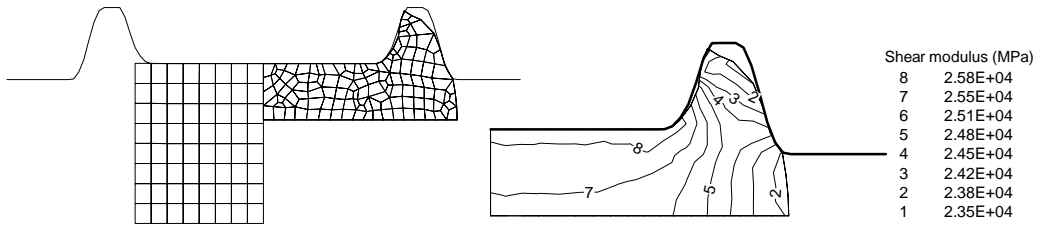


Figure 6: Initial guess preform shape, the final product using this guess and the distribution of shear modulus in the final product for a thermomechanical closed die forging process (Example 2).

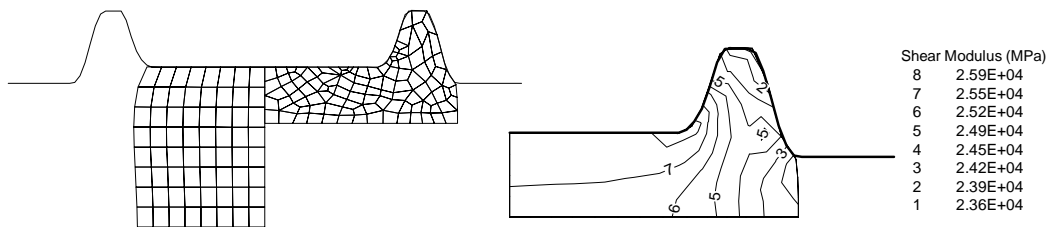


Figure 7: Optimal preform shape, the final product using this preform and the distribution of the shear modulus for a thermomechanical closed die forging process (Example 2).

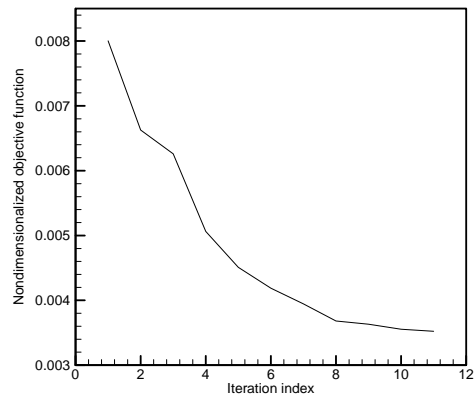


Figure 8: Variation of the nondimensionalized objective function with iteration index for a thermomechanical closed die forming process (Example 2).