

Spatial statistics models for stochastic inverse problems in heat conduction

Jingbo Wang^a and Nicholas Zabaras^b

Materials Process Design and Control Laboratory
Sibley School of Mechanical and Aerospace Engineering
Cornell University
188 Frank H.T. Rhodes Hall
Ithaca, NY 14853-3801

^ajjw332@cornell.edu, ^bzabaras@cornell.edu

Uncertainties such as measurement noise are unavoidable in inverse problems and often lead to unstable solutions due to the ill-posed nature of such problems. However, there is a rich statistical information contained in the actual data that is often not used. In this paper, we explore the solution of stochastic inverse heat conduction problems where the unknowns (e.g. boundary heat flux or distributed heat source) are computed in probabilistic spaces. A Bayesian statistical inference approach is presented here for the solution of such inverse problems. Spatial statistics models, in particular Markov Random Fields (MRF), are used to model the prior correlations of the unknown quantities at different sites and time points. The joint posterior probability density function (PPDF) of these unknown quantities is derived and then exploited using Markov Chain Monte Carlo (MCMC) algorithm, in particular Gibbs sampler. Both Maximum A Posteriori (MAP) and posterior mean estimates and associated statistics are computed using MCMC samples, and compared with the Maximum Likelihood Estimate (MLE). An augmented Bayesian formulation is also presented to estimate the statistics of measurement noise simultaneously with the unknown quantities. The intrinsic relations between Tikhonov regularization, spatial statistics models and Expectation-Maximization algorithm (EM) are revealed. Typical examples of reconstructing boundary heat flux and heat sources from thermocouple temperature readings are presented to demonstrate features of the proposed methodologies.

References

- [1] O. Alifanov, "Inverse Heat Transfer Problems", Springer-Verlag, Berlin Heidelberg, 1994.
- [2] J. Beck, B. Blackwell, and C. St-Clair Jr., "Inverse Heat Conduction: Ill-posed Problems", Wiley-Inter Science, New York, 1985.
- [3] C. Andrieu, N. Freitas, A. Doucet, and M. Gordan, "An introduction to MCMC for machine learning", *Machine Learning*, v. 50, p. 5-43, 2003.
- [4] J. Besag, P. Green, D. Higdon, and K. Mengersen, "Bayesian computation and stochastic systems", *Statistical Science*, v. 10, p. 3-41, 1995.
- [5] J. Wang, and N. Zabaras, "A Bayesian inference approach to the inverse heat conduction problem", *International Journal of Heat and Mass Transfer*, in press.