A Bayesian inference approach to inverse problems using an adaptive sparse grid collocation method[1]

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Abstract

A new approach to modeling inverse problems using Bayesian inference is presented. By introducing the concept of the stochastic prior state space to the Bayesian formulation, we reformulate the deterministic forward problem as a stochastic one. The adaptive hierarchical sparse grid collocation (ASGC) method is used for constructing an interpolant to the solution of the forward model in this prior space which is large enough to capture all the variability in the posterior distribution of the unknown parameters. This solution can be considered as a function of the random unknowns and serves as a stochastic surrogate model for the likelihood calculation. Hierarchical Bayesian formulation is used to derive the posterior probability density function (PPDF). The spatial model is represented as a convolution of a smooth kernel and a Markov random field. The state space of the PPDF is explored using Markov chain Monte Carlo algorithms to obtain statistics of the unknowns. The likelihood calculation is performed by directly sampling the approximate stochastic solution obtained through the ASGC method. The technique is assessed on two nonlinear inverse problems: source inversion and permeability estimation in flow through porous media.

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