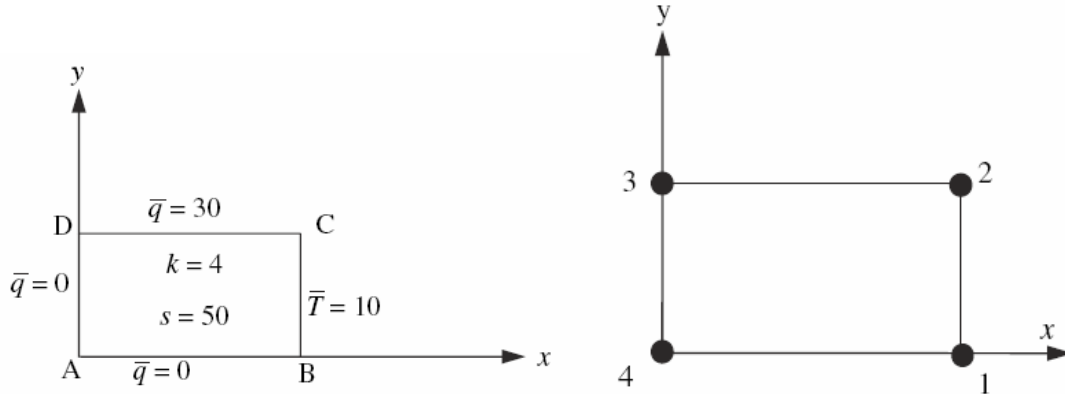


Due Monday, October 19th, 12:00 midnight

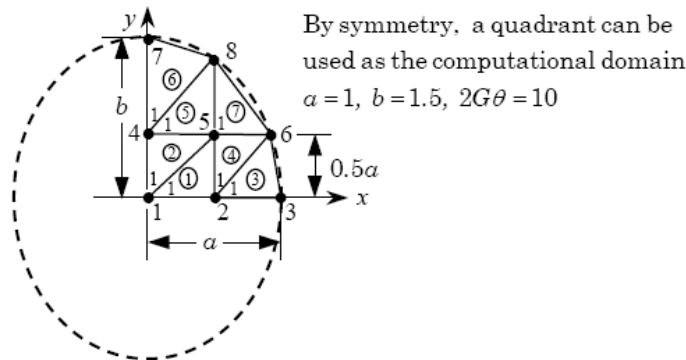
Problem 1 – 2D heat conduction in a rectangular domain (hand calculation)



Consider a problem on a rectangular (2 m × 1 m) domain as shown in the left figure. The conductivity is $k = 4 \text{ W } ^\circ\text{C}^{-1}$. $\bar{T} = 10 \text{ } ^\circ\text{C}$ is prescribed along the edge CB. Edges AB and AD are insulated, i.e. $\bar{q} = 0 \text{ W m}^{-1}$; along the edge DC, the boundary flux is $\bar{q} = 30 \text{ W m}^{-1}$. A constant heat source is given: $s = 50 \text{ W m}^{-2}$

Find the nodal temperature and nodal fluxes; evaluate the element matrices by Gauss quadrature. Use a single rectangular finite element with node numbering shown in the right figure so that the local and global node numbering coincide.

Problem 2 – Torsion of a cylindrical member (MatLab)



The Prandtl theory of torsion of a cylindrical member leads to

$$-\nabla^2 u = 2G\theta \quad \text{in } \Omega; \quad u = 0 \quad \text{on } \Gamma$$

where Ω is the cross section of the cylindrical member being twisted, Γ is the boundary of Ω , G is the shear modulus of the material of the member, θ is the angle of twist, and u is the stress function. Solve the equation for the case in which Ω is an elliptical section member using the mesh of linear triangular or quadrilateral elements implemented in the

MATLAB libraries provided. Compare the finite element solution with the exact solution (valid for elliptical sections with axes a and b):

$$u = \frac{G\theta a^2 b^2}{a^2 + b^2} \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} \right)$$

Use $a = 1, b = 1.5$, and $f_0 = 2G\theta = 10$.

Note the mesh shown in the figure is just for illustration purposes. You may first create the grid in Ansys and use the provided function loadFromGridFile.m to read the grid. You may want to check the convergence of the mesh. Once the stress function is determined, the stresses can be computed from:

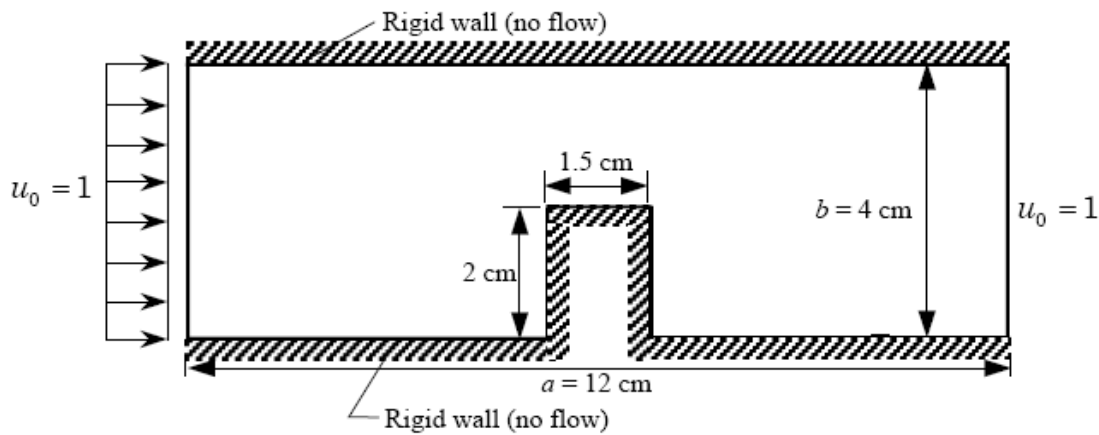
$$\sigma_{xz} = G\theta \frac{\partial u}{\partial y}, \quad \sigma_{yz} = -G\theta \frac{\partial u}{\partial x}$$

Plot the stress vector $\mathbf{v} = \bar{\sigma}_{xz} \hat{\mathbf{i}} + \bar{\sigma}_{yz} \hat{\mathbf{j}}$, where $\bar{\sigma}_{xz} = \sigma_{xz} / G\theta, \bar{\sigma}_{yz} = \sigma_{yz} / G\theta$.

Problem 3 – Torsion of a cylindrical member (Ansys)

Compute the solution of problem 2 using Ansys and compare your answers with those obtained with the MatLab formulation. Provide complete list of your implementation commands.

Problem 4 – Confined flow around a rectangular wall (MatLab)



The irrotational flow of an ideal fluid (i.e. a non-viscous fluid) about a vertical wall (see the above figure) is to be analyzed using the finite element method implemented in the MatLab libraries provided. The equation governing the flow is given as

$$-\nabla^2 u = 0 \text{ in } \Omega$$

where u can be any of the two functions: u is the stream function or (2) u is the velocity potential. If u is the stream function ψ , the velocity components of the flow field are given as

$$u_1 = \frac{\partial \psi}{\partial y}, \quad u_2 = -\frac{\partial \psi}{\partial x}$$

If u is the velocity potential ϕ , the velocity components can be computed from

$$u_1 = \frac{\partial \phi}{\partial x}, u_2 = \frac{\partial \phi}{\partial y}$$

In either case, the velocity field is not affected by a constant term in the solution u . By symmetry, only left half of the domain needs to be modeled. Choose the left bottom point as the origin. First create the grid in Ansys and use the function loadFromGridFile.m to read the grid to solve this problem using the two formulations. Compute the solution in each element and compare your results from the two formulations. Give the velocity vector plot and contour plot of the potentials.

You need to be careful with the type of boundary conditions that you use. Here are the basic justifications for the boundary conditions shown in the figures.

- a) **Stream function formulation:** Remember that flow perpendicular to a streamline is zero. Therefore the fixed walls are streamlines! Due to the symmetry about the vertical centerlines, only left half of the domain needs to be used in the analysis. Since the velocity field depends on the relative difference of two streamlines, we take the value of the streamline that coincides with bottom left point (say A) to be zero ($\psi_A = 0$) and then determine the value of ψ on the upper wall from the condition where u_0 is the velocity of the fluid parallel to the streamline. Since u_0 is given at the inlet, we determine the value of the streamline at the upper left point (say B) by integrating the above equation as follows:

$$\int_{\psi_A}^{\psi_B} d\psi = \psi_B - \psi_A = \int_{y_A}^{y_B} u_0 dy = u_0 (y_B - y_A) \text{ or}$$

$$\psi_B = \psi_A + u_0 (y_B - y_A)$$

Since the velocity at the inlet is constant, ψ varies linearly at the inlet from 0 at point A to the value $u_0 y_B$ at point B. These boundary conditions are summarized as: On the left boundary, ψ is specified to be $\psi = yu_0$ on the left boundary, $\psi = 4u_0$ on the top wall, and $\psi = 0$ on the bottom wall as well as on the rectangular boundary.

- b) **Velocity potential formulation:** The fact that $u_2 = \frac{\partial \phi}{\partial y} = 0$ on the upper wall and the bottom line of domain gives the boundary conditions there. Along AB the velocity $u_1 = \frac{\partial \phi}{\partial x} = -\frac{\partial \phi}{\partial n}$ is specified to be u_0 . Along vertical line of symmetry we

must know either ϕ or $\frac{\partial\phi}{\partial n} = \frac{\partial\phi}{\partial x}$. Since $u_1 = \frac{\partial\phi}{\partial x}$ is not known there, we assume that ϕ is known and set ϕ equal to a constant (e.g. zero) to complete the boundary conditions. Setting ϕ equal to zero on the vertical line of symmetry eliminates the rigid body motion in the solution! Since the velocity field depends on the gradient of ϕ , any constant for ϕ on this line will give the same results for $\nabla\phi$. These boundary conditions are summarized as: on the right boundary, ϕ is specified to be $\phi = 0$. $\frac{\partial\phi}{\partial n}$ is specified to be zero at the top and bottom walls as well as on the rectangular boundary. On the left boundary $\frac{\partial\phi}{\partial n} = -u_0$.