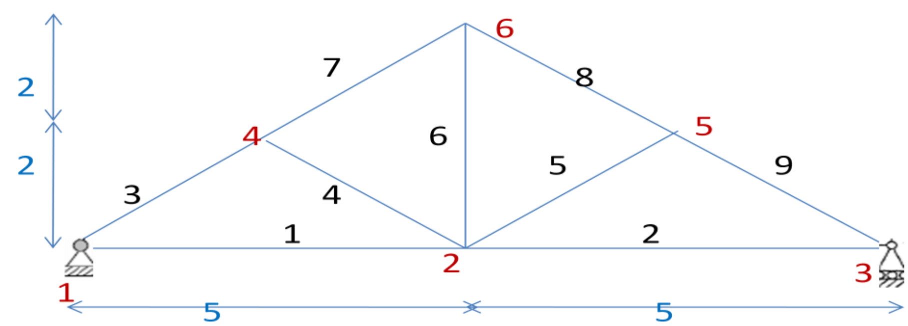
Due Monday, November 16th, 12:00 midnight

This homework is considering the finite element analysis of transient and dynamic FEM analysis. You are asked to include transient and/dynamic effects to MatLab software considered in earlier homework.

Problem 1 – Eigenmodes of a truss structure (MatLab)

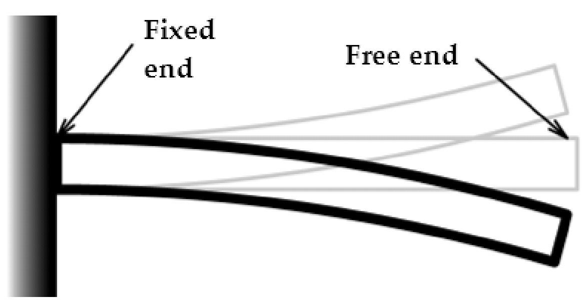
Use the eigensolver from MatLab to solve an eigenvalue problem for the truss structure shown below. For simplicity, normalize the stiffness and mass matrices by taking the values E=1, A=1 and ρ=1 for the material constants and cross-sectional areas of the bars. Physical dimensions of the truss elements are as shown in the figure. In this problem 9 eigenvalues exist. Report only the first 4 eigenvalues and free vibration modes. Plot these eigenmodes. The eigenvalues are sorted from smallest to largest.

Hint: You will need to program the truss element mass matrix, the assembled mass matrix and also set and solve the appropriate eigenvalue problem as discussed in lecture.



Problem 2 – Free vibration problem of a cantilever beam (MatLab)

Consider the free vibration of the cantilever beam shown in the figure below with EI=1, ρA=1 and L=1, where L is the length of the cantilever. In this case the analytically obtained first three eigenvalues are given as $\lambda_1 = (1.875)^4, \lambda_2 = (4.694)^4, \lambda_3 = (7.855)^4$. We need to verify these results using FEM analysis. Modify the beam software of HW3 to compute and assemble the mass matrix for beam elements and solve the appropriate eigenvalue problem deduced from the free vibration of the cantilever. Plot the corresponding eigenfunctions. The eigenvalues are sorted from smallest to largest.



Problem 3 – Dynamic response of a uniform simply supported beam (MatLab)

Consider the dynamic response of a uniform simply supported beam subjected to a central step function loading. Suppose that ρ and EI are constant and that the magnitude of the load is P_0. The deflection of the beam has been obtained analytically as

$$u_y(t) = \sum_{n=1}^{\infty} \frac{2P_0 a_n}{\rho A L \omega_n^2}\left[1-\cos\left(\omega_n t\right)\right]\sin\frac{n\pi x}{L}$$

where

$$\alpha_n = 1 \;\; if \;\; n = 1,5,9,\ldots$$
$$\alpha_n = -1 \;\; if \;\; n = 3,7,11,\ldots$$
$$\alpha_n = 0 \;\; if \;\; n \; even$$

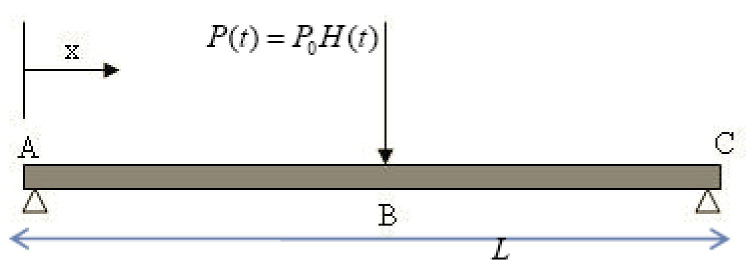
and

$$\omega_n = \left(\frac{n\pi}{2}\right)^2, n = 1,2,3,..$$

If the deflection is computed at x=L/2, we have:

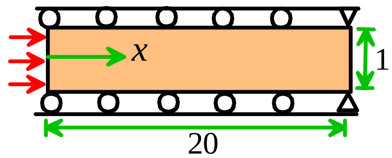
$$u_y(\frac{L}{2},t) = \frac{2P_0L^3}{\pi^4 EI}\left[\left(1-\cos\omega_1 t\right) - \frac{\left(1-\cos\omega_3 t\right)}{81} + \frac{\left(1-\cos\omega_5 t\right)}{625} + \ldots\right]$$

For the beam with $EI=1$, $\rho A=1$, $L=2$ and $P_0=20$, solve the dynamic problem using the Newmark method with $\beta_1=\beta_2=1/2$ and using four equal size beam elements. Show that the results compare well with the analytical solution even for large time step, e.g. $\Delta t=0.05$. Provide a plot of the computed and analytical deflection (using only first three terms as shown above) of the beam at the center (x=L/2) versus time. Note that deflection is the absolution value of the vertical displacement.



Problem 4 – 1D Wave Propagation in Solids (MatLab)

We consider 1D wave propagation in a bar with material properties $E = 10, \nu = 0.25, \rho = 1$ in arbitrary units. Mesh the bar with 4 node quadrilateral plane strain elements. Assume that the left hand end of the bar is subjected to a constant traction with magnitude *5* at time *t=0* and held constant for *t>0*.

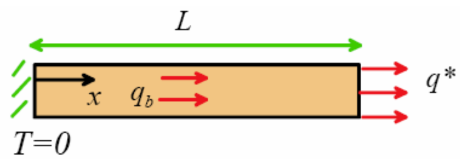


1. Calculate the maximum time step for a stable explicit computation.
2. Run an explicit dynamics computation (Newmark parameters $\beta_1 = 1/2, \beta_2 = 0$) with a time step equal to half the theoretical stable limit. Plot the time variation of the computed displacement, velocity and acceleration at point (*x=0, y=0*). Use ~200 time steps and compare the FEM solution with the exact solution if you can.
3. Repeat step 2 with a time step equal to twice the theoretical stable limit.
4. Repeat step 2 above with Newmark parameters $\beta_1 = \beta_2 = 1/2$.
5. Repeat step 2 with Newmark parameters $\beta_1 = \beta_2 = 1$ and use a time step equal to ten times the theoretical stable limit.
6. Comment on these results.

Hint: For this problem, you will need to modify the 2DStressAnalysis MatLab code.

Problem 5 – One-dimensional transient heat conduction (MatLab)

The figure below shows a bar of length 5, cross-section area = 1, with thermal conductivity k=50 and heat capacity *c=100*. At time *t=0* the bar is at uniform temperature, $T = 0$. The sides of the bar are insulated to prevent heat loss; the left hand end is held at fixed temperature $T = 0$ while a flux of heat $q^* = 10$ is applied into to the right hand end of the bar. The heat generated inside the bar at rate q_b per unit length is here taken as zero.



The temperature distribution in the bar is governed by the 1-D heat equation

$$c\frac{dT}{dt} = k\frac{d^2T}{dx^2} + q_b$$

and the boundary condition at $x = L$ is $k\frac{dT}{dx} = q^*$.

As discussed in class, the weak form of the problem after finite element discretization leads to the following semidiscrete finite element model:

$$\mathbf{M\dot{T}} + \mathbf{KT} = \mathbf{f}$$

Modify the MatLab software 1DBVP to solve the transient problem using θ-family of approximation schemes and consider the following cases:

1. To check the code, set the number of elements=15, $\Delta t = 0.1$, number of time steps=1000 and use lumped mass matrix M and explicit time stepping $\theta = 0$. You should find that this case run quickly, and that the temperature in the bar gradually rises until it reaches the expected linear distribution.
2. Consider the fully explicit case $\theta = 0$ with $\Delta t = 0.2$. To show that the integration scheme is conditionally stable run your code for 100 steps.
3. Show that the critical stable time step size reduces with element size with fully explicit case $\theta = 0$. Try running with $\Delta t = 0.1$ with 20 elements in the bar.
4. Run your code with a fully implicit time stepping scheme with $\theta = 1$, 15 elements and $\Delta t = 0.2$. Use a full consistent mass matrix for this calculation. Show that you can take extremely large time steps with $\theta = 1$ without instability (e.g. try Δt =10 for 10 or 100 steps). You will find out that the algorithm is unconditionally stable for $\theta = 1$.

For each case, plot the final temperature distribution of the bar and the time history of the temperature at $x = L$.