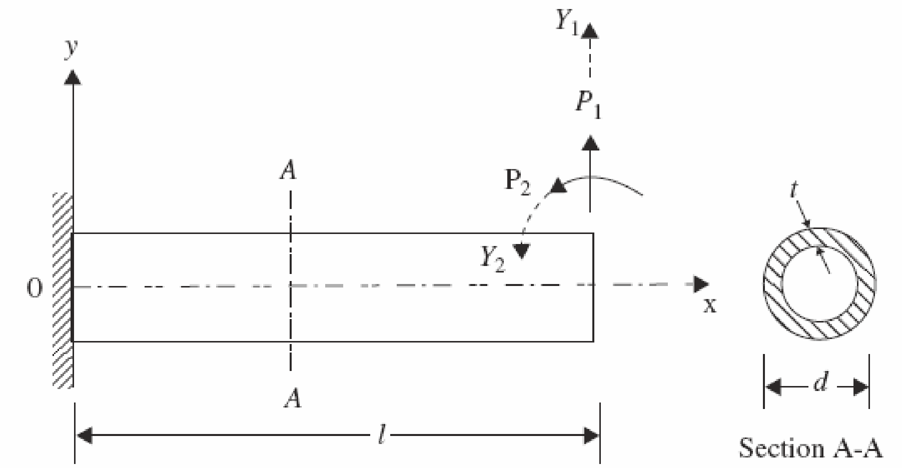
Due Tuesday, November 9th, 12:00 midnight

Problem 1 – Sensitivity of the solution (Hand calculation)

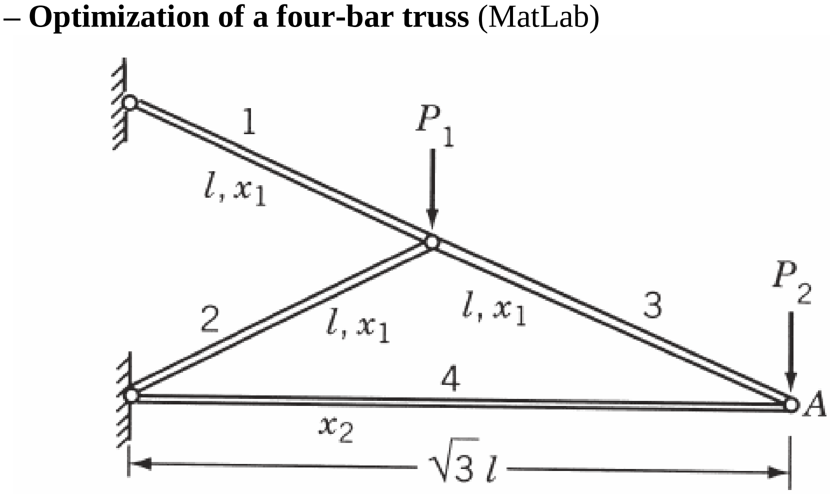


A cantilever beam with a hollow circular section with outside diameter d and wall thickness t is modeled with one beam finite element. The resulting static equilibrium equations can be expressed as

$$\frac{2EI}{l^3}\begin{bmatrix} 6 & -3l \\ -3l & 2l^2 \end{bmatrix}\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

where I is the area moment of inertia of the cross section, E is Young's modulus, and l the length. Determine the displacements, Y_i, and the sensitivities of the deflections, $\partial Y_i / \partial d$ and $\partial Y_i / \partial t$ $(i = 1, 2)$, for the following data: $E = 30 \times 10^6$ psi, $l = 20$ in, $d = 2$ in, $t = 0.1$ in, $P_1 = 100$ lb and $P_2 = 0$.

Problem 2 – Optimization of a four-bar truss (MatLab)



Consider the four-bar truss shown in the figure, in which members 1, 2, and 3 have the same cross-sectional area x_1 and the same length l, while member 4 has an area of cross section x_2 and length $\sqrt{3}l$. The truss is made of a lightweight material for which Young's modulus and the weight density are given by 30×10^6 psi and $0.03333\,\text{lb/in}^3$, respectively. The truss is subjected to the loads $P_1 = 10,000$ lb and $P_2 = 20,000$ lb. The weight of the truss per unit value of l can be expressed as

$$f = 3x_1(1)(0.03333) + x_2\sqrt{3}(0.03333) = 0.1x_1 + 0.05773x_2$$

The weight of the truss is to be minimized with constraints on the vertical deflection δ_A of the joint A and the stresses in members 1 and 4. The maximum permissible deflection of joint A is 0.001 in and the permissible stresses in members are $\sigma_{\max} = 8333.333$ psi (tension) and $\sigma_{\min} = -4948.5714$ psi (compression). Solve this optimization problem in Matlab, i.e. minimize f with $l = 1$ and the constraints $|\delta_A| \leq 0.001$ in, $\sigma_1 \leq 8333.333$ psi and $\sigma_2 \geq -4948.5714$ psi.

In order to further verify your results, resolve the optimization problem using the following analytical expressions:

The vertical deflection of joint A can be expressed as

$$\delta_A = \frac{0.006}{x_1} + \frac{0.003464}{x_2}$$

and the stresses in members 1 and 4 can be written as

$$\sigma_1 = \frac{5(10,000)}{x_1} = \frac{50,000}{x_1}, \quad \sigma_4 = -\frac{2\sqrt{3}(10,000)}{x_2} = -\frac{34,640}{x_2}$$

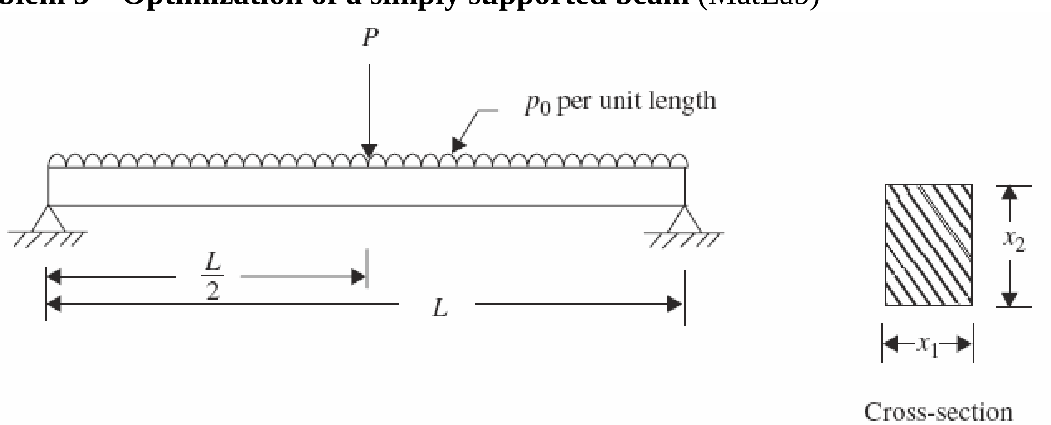
Therefore, the optimization problem can be stated as a separable programming problem as follows:

$$\text{Minimize } f\left(x_1, x_2\right) = 0.1x_1 + 0.05773x_2$$

Subject to

$$\frac{0.006}{x_1} + \frac{0.003464}{x_2} - 0.001 \leq 0, \quad \frac{50,000}{x_1} \leq 8333.333, \quad -\frac{34,640}{x_2} \geq -4948.5714$$

Problem 3 – Optimization of a simply supported beam (MatLab)



A simply supported beam, with a uniform rectangular cross section, is subjected to both distributed and concentrated loads as shown in the figure. It is desired to find the cross section of the beam to minimize the weight of the beam while ensuring that the maximum bending stress induced in the beam does not exceed the permissible stress (σ_0) of the material and the maximum deflection of the beam does not exceed a specified limit (δ_0). The data of the problem are $P = 10^5\,\text{N}$, $p_0 = 10^6\,\text{N/m}$, $L = 1\,\text{m}$, $E = 207\,\text{GPa}$, weight density $(\rho_w) = 76.5\,\text{kN/m}^3$, $\sigma_0 = 220\,\text{MPa}$ and $\delta_0 = 0.02\,\text{m}$. Solve the optimization problem assuming that the cross-sectional dimensions of the beam are restricted as $x_1 \le x_2$, $0.04m \le x_1 \le 0.12m$, and $0.06m \le x_2 \le 0.20m$.

Note: Due to symmetry, the maximum bending stress and deflection occur at the middle. The bending stress can be computed from $\sigma = \frac{1}{2}\frac{Mx_2}{I}$, where the moment of inertia is $I = \frac{1}{12}x_1 x_2^3$.

Problem 4 – Design optimization of a plate with a hole (Ansys)

A square plate with a hole in the center has a uniform pressure of 10 ksi applied to the hole. The objective of this problem is to use the design optimization feature in ANSYS to modify the dimensions of the plate and the hole in order to minimize the volume of the plate without exceeding a maximum von Mises stress of 32.5 ksi.

One quarter of the plate should be modeled and Plane 2 elements with a default mesh density should be used.

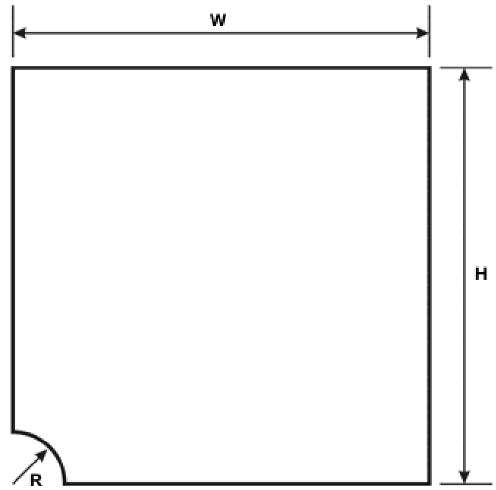
Design Variables:

This model will have the plate height, width, thickness and the radius of the hole parameterized. The ranges are as follows:

Parameter	Minimum Value	Maximum Value
Height (H)	10 in	15 in
Width (W)	10 in	15 in
Thickness (T)	0.1 in	0.25 in
Radius (R)	1.0 in	2.5 in

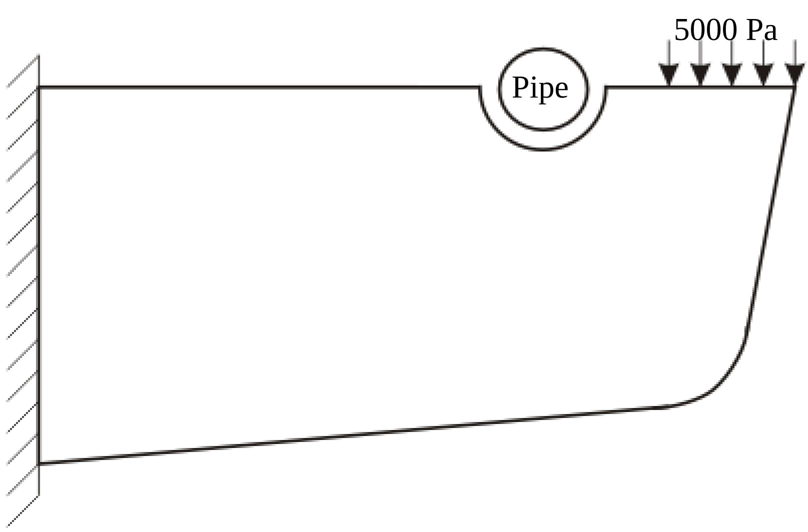
State Variable:
The maximum von Mises equivalent stress will be limited to 32.5 ksi.

Design Objective:
Minimize the volume of the plate.

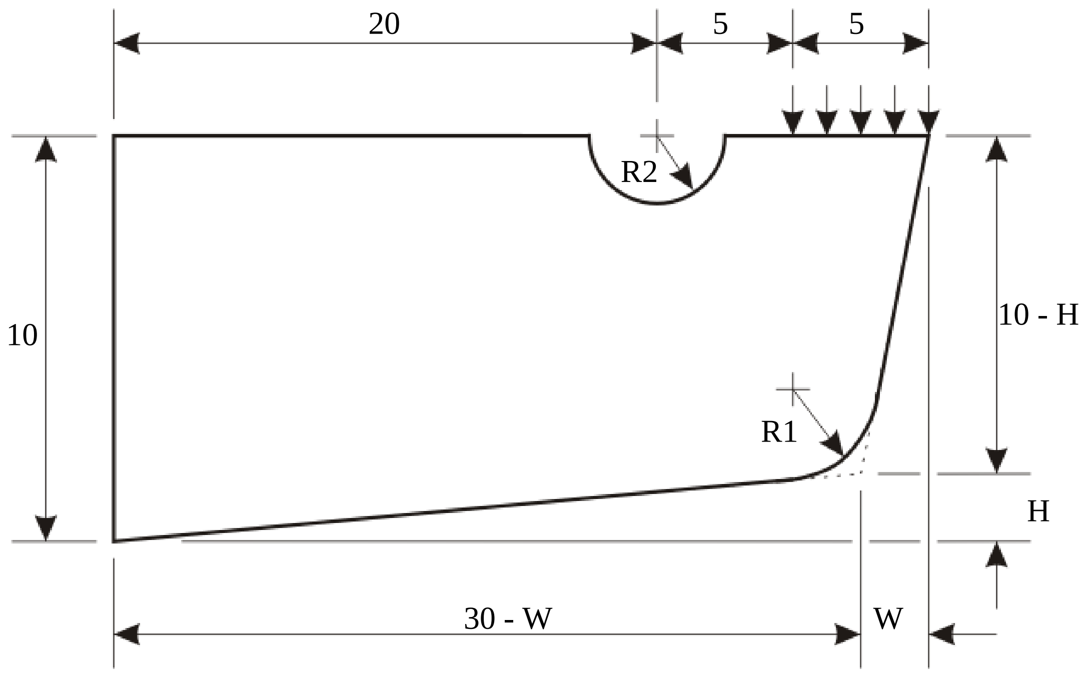


The plate has a Young's Modulus of $30x10^3$ ksi and Poisson's Ratio of 0.3.

Problem 5 – Design optimization of a support bracket (Ansys)
A bracket is to be designed that will support a 5000 Pa load. A 2 m diameter pipe is located between the applied load and the wall where the bracket is to be mounted. The center of the pipe is 20 m from the mounting wall. The bracket must be designed around the existing pipe and it must be at least 0.5 m from the pipe. The bracket is constructed from structural steel, which has a modulus of elasticity of 200 GPa and Poisson's ratio of 0.3.



The dimensions of the bracket (shown in the figure below) are to be optimized in order to minimize the volume of the bracket without exceeding a maximum von Mises stress of 100 kPa.



Note: All dimensions are in meters.

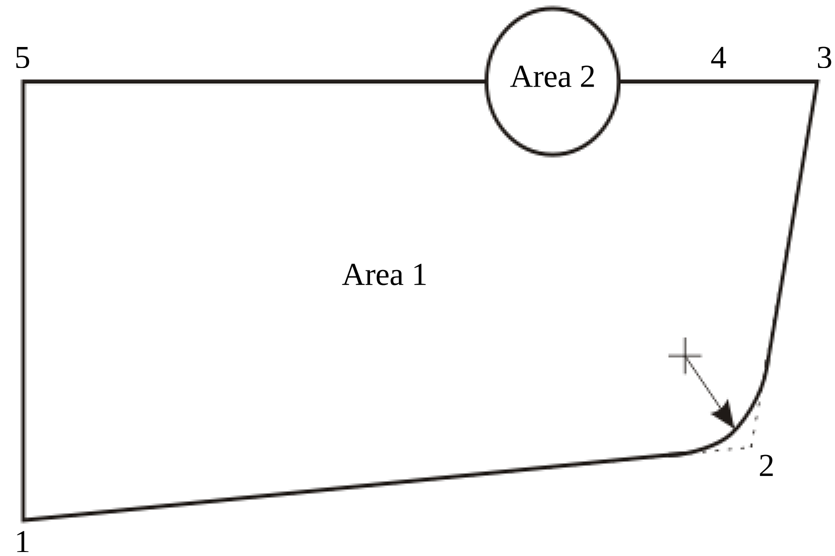
Initial Conditions:

The initial dimensions are listed in the table below.

Parameter:	Initial Value
H	0 m
R1	1 m
R2	2 m
T	0.3 m
W	0 m

Computational Domain and Sub-Domains:

It is recommended that the computational domain be constructed by creating keypoints with parameterized coordinates. Lines and the fillet radius will be created using the keypoints. Area 1 will be created using the lines and Area 2 is a solid circle. Boolean operations will be used to combine the areas to generate the computational domain.



The values that that should be used to parameterize the coordinates of the keypoints are shown in the table below.

Keypoint	X value	Y value
1	0	0
2	30-H	W
3	30	10
4	25	10
5	0	10

Design Optimization:
For this problem, the sub-problem formulation with a maximum of 30 iterations should be used. The design variables & their limits, the state variables & objective function are:

Design Variables and their Limits:

Parameter:	Lower Limit	Upper Limit
H	0 m	5 m
R1	0 m	5 m
R2	1.5 m	5 m
T	0.25 m	0.5 m
W	0 m	5 m

State Variables:
The maximum von Mises stress should not exceed 100 kPa.

Objective Function:
Minimize the volume of the bracket.

Report:
1. ANSYS Report Form.
2. Mesh Plot.
3. Von Mises equivalent stress plot for the initial conditions.
4. List of the design iterations.
5. List of parameters for the optimal design.
6. Von Mises equivalent stress plot for the optimal design.