MAE 212: SHEET METAL FORMING PROCESSES

N. Zabaras, Spring 2000

Elastic Plane Strain Bending

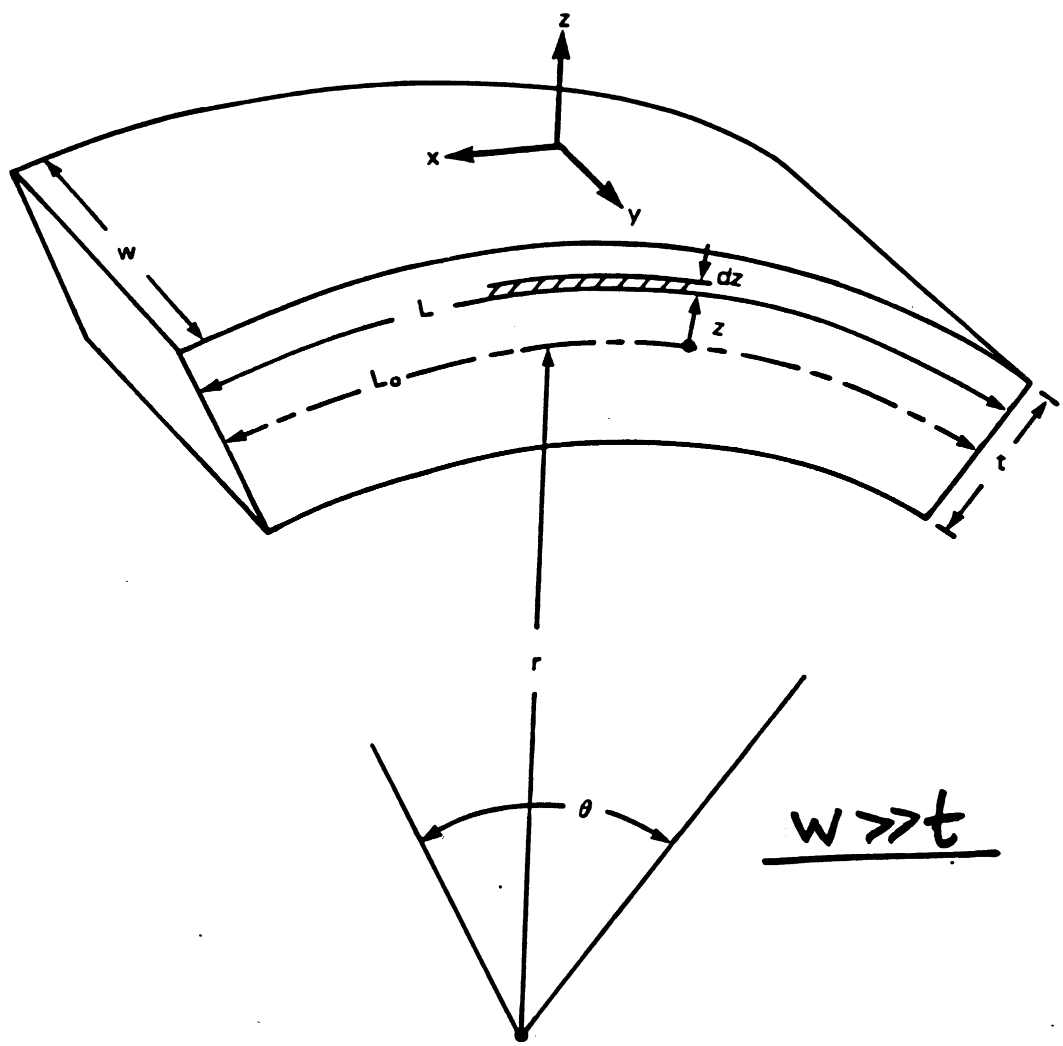

Figure 1: Coordinate System for Analysis of Bending

$$\left.\begin{array}{l}\epsilon_y = \frac{1}{E}\left(\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right) = 0 \\ \sigma_z = 0\end{array}\right\} \Rightarrow \sigma_y = \nu\sigma_x \qquad (1)$$

Let r be the radius of curvature measured to the mid-plane and z the distance of an element from the mid-plane.

$$e_x = \frac{(r+z)\,\theta - r\theta}{r\theta} = \frac{z}{r}, \qquad \epsilon_x = \ln\left(1 + \frac{z}{r}\right) \tag{2}$$

For small strains: $e_x \simeq \epsilon_x \simeq \frac{z}{r}$

But:

$$\begin{aligned}
\epsilon_x &= \frac{1}{E}\left(\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right) = \frac{1-\nu^2}{E}\sigma_x \to \\
\sigma_x &= \frac{E}{1-\nu^2}\frac{z}{r}, \quad \sigma_y = \frac{-E}{1-\nu^2}\frac{\nu z}{r} \\
E' &= \frac{E}{1-\nu^2} = \text{ plane strain elastic modulus.}
\end{aligned} \tag{3}$$

<u>Springback in Sheet Bending</u>

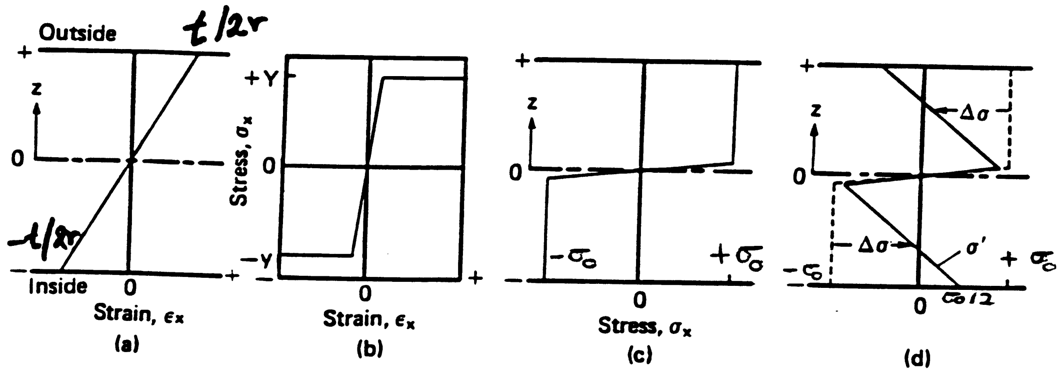

Figure 2: Strain and stress distribution across sheet thickness. Bending strain (a) varies linearly across the section. For the non-work hardening stress-strain relation (b), the bending moment causes the stress distribution in (c). Elastic unloading after removal of the loads results in the residual stresses shown in (d).

Let us here assume an ideally elastic-plastic material (yield stress Y=constant). For plane strain, $\epsilon_y = 0$, the flow rule gives $\sigma_y = \frac{\sigma_x + \sigma_z}{2}$. With the assumption of thin sheet ($\sigma_z = 0$), we have: $\sigma_y = \frac{\sigma_x}{2}$. Substitution of these expressions in the von-Mises equivalent stress gives $\bar{\sigma} = \frac{\sqrt{3}}{2}\,\sigma_x$. Thus the yield condition results in the following:

$$\sigma_x = \frac{2}{\sqrt{3}} Y \tag{4}$$

To calculate the bending moment, M, needed to create the stress distribution in (c) of the figure above, we have:

$$M = \int_{-t/2}^{+t/2} w\sigma_x z dz = 2\int_o^{t/2} w\sigma_x z dz \tag{5}$$

For the ideally plastic material with a negligible elastic core, $\sigma_x = \frac{2}{\sqrt{3}}Y \equiv \sigma_o$ (σ_o is here introduced to simplify the notation), and

$$M = 2w\sigma_x \int_o^{t/2} z dz = w\sigma_o \frac{t^2}{4} \tag{6}$$

Example:

A steel sheet, 0.036 inches thick, is bent to a radius of curvature of 5.0 inches. The flow stress $Y = \frac{\sqrt{3}}{2}\, 33 \times 10^3$ psi (i.e. $\sigma_o = \ 33 \times 10^3$ psi). $E' = 33 \times 10^6$ psi.

1. What fraction of the cross section remains elastic?

2. What percent error does neglecting the elastic core cause in the calculation of the bending moment?

Solution

1. The elastic strain at yielding is $\epsilon_x = \sigma_o/E'$, where E' is the plane-strain modulus, $E/\left(1-\nu^2\right)$. The limit of the elastic core will be at $z = r\epsilon_x = r\sigma_o/E'$. Taking E' as 33×10^6 psi, $z = 5 \times 33 \times 10^3/33 \times 10^6 = 0.005$in. The elastic fraction is $2 \times 0.005/0.036 = 0.28$ or 28%.

2. To calculate the bending moment, for the elastic portion $(0 \leq z \leq 0.005)\,, \sigma_x = \epsilon_x E' = zE'/r$, and for the plastic portion $(0.005 \leq z \leq 0.018)\,, \sigma_x = \sigma_o$.

$$\begin{aligned} M &= 2\int_0^{0.005} w\frac{E'}{r} z^2 dz + 2\int_{0.005}^{0.018} w\sigma_o z dz \\ &= \frac{2}{3}\frac{33 \times 10^6}{5}(0.005)^3 w + 33 \times 10^3 \left(0.018^2 - 0.005^2\right) w = 10.42w \end{aligned} \tag{7}$$

Using the equation which neglects the elastic core,

$$M = \left(33 \times 10^3\right) \frac{\left(0.036^2\right) w}{4} = 10.96w \tag{8}$$

The error is $(10.69 - 10.42)/10.42 = 0.026$ or 2.6%.

Let us denote with $\Delta(\alpha)$ =(the value of α after unloading) - (the value of α before unloading), for any quantity α. Let us also denote with α' the quantity α after unloading.

When the external moment is released, the internal moment must also vanish. As the material unbends (springs back) elastically, the internal stress distribution results in a zero bending moment, i.e.

$$M + \Delta M = 0 \tag{9}$$

Since the unloading is elastic,

$$\Delta\sigma_x = E'\Delta\epsilon_x \tag{10}$$

where, because of plane strain, $E' = E/\left(1-\nu^2\right)$. The change in strain is given by

$$\Delta\epsilon = \frac{z}{r'} - \frac{z}{r} \tag{11}$$

where r' is the radius of curvature after springback. This causes a change in bending moment, ΔM, of

$$\begin{aligned} \Delta M = 2w\int_o^{t/2}\Delta\sigma_x z dz &= 2w\int_o^{t/2} E'\left(\frac{1}{r'}-\frac{1}{r}\right)z^2 dz \\ \Delta M &= \frac{wE't^3}{12}\left(\frac{1}{r'}-\frac{1}{r}\right) \end{aligned} \tag{12}$$

Since $M' = M + \Delta M = 0$ after springback,

$$\frac{wE't^3}{12}\left(\frac{1}{r'}-\frac{1}{r}\right) + \frac{w\sigma_o t^2}{4} = 0 \tag{13}$$

or

$$\boxed{\tfrac{1}{r} - \tfrac{1}{r'} = \tfrac{3\sigma_o}{tE'}} \tag{14}$$

The resulting residual stress, $\sigma'_x = \sigma_x + \Delta\sigma_x = \sigma_o + E'\Delta\epsilon_x = \sigma_o + E'z\left(\frac{1}{r'}-\frac{1}{r}\right) = \sigma_o - E'z\left(\frac{3\sigma_o}{tE'}\right)$

$$\boxed{\sigma'_x = \sigma_o\left(1-\tfrac{3z}{t}\right)} \qquad 0 \le z \le t/2 \tag{15}$$

This is plotted in case d. Note that on the outside surface where $z = t/2$, the residual stress is compressive, $\sigma'_x = -\sigma_o/2$, and on the inside surface $z = -t/2$ it is tensile $\sigma'_x = +\sigma_o/2$.

A similar development can be made for a work-hardening material. If $\bar{\sigma} = K\bar{\epsilon}^n$, then $\sigma_x = K'\epsilon_x^n = K'\left(z/r\right)^n$, where $K' = K\left(\frac{4}{3}\right)^{(n+1)/2}$. Finally,

$$M = \left(\frac{2}{2+n}\right)\frac{wK'}{r^n}\left(\frac{t}{2}\right)^{2+n} \tag{16}$$

Since ΔM is still described as before and $M' = M + \Delta M = 0$ after springback,

$$\left(\frac{1}{r} - \frac{1}{r'}\right) = \left(\frac{6}{2+n}\right)\left(\frac{K'}{E'}\right)\left(\frac{t}{2r}\right)^n\left(\frac{1}{t}\right) \tag{17}$$

Finally,

$$\sigma'_x = K'\left(\frac{z}{r}\right)^n\left[1 - \left(\frac{3}{2+n}\right)\left(\frac{2z}{t}\right)^{1-n}\right] \tag{18}$$

The variations of $\sigma_x, \Delta\sigma_x,$ and σ'_x through the section are shown below. The magnitude of the springback predicted can be very large.

Example

Find the tool radius necessary to produce a final bend radius of $r' = 10$ in. in a part made from a steel of thickness 0.03 inches. Assume a yield stress of $\frac{\sqrt{3}}{2}45,000$ psi ($\sigma_o = 45,000$ psi).

Solution

¿From $\frac{1}{r} - \frac{1}{r'} = \frac{3\sigma_o}{tE}, r' = 10, t = 0.03, \sigma_o = 45,000, E' = 33 \times 10^6 \Rightarrow r = 4.2$ in.

If the bend was in a portion of a complex stamping it would be almost impossible to design tooling for so much of an overbend.

Note: The springback problem is actually greater, since at a bend radius of 4.2 inches, the elastic core is $z = r\sigma_o/E' = 4.2 \times 45 \times 10^3/33 \times 10^6 = 0.0057$ in., i.e., 38% of the cross section. This introduces $\approx 5\%$ error in the moment value.

Bending with Superimposed Tension

Such allowances for springback would cause severe problems in tool design, but fortunately there is a relatively simple solution. Often, as in stretch forming, the tooling does not apply a pure bending moment as assumed above. Rather, tension is applied simultaneously with bending. With increasing tensile forces, F_x, the neutral plane shifts towards the inside of the bend and in most

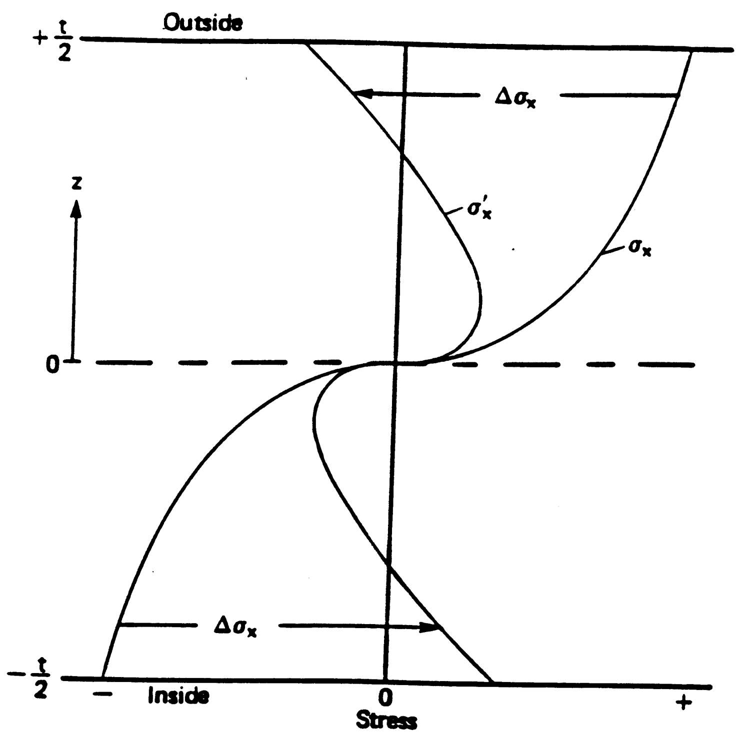

Figure 3: Stress distribution under bending moment and after unloading for a work-hardening material.

operations, this tension is sufficient to move the neutral plane completely out of the sheet so that the entire cross section yields in tension. For such a case, the strain and stress distributions are sketched in Figure 4.

Sheet Bendability

If bend radius is too sharp, excessive tensile strain on outside surface can cause cracking, while buckling can occur on the inside surface.

The limiting values of $\frac{r}{t}$ have been shown to be function of the tensile ductility ($\%$ of elongation at fracture or $\%$ of area reduction at fracture).

$$\frac{R}{t} = \frac{1}{2A_r} - 1 \qquad \text{(solid line on the graph)} \tag{19}$$

where R = inside radius of curvature (i.e. $\frac{R}{t} = \frac{r}{t} - \frac{1}{2}$) and $A_r = \frac{A_o - A_f}{A_o}$ (we don‘t use the notation r for area reduction here to avoid confusion with the radius r).

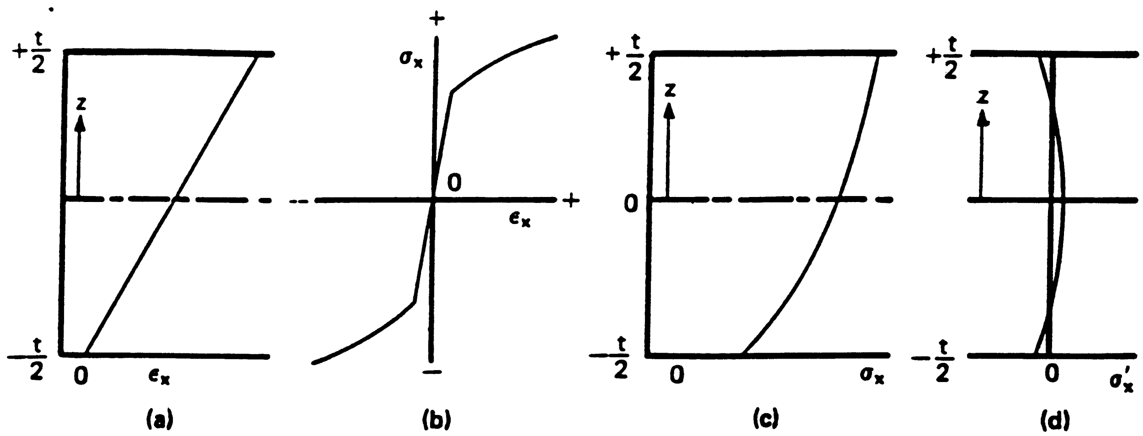

Figure 4: Bending with superimposed tension. With sufficient tension, the neutral axis moves out of the sheet so the strain is tensile across the entire section, (a). With the stress-strain curve shown in (b), the stress distribution in (c) results. After removal of the moment, elastic unloading leaves very minor residual stresses, as shown in (d).

The above correlation is not accurate for sharp bends (low $\frac{r}{t}$) because the neutral axis shifts from the mid-plane and the amount of shift depends upon the applied tension and the frictional conditions. With tight bends (small $\frac{r}{h}$), the neutral axis shifts toward the inside; there are several reasons for this.

The cross section at the inside will increase while the outside decreases and the magnitude of the true strain (and hence the flow stress in a work-hardening material) increases faster with z in compression than tension. As a consequence, the neutral axis moves inward to compensate for the higher stresses and greater cross section. In non-symmetric sections, transition from elastic to plastic flow will not occur simultaneously on both sides of the bend and, consequently, as yielding starts, there will be a shift of the neutral axis toward the heavier sections.

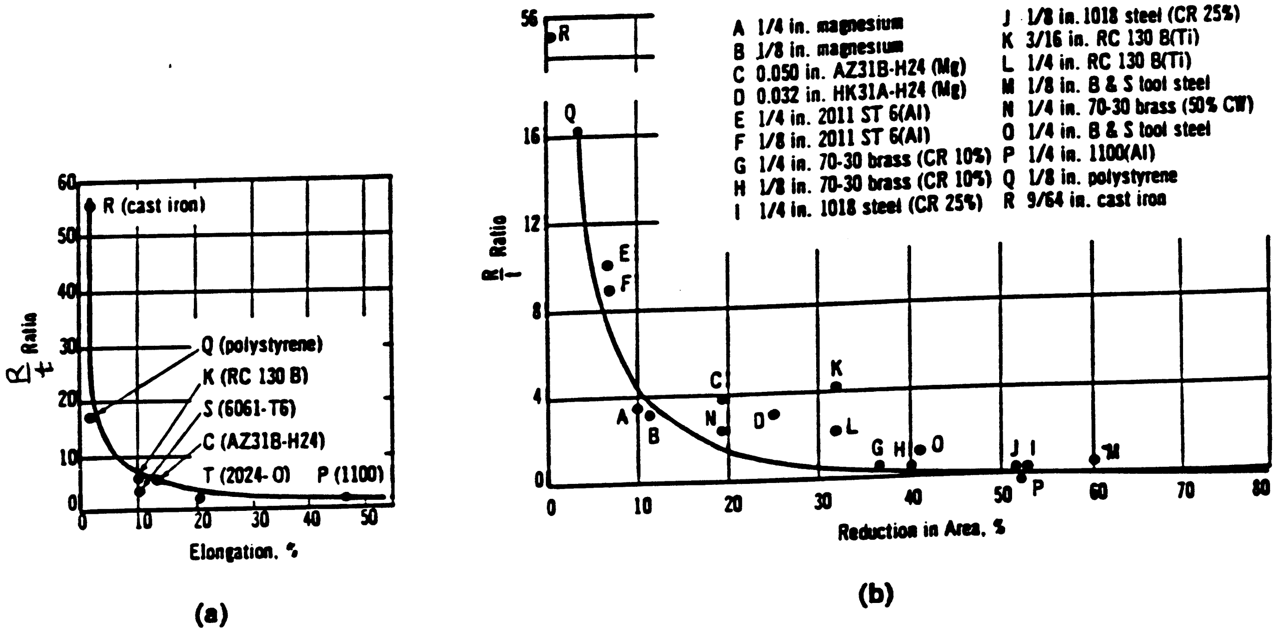

Figure 5: Correlation of limiting bend severity, $\frac{R}{t}$, with tensile ductility