MAE212 - Exam I

March 1st, 2001

1 $$\left.\begin{aligned}\varepsilon_1 &= \frac{1}{E}\left(\sigma_1 - \nu\left(\sigma_2 + \cancel{\sigma_3}\right)\right)\\ \epsilon_2 &= \frac{1}{E}\left(\sigma_2 - \nu\left(\sigma_1 + \cancel{\sigma_3}\right)\right)\end{aligned}\right\} \Rightarrow \epsilon_1 + \epsilon_2 = \frac{1}{E}\left(\sigma_1 + \sigma_2 - \nu\left(\sigma_1 + \sigma_2\right)\right)$$

$$\Downarrow$$

$$\epsilon_1 + \epsilon_2 = \frac{1-\nu}{E}\left(\sigma_1 + \sigma_2\right) \quad (1)$$

But $\epsilon_3 = \frac{1}{E}\left(\cancel{\sigma_3} - \nu\left(\sigma_1 + \sigma_2\right)\right) \Rightarrow$

$$\epsilon_3 = -\frac{\nu}{E}\left(\sigma_1 + \sigma_2\right) \xrightarrow{(1)} \epsilon_3 = -\frac{\nu}{E}\frac{E}{1-\nu}\left(\epsilon_1 + \epsilon_2\right) \Rightarrow$$

$$\boxed{\epsilon_3 = -\frac{\nu}{1-\nu}\left(\epsilon_1 + \epsilon_2\right)}$$

2 $$\left.\begin{aligned}\varepsilon_u &= n\\ \sigma_u &= Kn^n\end{aligned}\right\} \Rightarrow S_u = \frac{F_u}{A_o} = \frac{F_u}{A_u}\frac{A_u}{A_o} = \sigma_u e^{-\varepsilon_u} \Rightarrow$$

$$\varepsilon_u = \ln\frac{A_o}{A_u}$$

$$S_u = \sigma_u e^{-\varepsilon_u} = Kn^n e^{-n} \Rightarrow \boxed{S_u = K\left(\frac{n}{e}\right)^n}$$

3 (a) $$\bar{\sigma} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right)} \equiv Y$$

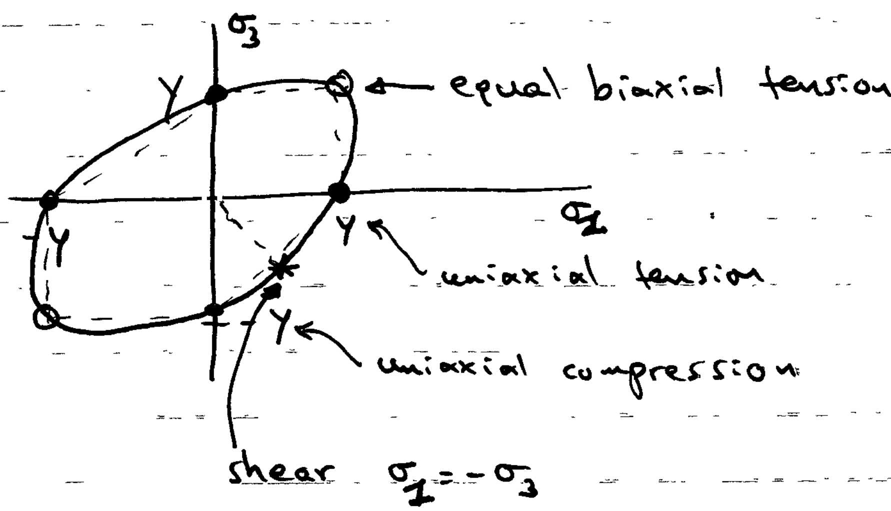
Consider that all stress components are zero except τ_{xy} :

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\sqrt{6\tau_{xy}^2} = Y \longrightarrow \tau_{xy} = Y/\sqrt{3} \quad \text{or} \quad \boxed{k = \frac{Y}{\sqrt{3}}}$$

(b) $\bar{\sigma} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}$

For $\sigma_2 = 0 \rightarrow \bar{\sigma} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-0)^2+(0-\sigma_3)^2+(\sigma_3-\sigma_1)^2} = \sqrt{\sigma_1^2 - \sigma_1\sigma_3 + \sigma_3^2}$

For yielding $\bar{\sigma} = Y \Rightarrow \boxed{\sigma_1^2 - \sigma_1\sigma_3 + \sigma_3^2 = Y^2}$

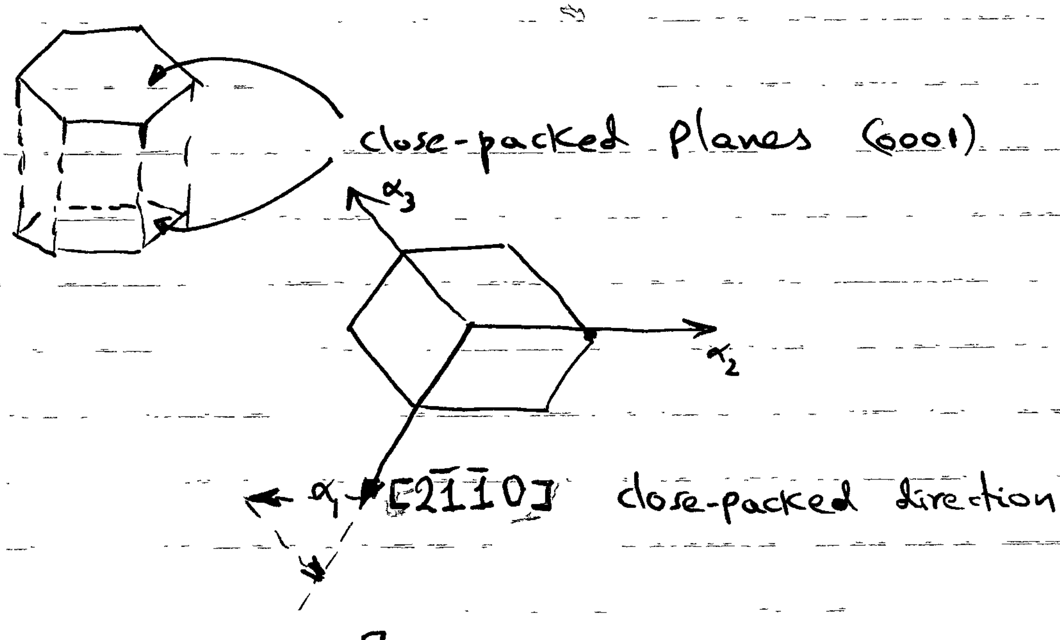


inside the yield surface $\Longrightarrow$ elastic

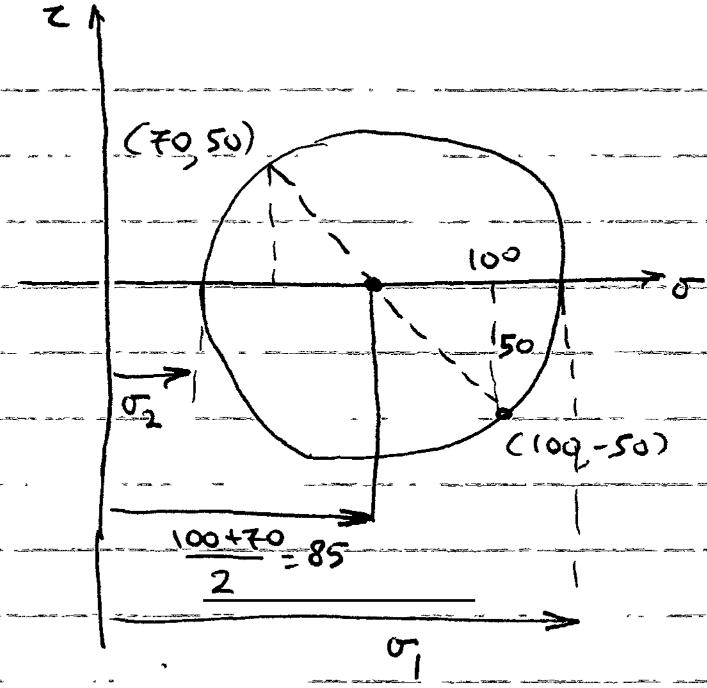
on the yield surface $\Longrightarrow$ plastic

outside the yield surface $\Longrightarrow$ Not allowed!

4



5



$$R = \sqrt{\left(\frac{100-70}{2}\right)^2 + 50^2} = 52.2$$

Principal stresses: $\sigma_1 = 85 + R = 137.2$

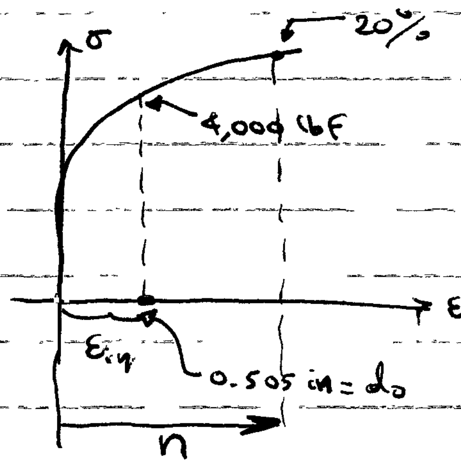
$\sigma_2 = 85 - R = 32.8$

σ_3

Tresca: $\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{Y}{2} \Rightarrow$

$$\sigma_1 - 0 = Y \Rightarrow \boxed{Y = 137.2 \text{ MPa}}$$

6



(a) $\boxed{Y = \frac{4{,}000}{\frac{\pi}{4} 0.505^2} = 19{,}970 \text{ psi}}$

(b) Strain up to the ultimate point during the tensile test: $\ln\frac{1}{1-0.2}$

Total strain up to the ultimate point $= n = 0.45$

Thus: $\varepsilon_{in} = 0.45 - \ln\frac{1}{1-0.2} =$

$\Downarrow$

$$\boxed{\varepsilon_{in} = 0.22686}$$

(c) $Y = K\epsilon_{in}^n \Longrightarrow K = \dfrac{Y}{\epsilon_{in}^n} = \dfrac{19{,}970}{0.22686^{0.45}}$

$$\Downarrow$$

$$\boxed{K = 38{,}930 \text{ psi}}$$

7 (a) From $\epsilon_2 = 0 \xrightarrow[\text{rule}]{\text{flow}} \sigma_2' = 0 \rightsquigarrow \left.\begin{array}{l} \sigma_2 = \dfrac{\sigma_1+\sigma_3}{2} \\ \sigma_3 = 0 \end{array}\right\} \rightsquigarrow \sigma_2 = \dfrac{\sigma_1}{2}$

From $\bar{\sigma} = \dfrac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2} \Longrightarrow$

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\sqrt{\left(\sigma_1-\frac{\sigma_1}{2}\right)^2+\left(\frac{\sigma_1}{2}-0\right)^2+(0-\sigma_1)^2} \Longrightarrow$$

$$\boxed{\bar{\sigma} = \frac{\sqrt{3}}{2}|\sigma_1|}$$

(b) $\left.\begin{array}{l} d\epsilon_2 = 0 \\ d\epsilon_3 = -d\epsilon_1 \text{ (incompressibility)} \end{array}\right\} \rightsquigarrow$

$$d\bar{\epsilon} = \sqrt{\frac{2}{3}(d\epsilon_1^2 + d\epsilon_2^2 + d\epsilon_3^2)} =$$

$$= \sqrt{\frac{2}{3}(d\epsilon_1^2 + 0 + d\epsilon_1)^2} \Longrightarrow \boxed{d\bar{\epsilon} = \frac{2}{\sqrt{3}}|d\epsilon_1|}$$

(c) $\left.\begin{array}{l} \text{From } \bar{\epsilon} = \dfrac{2}{\sqrt{3}}|\epsilon_1| \\ |\epsilon_1| = \ln\dfrac{1}{1-r} \end{array}\right\} \rightsquigarrow \bar{\epsilon} = \dfrac{2}{\sqrt{3}}\ln\dfrac{1}{1-r}$

$$\text{yield stress} = K\bar{\epsilon}^n = K\left(\frac{2}{\sqrt{3}}\ln\frac{1}{1-r}\right)^n$$

(d) $\bar{\sigma} = \text{yieldstress} \overset{(a)}{\Longrightarrow} \dfrac{\sqrt{3}}{2}|\sigma_1| = K\left(\dfrac{2}{\sqrt{3}}\ln\dfrac{1}{1-r}\right)^n \rightsquigarrow |\sigma_1| = \dfrac{2}{\sqrt{3}}K\left(\dfrac{2}{\sqrt{3}}\ln\dfrac{1}{1-r}\right.$

8 $U(r) = -\frac{A}{r^m} + \frac{B}{r^n}$

(a) At equilibrium:

$$\left.\begin{aligned} F = \left.\frac{dU}{dr}\right|_{r=r_0} &= \frac{Am}{r_0^{m+1}} - \frac{Bn}{r_0^{n+1}} = 0 \\ U(r_0) = -U_b \rightsquigarrow & -\frac{A}{r_0^m} + \frac{B}{r_0^n} = -U_b \end{aligned}\right\} \Longrightarrow$$

$$A = -\frac{n r_0^m U_b}{m-n}$$

$$B = -\frac{U_b m r_0^n}{m-n}$$

(b) $S = \left.\frac{d^2U}{dr^2}\right|_{r_0} = -\frac{Am(m+1)}{r_0^{m+2}} + \frac{Bn(n+1)}{r_0^{n+2}}$

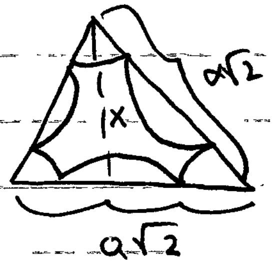
with A, B given above:

(c) $E = \frac{d\sigma}{d\varepsilon} = \frac{d\,\frac{F}{r_0^2}}{d\,\frac{r-r_0}{r_0}} = \frac{dF}{dr}\frac{1}{r_0} = \frac{S}{r_0} \Longrightarrow$

$E = -\frac{Am(m+1)}{r_0^{m+3}} + \frac{Bn(n+1)}{r_0^{n+3}} = \cdots$ substitute A & B $\Rightarrow$

$$\boxed{E = \frac{mn}{r_0^3} U_b}$$

9



Number of atoms: $3\,\frac{1}{6} + 3\,\frac{1}{2} = 2$ atoms

$$\left.\begin{aligned} &\text{Area} = \frac{1}{2}(a\sqrt{2})\,x \\ &x^2 = (a\sqrt{2})^2 - \left(\frac{a\sqrt{2}}{2}\right)^2 \rightsquigarrow x = \frac{a\sqrt{3}}{\sqrt{2}} \end{aligned}\right\}$$

$\text{Area} = \frac{1}{2}a\sqrt{2}\,\frac{a\sqrt{3}}{\sqrt{2}} = \frac{a^2\sqrt{3}}{2}$, Finally $\rho_p = \frac{2}{a^2\sqrt{3}/2} \Rightarrow \boxed{\rho_p = 14.1\ \frac{\text{atoms}}{\text{nm}^2}}$