MAE 212: Spring 2001
PRELIM 1
Thursday, March 1
7:30 - 9:15 p.m.

Closed books and notes. Answer all questions. Make sure your answers are legible. Circle your final answer and show all work. Solutions not supported by appropriate development will not be accepted.

The TAs and instructor will not respond to any questions during the exam. If you think that something is wrong in one or more of these problems, please state your concern in your exam book.

1. (10 points)

   Consider that the axes $1, 2, 3$ are principal stress axes and that $\sigma_3 = 0$ (i.e. assume plane stress conditions). Using two linear strain gages on the plane 12, one can measure ϵ_1 and ϵ_2. Assuming that the deformation is elastic, show that the out of plane strain ϵ_3 can be calculated as follows (ν is the Poisson's ratio):

$$\epsilon_3 = -\frac{\nu}{1-\nu}(\epsilon_1 + \epsilon_2)$$

2. (10 points)

   Assume a uniaxial tensile test using a specimen from a material that obeys $\bar{\sigma} = K\bar{\epsilon}^n$. Compute the ultimate stress in terms of K and n alone.

3. (10 points)

   (a) (3 points)
   For the von-Mises yield criterion, compute the yield stress κ in pure shear in terms of the yield stress Y in tension.

   (b) (7 points)
   Consider plane stress conditions ($\sigma_2 = 0$) for a non-hardening material with yield stress Y. Show that the von-Mises yield condition takes the form $\sigma_1^2 - \sigma_1\sigma_3 + \sigma_3^2 = Y^2$. Plot the yield surface in the σ_1–σ_3 and mark (identify) the coordinates of the following key points: (a) yielding in uniaxial tension/compression; (b) yielding in equal biaxial tension/compression and (c) pure shear. Give a physical interpretation for points inside the yield surface, on the yield surface and outside the yield surface.

4. (5 points)

   Sketch the hexagonal lattice and show one close-packed plane and one close-packed direction for HCP crystals. Provide the Miller indices for this plane and direction.

5. (10 points)

For the plane stress state of Figure 1, find the yield stress Y using the Tresca criterion if you know that the material is yielding.

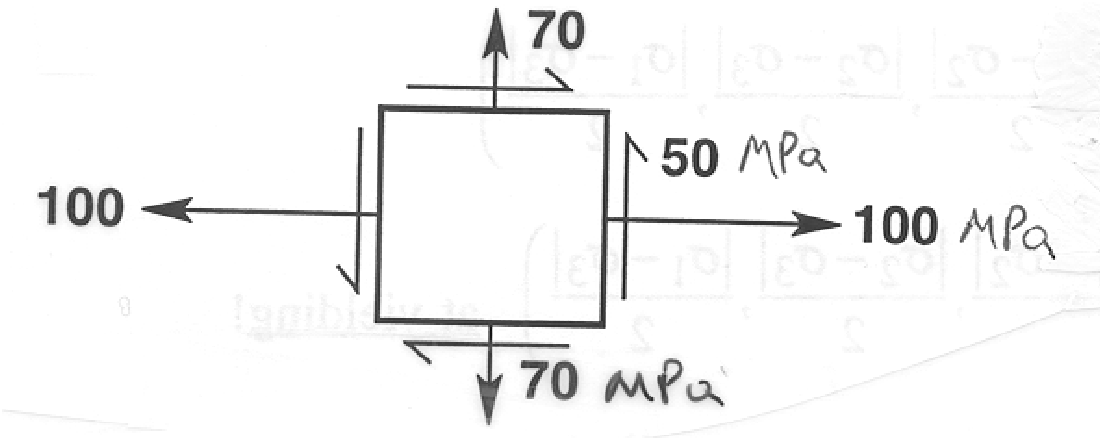

Figure 1: A plane stress state.

6. (15 points)

A tensile specimen is machined to a gage diameter of 0.505-in. When subjected to a tensile test, the following results were found:

yield load = 4,000 lbf

area reduction at the maximum load = 20 %

(a) (5 points) What is the yield stress for this specimen?

(b) (5 points) After completing this test, you are informed that the tensile specimen had been cold-worked (plastically deformed) some amount before it was machined and tested, and that in the 'annealed' state $\sigma = K\ \epsilon^{0.45}$. How much strain was induced by the unknown amount of cold work?

(c) (5 points) Compute K.

7. (15 points)

Consider the plane strain compression (forging) process shown in Fig. 2 where we assume the deformation to be homogeneous. The workpiece is compressed in direction 1 through a downwards moving die (not shown in the Fig). Let h_0 be the initial height of the workpiece. Assume that the workpiece yields at all times during the process and obeys a power law model of the form $\bar{\sigma} = K\bar{\epsilon}^n$. Consider von-Mises yield conditions.

(a) (3 points) Prove that $\bar{\sigma} = \frac{\sqrt{3}}{2}|\sigma_1|$.

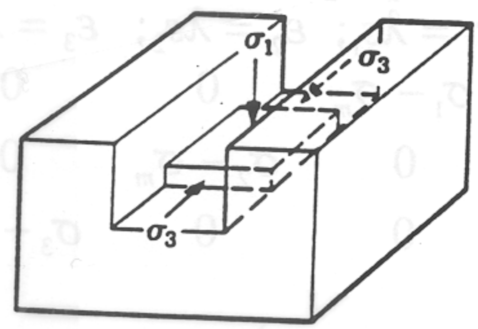

Figure 2: A plane strain compression (forging) process. The workpiece is constrained in direction 2 and it is free to expand in direction 3.

(b) (3 points) Prove that $d\bar{\epsilon} = \frac{2}{\sqrt{3}}|d\epsilon_1|$ (Recall that $d\bar{\epsilon} = \sqrt{\frac{2}{3}(d\epsilon_1^2 + d\epsilon_2^2 + d\epsilon_3^2)}$).

(c) (5 points) Assuming that you can integrate the result in (b) and write that $\bar{\epsilon} = \frac{2}{\sqrt{3}}|\epsilon_1|$, compute the yield stress (flow stress) of the workpiece after compression to a height reduction $r = \frac{h_0 - h}{h_0}$ (at height h) in terms of K, n and r.

(d) (4 points) Apply the von-Mises yield condition using the yield stress of the workpiece obtained in (c) above and compute (in terms of K, n and r) the value of $|\sigma_1|$ (i.e. the die pressure) when the workpiece is compressed to a reduction r.

8. (15 points)

Let us approximate the potential energy curve for the covalent bond between two atoms by the equation:

$$U(r) = \frac{-A}{r^m} + \frac{B}{r^n} \tag{1}$$

where the integers m and n are given. The equilibrium distance r_0 between the two atoms and the binding energy $U_b > 0$ are also assumed known.

(a) (5 points) Determine the constants A and B.

(b) (5 points) Compute the stiffness of the bond S in terms of m, n, r_0 and U_b.

(c) (5 points) Show that the elastic modulus E of a material made of atoms with such type of bonds is given as $E = \frac{nm}{r_0^3}U_b$. Assume that r_0^2 is the area corresponding to each bond.

9. (10 points)

Show that the planar density ρ_p of atoms in the (111) plane of fcc aluminum is 14.1 atoms per nm^2. The lattice constant a of aluminum is 0.404 nm (Be sure to only count those atoms centered on this plane).