MAE 212: OPEN DIE FORGING PROCESSES

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Direct Compression in Plane Strain

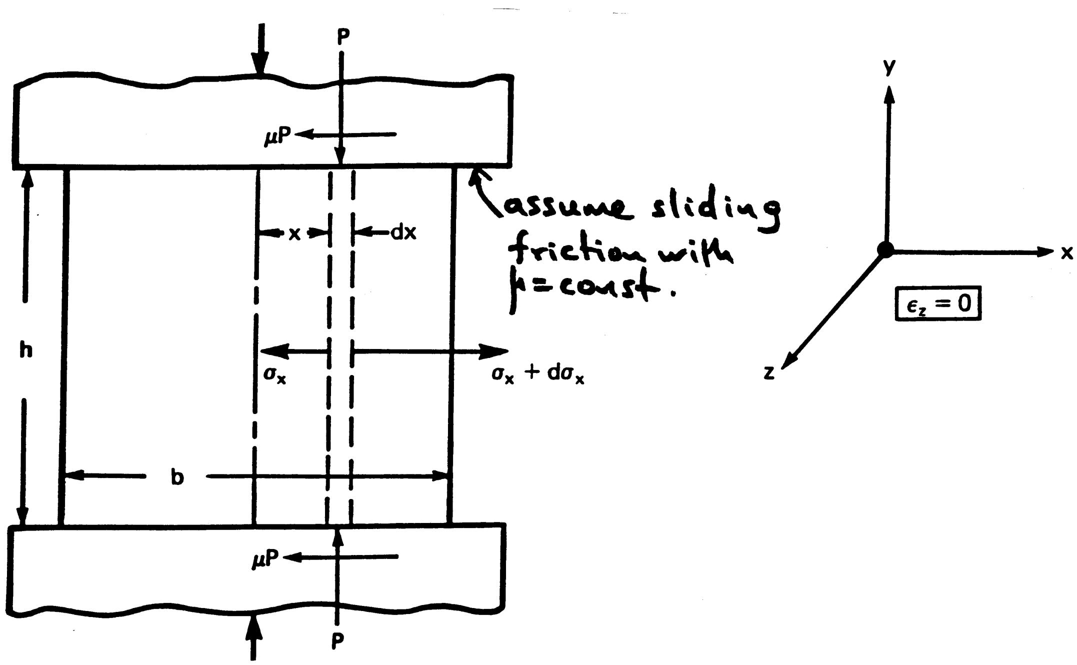

Figure 1: Force balance in a slab.

Let us consider the workpiece at a given height h. Lets assume that the average yield stress of the material at this stage is Y.

Take σ_x, $\sigma_y = -p$, $\sigma_z = \frac{\sigma_x - p}{2}$ as the principal stresses. The expression for σ_z is the result of the plane strain conditions and p is the pressure at the die/workpiece interface. Assume little effect of friction on principal stresses.

For plane strain conditions, the von-Mises criterion becomes:

$$\sigma_x - \sigma_y = \frac{2Y}{\sqrt{3}} \;\Rightarrow\; \sigma_x = \frac{2Y}{\sqrt{3}} - p \tag{1}$$

The force equilibrium equation for the slab in the x direction is the following:

$$(\sigma_x \,+\, d\sigma_x)\, h\, w \,-\, \sigma_x\, h\, w \,-\, 2\,\mu\, p\, dx\, w \,=\, 0 \tag{2}$$

or after simplification

$$h\, d\sigma_x \,=\, 2\,\mu\, p\, dx \tag{3}$$

But from equation (1), $d\sigma_x = -dp$ (note that Y is constant for a given h). Substitution of the above equation in equation (3) results in the following:

$$\frac{dp}{p} = \frac{-2\mu}{h} dx \tag{4}$$

We need some boundary conditions to integrate the above equation: At $x = \frac{b}{2}$ we know that $\sigma_x = 0$ (free surface). Using the von-Mises criterion at the free surface results in the following:

$$p(\frac{b}{2}) = \frac{2Y}{\sqrt{3}} - \sigma_x(\frac{b}{2}) = \frac{2Y}{\sqrt{3}} \tag{5}$$

Using equation (5) and integrating equation (4) from position x to position $\frac{b}{2}$ results in the following:

$$\frac{p}{\frac{2Y}{\sqrt{3}}} = \exp\left[\frac{2\mu}{h}\left(\frac{b}{2} - x\right)\right], \qquad 0 \le x \le \frac{b}{2} \tag{6}$$

which governs from $x = 0$ to $x = \frac{b}{2}$.
The maximum value for p occurs at the centerline where

$$\left(\frac{p}{\frac{2Y}{\sqrt{3}}}\right)_{\max} = \exp\left(\frac{\mu b}{h}\right) \tag{7}$$

Also of great interest is p_{avg}, i.e. the average or mean pressure at the tool-workpiece interface (for a given height h).
For simplicity, let $a = \frac{b}{2}$ and $c = \frac{2\mu}{h}$ in the following derivation, and note that e is the base of natural logarithms.

$$p_{\text{avg}} = \frac{1}{a}\int_0^a p dx = \frac{1}{a}\int_0^a \frac{2Y}{\sqrt{3}} e^{c(a-x)} dx = \frac{2Ye^{ca}}{\sqrt{3}a}\int_0^a e^{-cx} dx \tag{8}$$

so

$$p_{\text{avg}} = \frac{2Ye^{ca}}{\sqrt{3}a}\left[\frac{-e^{-cx}}{c}\right]_{x=0}^{x=a} \tag{9}$$

Thus,

$$p_{\text{avg}} = \frac{\frac{2Y}{\sqrt{3}}}{\frac{\mu b}{h}}\left(e^{\frac{\mu b}{h}} - 1\right) \tag{10}$$

Now

$$\exp\left(\frac{\mu b}{h}\right) - 1 = 1 + \frac{\mu b}{h} + \frac{\left(\frac{\mu b}{h}\right)^2}{2!} + \ldots - 1 \tag{11}$$

so

$$p_{\text{avg}} \approx \frac{2Y}{\sqrt{3}}\left(1 + \frac{\frac{\mu b}{h}}{2} + \ldots\right) \approx \frac{2Y}{\sqrt{3}}\left(1 + \frac{1}{2}\frac{\mu b}{h}\right) \text{ for small } \frac{\mu b}{h} \tag{12}$$

Recall that $\kappa = \frac{Y}{\sqrt{3}}$ is the yield stress in shear for the von-Mises criterion. The earlier results for the pressure, maximum pressure and average pressure can now be written in terms of κ as follows:

$$\frac{p}{2\kappa} = \exp\left[\frac{2\mu}{h}\left(\frac{b}{2} - x\right)\right] \tag{13}$$

$$\left(\frac{p}{2\kappa}\right)_{\max} = \exp\left(\frac{\mu b}{h}\right) \tag{14}$$

$$p_{\text{avg}} \approx 2\kappa\left(1 + \frac{1}{2}\frac{\mu b}{h}\right) \text{ for small } \frac{\mu b}{h} \tag{15}$$

¿From equation (13), it is obvious (see also Fig. 2)

$$p \geq 2\kappa \tag{16}$$

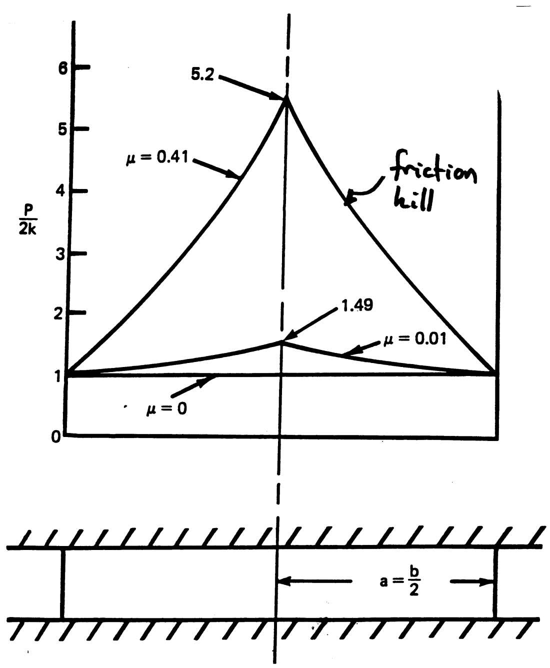

Figure 2: Illustration of the friction hill in plane strain compression for different values of the friction coefficient.

There is an upper limit on the shear stress at the interface, $\mu\, p \leq \kappa$. Using equation (16), $p \geq 2\kappa$, results in $\mu\, 2\, \kappa \leq \kappa$ or $2\, \mu \ \leq\ 1$ or $\mu \leq \frac{1}{2}$ if sliding friction is to occur.

Example

Plane-strain compression is conducted on a slab of metal whose yield shear strength, κ, is 15,000 psi. The width of the slab is 8 inches while its height is 1 inch. Assuming the average coefficient of friction at each interface is 0.10,

1. Estimate the maximum pressure at the onset of plastic flow, and
2. Estimate the average pressure at the onset of plastic flow.

Solution:

1.
$$p_{\max} = 2\kappa \exp\left(\frac{\mu b}{h}\right) = 30,000 \exp\left(\frac{0.1 \times 8}{1}\right) \tag{17}$$
so, $p_{\max} = 30,000\,(2.226) = 66,800$ psi.

2. First, use the exact solution.
$$p_{\text{avg}} = \frac{30,000}{0.8}\left(e^{0.8} - 1\right) = 46,000 \text{ psi} \tag{18}$$

Sticking friction approximation for plane strain forging

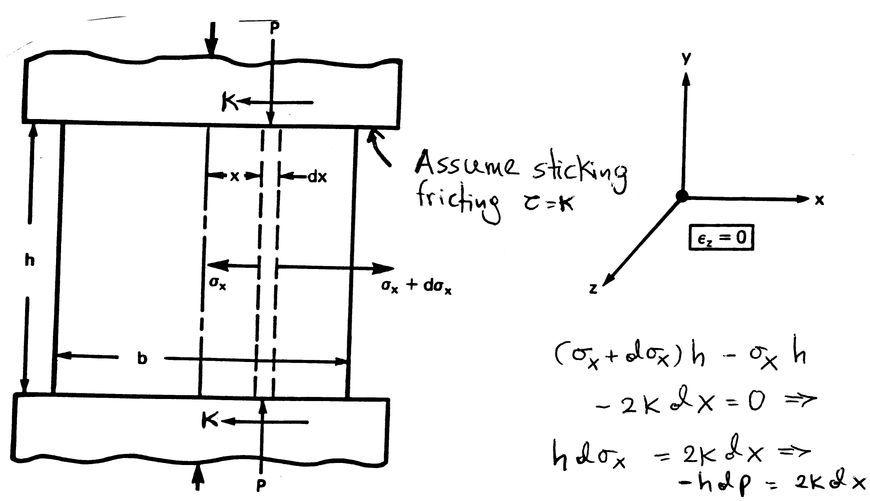

Figure 3: Force balance in a slab for the sticking friction approximation (plane strain).

As just indicated, there is a limit at which sliding friction can exist at the tool-workpiece interface: if that is reached, then interfacial shear of the workpiece occurs and the frictional forces indicated as μp are replaced by the yield shear stress κ. Following the previous analysis the result assuming sticking friction in the whole workpiece/die interface is given as (see Figure 4 for a slab analysis pictorial):
$$\frac{p}{2\kappa} = 1 + \frac{\frac{b}{2} - x}{h} \tag{19}$$
which predicts a linear variation of p from the outer edge to the centerline. The maximum value, which occurs at the centerline, is
$$p_{\max} = 2\kappa\left(1 + \frac{b}{2h}\right) \tag{20}$$

The average pressure is given as follows:

$$p_{\text{avg}} = 2\kappa \left(1 + \frac{b}{4h}\right) \tag{21}$$

Figure 5 shows the friction hill for plane-strain compression with sticking friction.

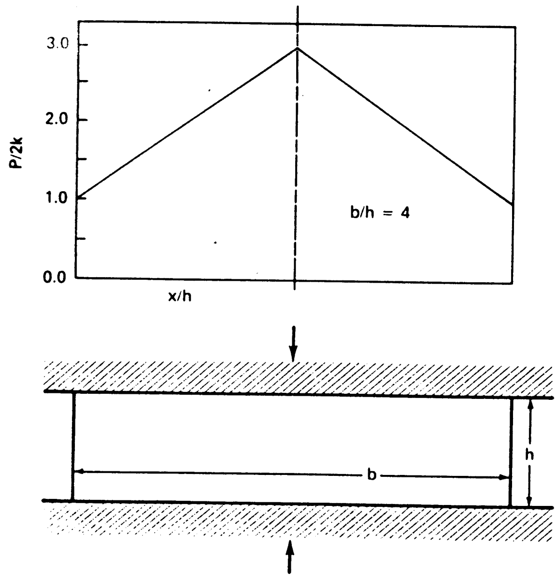

Figure 4: Friction hill in plane-strain compression for sticking-friction.

Example

Repeat previous example assuming sticking friction at each interface.

Solution:

1. $P_{\text{max}} = 30,000 \left(1 + \frac{8}{2}\right) = 150,000$ psi.
2. $P_a = 30,000 \left(1 + \frac{8}{4}\right) = 90,000$ psi.

Note: When lubrication is used between the tools and the workpiece, $\tau = mk, 0 < m < 1$. In this case, $\frac{P}{2\kappa} = 1 + \frac{\frac{mb}{2} - x}{h}$.

Axisymmetric Compression

For a constant value of μ, a force balance in the radial direction gives

$$\sigma_r hrd\theta + 2\mu prd\theta dr + \frac{2\sigma_\theta hdrd\theta}{2} - (\sigma_r + d\sigma_r)\, hd\theta\, (r + dr) = 0 \tag{22}$$

or

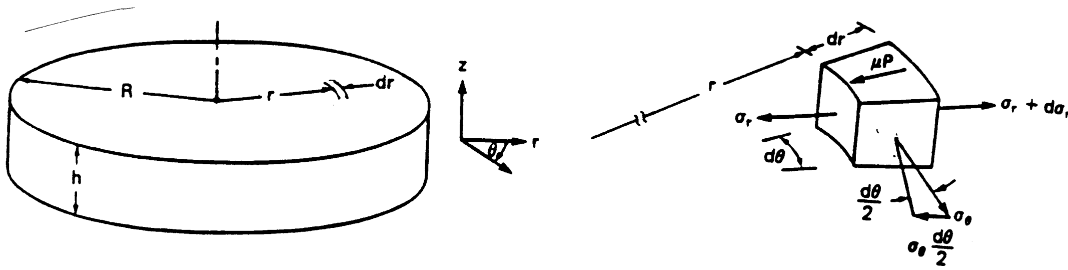

Figure 5: Slab for radial force balance in axisymmetric forging.

$$2\mu prdr + h\sigma_\theta dr - h\sigma_r dr - hrd\sigma_r = 0 \tag{23}$$

if higher order terms are neglected.
With axisymmetric flow, $\epsilon_\theta = \epsilon_r$, so $\sigma_\theta = \sigma_r$, and for yielding, $\sigma_r + p = Y$ or $d\sigma_r = -dp$. Inserting these into equation (22) gives

$$2\mu prdr = -hrdp \tag{24}$$

or

$$\frac{dp}{p} = -\frac{2\mu}{h}dr \tag{25}$$

Since $\sigma_r = 0$ when $r = R$, we have that: $p = Y$ when $r = R$. Integration of the above equation from r to R gives the following:

$$p = Y \ \exp\ \left[\frac{2\mu}{h}\left(R - r\right)\right] \tag{26}$$

The average pressure is defined as follows:

$$p_{\text{avg}} = \frac{1}{\pi R^2}\int_0^R p \cdot \ \ 2\ \pi\ r\ dr \tag{27}$$

from which we calculate that:

$$p_{\text{avg}} = \frac{1}{2}\left(\frac{h}{\mu R}\right)^2 Y\left[\exp\ \left(\frac{2\mu R}{h}\right) - \frac{2\mu R}{h} - 1\right] \tag{28}$$

or using a Taylor series expansion to approximate the exponential term in the equation above (how many terms should you keep?)

$$p_{\text{avg}} = Y[1 + \frac{2\ \mu\ R}{3h}] \tag{29}$$

The equation above is a good approximation to p_{avg} for small values of μ and moderate values of R/h.

Pressure distribution under sticking friction conditions

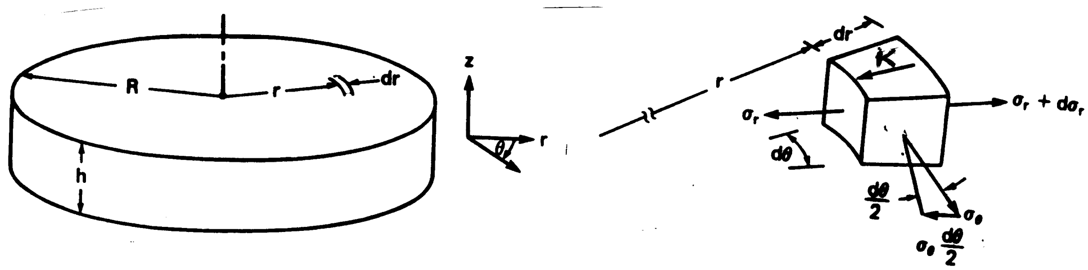

Figure 6: Slab for radial force balance in axisymmetric forging with sticking friction.

If sticking friction occurs, you can easily show that:

Force balance in the r direction: $2\kappa dr = -hdp$

Pressure distribution: $p = Y + \frac{2\kappa}{h}(R - r)$

Average pressure: $p_{\text{avg}} = Y + \frac{2\kappa R}{3h}$

Note in the last 3 formulas, the term Y comes from the von-Mises yield condition, whereas the term κ results from the sticking friction shear stress. Of course, do not forget that for the von Mises criterion, $\kappa = \frac{Y}{\sqrt{3}}$.

Example

A solid disc of 4 inch diameter and 1 inch height is to be compressed. If the tensile and shear yield stresses for this metal are 50,000 and 25,000 psi respectively, estimate the force needed to start plastic flow.

Solution

The average pressure at the start of flow is found: $P_a = 50,000 + \frac{2(25,000)(2)}{3(1)} = 83,333$ psi. Then $F = 83,333\ \frac{\pi}{4}4^2 \simeq 1.05 \times 10^6$ lbf.

Barreling

A workpiece under an open-die forging process develops a barrel shape. This is caused primarily by frictional forces that oppose the outward flow of the material at the contact interfaces. Barreling also occurs in upsetting hot workpieces between cold dies. The material at and near the interfaces cools rapidly, while the rest of the specimen is relatively hot. Because the strength of the material decreases with temperature, the ends of the specimen show greater resistance to deformation than does the center. Thus the central portion of the cylinder deforms to a greater extent than do its ends.

How do you handle the above calculations for a hardening material: $\bar{\sigma} = K\bar{\epsilon}^n$

For simplicity let us also discuss the plane strain compression of a block. Let us consider the block at an intermediate height h ($h_o \leq h_f$). Note that due to incompressibility $hb = h_o b_o = \text{constant}$, i.e. b is uniquely determined given h.

- From the plane strain conditions we can write the effective strain as: $\bar{\epsilon} = \frac{2}{\sqrt{3}} \ln \frac{h_o}{h}$
- The (current) yield stress of the workpiece of height h is given then as follows: $Y = K\bar{\epsilon}^n = K(\frac{2}{\sqrt{3}} \ln \frac{h_o}{h})^n$
  Notice how the yield stress varies with h.
- You can now use this value of Y in any of the formulas given above for the pressure.

Additional calculations you should be able to perform

Using the above formulas, one should be able to calculate:

- The maximum force required to decrease the height of a workpiece from h_o to h_f (at which h is the force maximum?)
- The total work necessary to decrease the height of a workpiece from h_o to h_f. Recall that work = $|\int_{h_o}^{h_f} F dh| = |\int_{h_o}^{h_f} p_{\text{avg}} b w dh| = |\int_{h_o}^{h_f} p_{\text{avg}} \frac{h_o b_o}{h} w dh|$. The last integral can be easily calculated using the derived expressions for p_{avg}.
- The location of transition from sticking to sliding ($k = \mu p$) if any.