MAE 212: Spring 2001
Module III: Material Property Characterization Using Strain Gage Instrumentation

Abstract

In this module, we will learn the fundamentals of applying resistance-type strain gages. Strain components will be measured on the lateral surface of an aluminum compression specimen and the strain/load data will be used to determine the Young's modulus, E, Poisson's ratio, ν, and the 0.2% yield strength of the material. The elastic properties of the Aluminum wall of a soda can will also be estimated by performing load/strain measurements for two specimens cut from the walls – one along the length and another along the circumference.[1] Finally strain measurements will be performed on the lateral surface of a soda Aluminum can while opening the top. These measurements will be used to estimate the pressure of the soda in the can!

1 Resistance-type strain gages

Electrical resistance strain gages are thin metal foils that are bonded to the surface of a material specimen. The resistance R of a metallic conductor is dependent on its geometry. For example, if the conductor is in the form of a wire and a load is applied along the axis of the wire, the length increases and the cross-sectional area decreases. Both effects serve to increase the resistance. Mechanical strains that develop in the surface of a component are sensed by the gage. The electrical resistance of a resistance-type gage changes in proportion to the strain the gage senses.

The fundamentals of resistance-strain gages are given in chapter 5 of *Instrumentation for engineering measurements*, by J.W. Dally et al.

Strain gages are produced to ensure the following linear relationship:

$$\frac{\Delta R}{R} = S_g \epsilon \tag{1}$$

where ϵ is the strain along the longitudinal axis of the gage and S_g is known as the gage factor or sensitivity. The gage factor is a gage characteristic and is typically included in the product information for the strain gages (e.g. see the strain gave manual given to you in the lab). Typically S_g is close to 2. Since ϵ is small for elastic strains, you see that $\Delta R/R$ is going to be quite small. Thus a special circuit is needed to sense this small change.

The Wheatstone bridge is a commonly used circuit for converting a change in resistance to a change in voltage. Thin voltage can then be further amplified and measured. The Wheatstone bridge is shown in Figure 1.

There are various strain gage arrangements in a Wheatstone bridge. For a $\frac{1}{4}$ bridge (case 1 in Fig. 2), one of the resistances in Fig. 1 is substituted with a strain gage.

For an initially balanced bridge and with all resistances equal, i.e. $R_i = R$, for all $i = 1, 2, 3, 4$, the total sensitivity S_s for this $\frac{1}{4}$ strain gage-Wheatstone bridge system is given as follows:

$$S_s = \frac{v_o}{\epsilon} = \frac{v_s S_g}{4} \tag{2}$$

[1] This will motivate us to speak about the anisotropy of rolled Al.

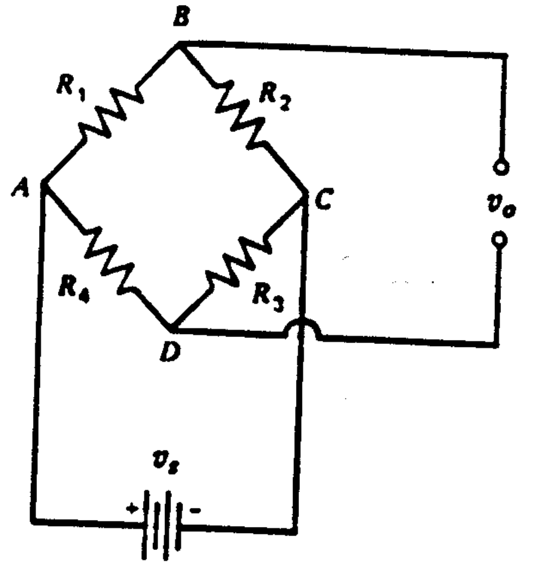

Figure 1: The constant-voltage Wheatstone bridge circuit. Here V_s is the source or excitation voltage and V_o is the measured output voltage. We can see that V_o vanishes if $R_1R_3 = R_2R_4$ – the bridge is in a balanced condition. This is the condition with which we will begin our measurements.

It is clear from the equation above that by increasing the excitation voltage v_s, the sensitivity is increased. Note tat the maximum value of v_s is limited by the power that the strain gage can dissipate without significant increase in temperature (which changes the strain and hence your readings). Typically, v_s is limited to around 5 V. Note also that v_o is small, around $1 - 10$ mV, thus strain gage outputs are usually amplified using differential amplifiers.

Other common strain gage arrangements in a Wheatstone bridge are shown in Figure 2. For generic information on the Wheatstone bridge for signal conditioning, consult chapter 6 of the text by J.W. Dally et al.

Strain gages are sensitive to other components of strain in addition to the normal strain along the gage axis. The effect is usually referred to as *cross sensitivity.*

As an example, let us assume a plane strain state in which we measure the strain components ϵ'_{xx} and ϵ'_{yy} in two orthogonal directions x and y. The $()'$ prime here indicates measured uncorrected strain components. The corrected for cross sensitivity strain components ϵ_{xx} and ϵ_{yy} are given as follows:

$$\epsilon_{xx} = \frac{1 - \nu_o K_t}{1 - K_t^2} \left[\epsilon'_{xx} - K_t \epsilon'_{yy}\right] \tag{3}$$

and

$$\epsilon_{yy} = \frac{1 - \nu_o K_t}{1 - K_t^2} \left[\epsilon'_{yy} - K_t \epsilon'_{xx}\right] \tag{4}$$

where $\nu_o = 0.285$ and K_t is the so called transverse sensitivity factor. The manufacturers of strain gages provide K_t for each gage.[2] The gage sensitivity to shearing strain is here assumed to be small and is neglected.

[2]Usually K_t is given in a percentage format. If for example, the transverse sensitivity is given as $+1.3$ then in the equations above, you should use $K_t = 0.013$.

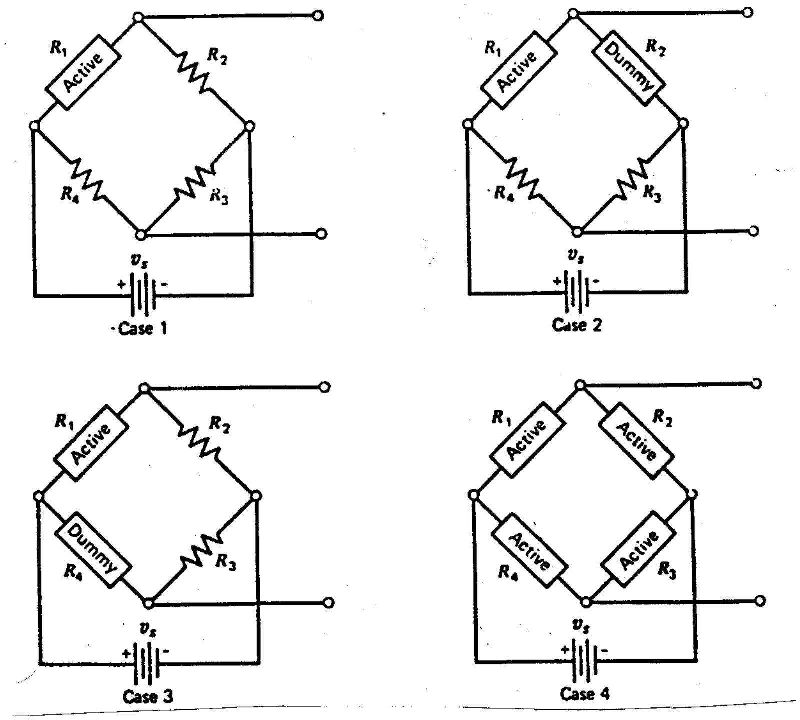

Figure 2: Four common strain-gage arrangements in a Wheatstone bridge. Case 1 is the $\frac{1}{4}$, cases 2 and 3 are known as $\frac{1}{2}$ bridge configurations and case 4 as a full bridge configuration.

2 Analysis of Strain Data

Strain gages are normally bonded on a free surface of a workpiece, $\sigma_{zz} = \tau_{xz} = \tau_{yz} = 0$, where z is normal to the surface. This means that strain gages properly configured will only allow you to determine the in-plane components of strain, i.e. $\epsilon_{xx}, \epsilon_{yy}$ and γ_{xy}. The remaining strain components cannot be calculated from this strain gage arrangement. Depending on which strain components are desired, a different placement of gages may be required.

In general we need three individual strain gages in order to determine $\epsilon_{xx}, \epsilon_{yy}$ and γ_{xy}. This can be seen by applying the strain transformation equation that you have seen in ENGR 202 (see p. 386 of Beer and Johnston's *Mechanics of Materials*)

$$\epsilon_{x'x'} = \frac{\epsilon_{xx} + \epsilon_{yy}}{2} + \frac{\epsilon_{xx} - \epsilon_{yy}}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta \tag{5}$$

This equation provides the strain $\epsilon_{x'x'}$ in terms of the strains $\epsilon_{xx}, \epsilon_{yy}$ and γ_{xy}, where θ is the angle between the axes x' and x. Applying this equation to the directions of the gages given in Figure 3, provides the following system of equations:

$$\epsilon_A = \frac{\epsilon_{xx} + \epsilon_{yy}}{2} + \frac{(\epsilon_{xx} - \epsilon_{yy})}{2} \cos 2\theta_A + \frac{\gamma_{xy}}{2} \sin 2\theta_A \tag{6}$$

$$\epsilon_B = \frac{\epsilon_{xx} + \epsilon_{yy}}{2} + \frac{(\epsilon_{xx} - \epsilon_{yy})}{2} \cos 2\theta_B + \frac{\gamma_{xy}}{2} \sin 2\theta_B \tag{7}$$

$$\epsilon_C = \frac{\epsilon_{xx} + \epsilon_{yy}}{2} + \frac{(\epsilon_{xx} - \epsilon_{yy})}{2} \cos 2\theta_C + \frac{\gamma_{xy}}{2} \sin 2\theta_C \tag{8}$$

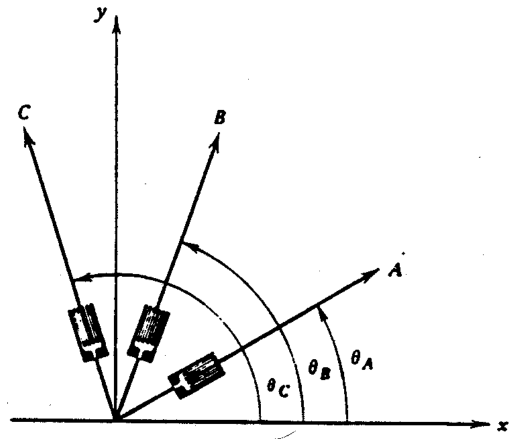

Figure 3: Three gages oriented at angles $\theta_A, \theta_B, \theta_C$ with respect to the x axis.

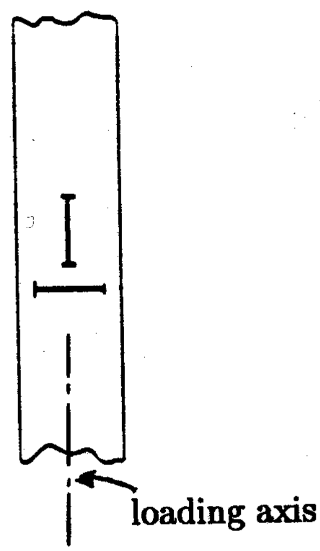

Figure 4: Strain gage orientation for Young's modulus and Poisson's ratio determination.

Equations (6)–(8) can be solved simultaneously for ϵ_{xx}, ϵ_{yy} and γ_{xy} in terms of the measurements ϵ_A, ϵ_B and ϵ_C. By judiciously choosing θ_A, θ_B and θ_C we can greatly simplify this process.

By arranging two strain gages on a tensile specimen along the tensile axis and perpendicular to it (see Figure 4) we can measure the longitudinal strain, ϵ_l, and the transverse strain, ϵ_t. By also recording the load we can calculate the tensile stress, σ. Young's modulus is just the ratio of σ to ϵ_l in the elastic regime. Therefore, by loading the specimen to several different loads we can determine (σ, ϵ_l) data pairs which can be regressed for E. Similarly, we can plot the longitudinal and transverse strains versus load (see Figure 5) and determine the Poisson's ratio $\left(\nu = -\frac{\epsilon_t}{\epsilon_l}\right)$ by regressing straight lines through the strain/load pairs. Poisson's ratio can then be calculated by

$$\nu = \frac{\frac{d\epsilon_t}{dp}}{\frac{d\epsilon_{\text{long}}}{dp}} \tag{9}$$

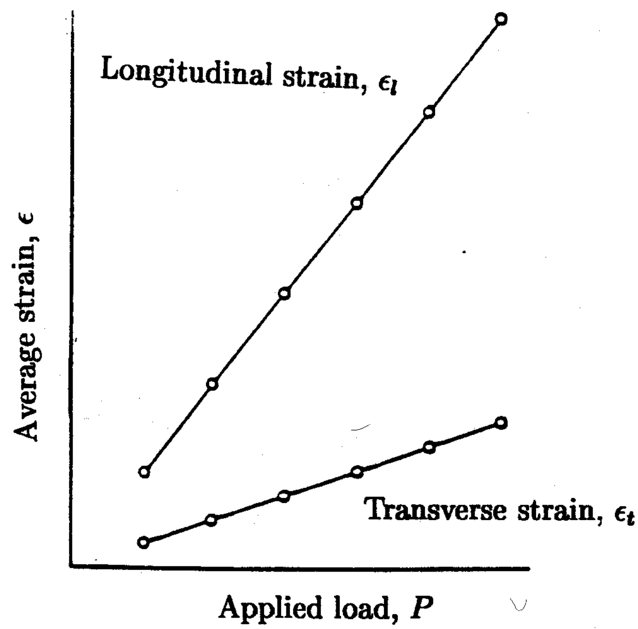

Figure 5: Axial and circumferential strains versus load plots.

3 Anisotropic Elasticity

For the two tensile specimens cut out from the walls of an Aluminum soda can,[3] we should be able to evaluate the elastic anisotropy of rolled aluminum. Two sets of Young's modulus and Poisson's ratio can be determined both in the rolling and transverse directions.

For the wall of a soda can (thin sheet under plane stress conditions), we can assume plane stress and relate the in-plane stress components to the in-plane strain components using an orthotropic version of Hooke's law, i.e.,

$$\epsilon_{zz} = \frac{\sigma_{zz}}{E_z} - \frac{\nu_{\theta z}}{E_\theta}\sigma_{\theta\theta} \tag{10}$$

$$\epsilon_{\theta\theta} = \frac{\sigma_{\theta\theta}}{E_\theta} - \frac{\nu_{z\theta}}{E_z}\sigma_{zz} \tag{11}$$

$$\gamma_{z\theta} = \frac{\tau_{z\theta}}{G_{z\theta}} \tag{12}$$

Here E_z and E_θ are Young's moduli measured in the z (axial) and θ (hoop) directions, respectively. The Poisson's ratios are now

$$\nu_{ij} = -\frac{\epsilon_{jj}}{\epsilon_{ii}} \tag{13}$$

where the stress is applied in the i direction.

In our two tensile experiments, therefore, we will be measuring $E_z, E_\theta, \nu_{z\theta}$ and $\nu_{\theta z}$.

[3] One in the rolling direction of the aluminum sheet and another in the transverse direction.

4 Procedure:

- Week I:
  - The TA will introduce the fundamentals of resistance-type strain gages.
  - The TA will demonstrate how strain gages are applied, wired and conditioned (using both the $6061-T6$ Aluminum compression specimen and the soda Aluminum can).
  - Each student will be given an Aluminum can and she/he will apply and wire a rosette (this is a time consuming process and some of you will have to complete it outside the regular Lab time – but before Week II of your Lab).
- Week II:
  - The TA will conduct the two strain gage instrumented tensile tests for the specimens cut out of the walls of the soda can. Load/strain data will be collected for further analysis.
  - The TA will demonstrate how the instrumented soda can be used to evaluate the soda pressure from measurements of strains after opening the can. The TA will balance the bridge and release the pressure. The compressive strains read are due to the unloading of the can walls by releasing the pressure.
  - Each student will repeat the above experiment using their soda can that already has been prepared with bonded and wired rosettes. Be sure that your gages have been prepared carefully since this is the only chance you have to run this experiment!
- Week III
  - The TA will conduct the compression test for the 6061-T6 Aluminum specimen using the Instron machine. Small strain data will be obtained using a rosette at some arbitrary angle and collected using a PC running *LabView*. The Instron load cell will be loaded under displacement control conditions up to a specified amount of strain.
  - Each of two groups of students will repeat the above experiment and collect the strain/load data for further processing.

5 Your Written Report

In addition to briefly summarizing the performed experiments and procedures, your report should include the following:

- A brief discussion of the procedure for mounting strain gages and how to instrument a component using them.
- The load/strain data collected from the two tensile tests performed for the wall material of the soda can. The strains reported should account for transverse sensitivity when appropriate.

- Calculation of the elastic properties of the wall material (i.e. of $E_z, E_\theta, \nu_{z\theta}$ and $\nu_{\theta z}$). Some discussion and explanation of anisotropy will be needed here.

- From your 3 strains measured (corrected for transverse sensitivity) from your rosette in the soda can experiment and using equations (6) – (8), you can calculate the radial and hoop strain components. Using Hooke's law and the measured elastic properties you can then calculate the radial and hoop stresses (consider the axial, hoop and radial stress directions to be principal stress directions!). Recall from ENGR202 that for thin wall cylinders under internal pressure $\sigma_\theta = \frac{pr}{t}$, $\sigma_z = \frac{pr}{2t}$ and $\sigma_r = 0$, respectively (section 6.9 of Beer and Johnston's *Mechanics of Materials*). Here p is the internal pressure, r is the radius of the can and t is the wall thickness. By knowing both σ_z and σ_θ we can calculate p in two ways. Average the two results if they are different. The thickness of the wall material is measured to be 0.005 in and the radius of the can $r = 1.300$ in.

- For the compression test, you should include: plots of the measured strain data versus load (correct the strain data for transverse sensitivity). Assuming that the stran gages are oriented along the loading axis and perpedicular to the loading axis, you can easily obtain the strains in the axial (ϵ_z) and circumferential (ϵ_θ) directions. Derive plots of the axial and circumferential strains vs. load in the elastic regime to determine ν. Also plot the axial strain vs. stress to determine E and the 0.2% yield strength. Compare your values of ν, E, and 0.2% yield strength to values given in the literature. Explain differences.

6 Suggested Readings

- The strain gage manual given to you in the Lab.

- The book *Instrumentation for engineering measurements*, by J.W. Dally et al. is an excellent strongly recommended reference. This book is on reserve in the Engr. library and you may find it useful while preparing your lab report.