MAE 212: Spring 2001
Module I: Deformation Processing

Abstract

An axisymmetric frictionless compression test is contacted to evaluate the hardening behavior of 2024-0 Aluminum. The uniaxial true stress/true strain data are used to calculate the parameters K and n in a strain hardening model of the form $\sigma = K\epsilon^n$. The same axisymmetric compression test but without lubrication is then performed to study the effects of friction on the efficiency of deformation processes. Finally, a plane strain forging process and a flat rolling process are conducted. The measured material responses are compared to the ones predicted by analysis for the power law material model obtained in the earlier experiments.

1 Uniaxial stress-strain data

In ENGR 202 all deformations were assumed to be very small, thus changes in cross sectional areas can be neglected. In this lab and in many materials processes the deformations are large and can no longer be neglected in the computation of stress and strain. Thus we will introduce new stress and strain measures. Let us review some simple concepts related to the uniaxial tension (or compression) test. Recall the definition of engineering and true stresses and strains as:

$$\text{Engineering Strain:,} \quad e = \frac{\ell - \ell_o}{\ell_o} \quad \text{True Strain:} \quad d\epsilon = \frac{(\ell + d\ell) - \ell}{\ell} \text{ and } \epsilon = \ell n \left(\frac{\ell}{\ell_o} \right) \tag{1}$$

$$\text{Engineering Stress:,} \quad S = \frac{\text{load}}{A_o} \quad \text{True Stress:} \quad \sigma = \frac{\text{load}}{A} = S\,(1+e) \tag{2}$$

where ℓ and A are the instantaneous length and area, respectively, and ℓ_o and A_o are the length and area of the initial (undeformed) specimen.

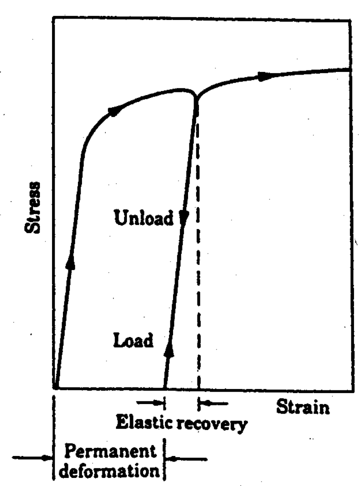

Figure 1: Schematic illustration of loading and unloading of a tensile test specimen. Note that during unloading the curve follows a path parallel to the original elastic slope.

If the material is loaded in tension or compression beyond the initial yield strength, the stress-strain curve becomes non-linear and the material behavior becomes elastic-plastic (see

Figure 1). In this region of deformations, the material will not return to its original state upon unloading and permanent deformation will remain. Note that the yield strength of the material (flow stress) is increasing as the material is deformed (think what will happen if you unload a specimen that was passed his initial yield strength and then you re-load, see Fig. 1).

At large ($> 10\%$) strains, elasticity may be ignored and the material behavior is typically well-characterized by a power law hardening equation, i.e.,

$$\sigma = K\epsilon^n \tag{3}$$

In equation (1), K is known as the strength coefficient and n is known as the strain hardening exponent (see Fig. 2). The first part of this lab deals with determining K and n for 2024-0 (annealed) aluminum by conducting a uniaxial compression test and analyzing the data.

2 Multiaxial yielding

In a uniaxial stress state (one normal stress component), one compares the stress level with the current yield strength (flow stress) to determine whether yielding has occurred. Similarly at large strains, equ. (1) relates the true normal strain to the true normal stress level.

How do we account for multiple stress components with respect to yielding? As it will be discussed in class, we use the concept of a yield criterion and introduce an effective stress from the stress components that is compared to the uniaxial yield strength. Using the von Mises criterion, the effective stress is defined as

$$\bar{\sigma} = \frac{1}{\sqrt{2}} \left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right) \right]^{\frac{1}{2}} \tag{4}$$

(See Beer and Johnston's *Mechanics of Materials*, p. 79 to review the notation of stresses in three dimensions. We can also define an effective strain, i.e.,

$$\bar{\epsilon} = \frac{\sqrt{2}}{3} \left[(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2 \right]^{\frac{1}{2}} \tag{5}$$

One can also hypothesize the existence of a "universal stress/strain curve" by replacing σ and ϵ in equ. (1) with their effective counterparts, i.e.,

$$\bar{\sigma} = K\bar{\epsilon}^n \tag{6}$$

In essence, this expression proposes that the effective stress-strain curves for any deformation state will overlay the uniaxial stress/strain curve since by equs. (4) and (5), $\bar{\sigma} = \sigma$ and $\bar{\epsilon} = \epsilon$ for a uniaxial state of deformation.[1]

[1] We will employ the concept of a universal stress/strain curve later in this Module to predict the material behavior during plane strain compression using the constants K and n determined from the uniaxial compression tests. We can also employ the same idea to predict the separating force and rolling torque in the flat rolling experiment.

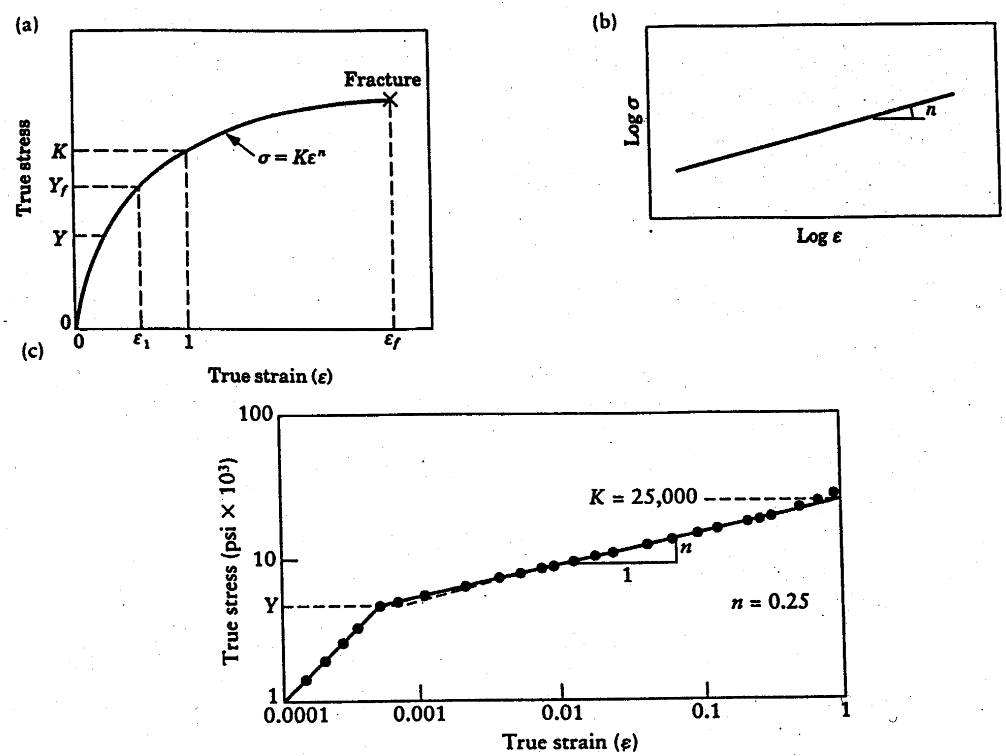

Figure 2: (a) True stress-true strain curve in tension. Note that, unlike in an engineering stress-strain curve, the slope is always positive and that the slope decreases with increasing strain. The total curve can be approximated by the power law shown, (b) True stress-true strain on a log-log scale. Here, Y is the initial yield stress and Y_f the flow stress of the material (c) True stress-true strain curve in tension for 1100-0 aluminum plotted on a log log scale (reproduced from S. Kalpakjian, Manufacturing Processes for Engr. Materials).

3 Effects of friction in deformation processing

In the absence of friction, the deformation of a workpiece in a forming process tends to be homogeneous and all the work performed by the external loads is used to uniformly deform the material (see Fig. 3b). In addition to generation of heat, friction in the workpiece/tool interface contributes to the so called redundant (non-homogeneous) deformation (see Fig. 3c). The stresses and strains in the workpiece are no longer uniform, hence, there is no simple relationship between the stresses and strains in the workpiece.

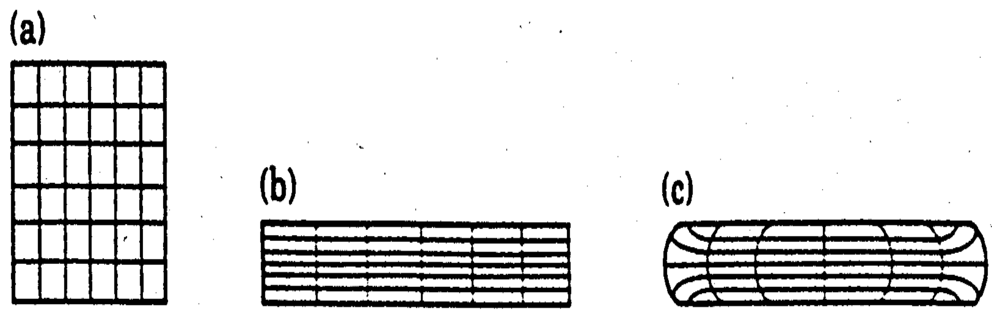

Figure 3: Deformation of grid patterns during forging of a workpiece: (a) Original pattern (b) after ideal (homogeneous) deformation and (c) after inhomogeneous deformation requiring redundant work of deformation. The case (c) requires greater work of deformation than (b).

The ideal work for a particular process is that work which will take the workpiece from an initial shape to a final shape via a uniform, frictionless deformation (Fig. 3a). The ideal work per unit volume is given as follows:

$$w_i = \int_0^{\bar{\epsilon}_f} \bar{\sigma} d\bar{\epsilon} \tag{7}$$

where $\bar{\epsilon}_f$ is the imposed equivalent strain at the end of the process.

In real processes there are losses due to friction and nonhomogeneous deformation (redundant work). So the total work (per unit volume) is given by

$$w_t = w_i + w_f + w_r \tag{8}$$

where w_f = frictional work and w_r = redundant work.

The work contributions w_r and w_f are difficult to quantify and we generally calculate the total work by introducing a process efficiency factor η, i.e.

$$w_i = \eta w_t \tag{9}$$

For the whole volume of the material, we can similarly write the following:

$$W_i = \eta W_t \tag{10}$$

For simple processes like an axisymmetric forging process, we can estimate the efficiency η by calculating the work terms W_i and W_t. The ideal work is calculated by integrating the load-displacement curve in a frictionless forging process, whereas W_t is evaluated similarly for an axisymmetric forging process with an unlubricated workpiece/die interface.

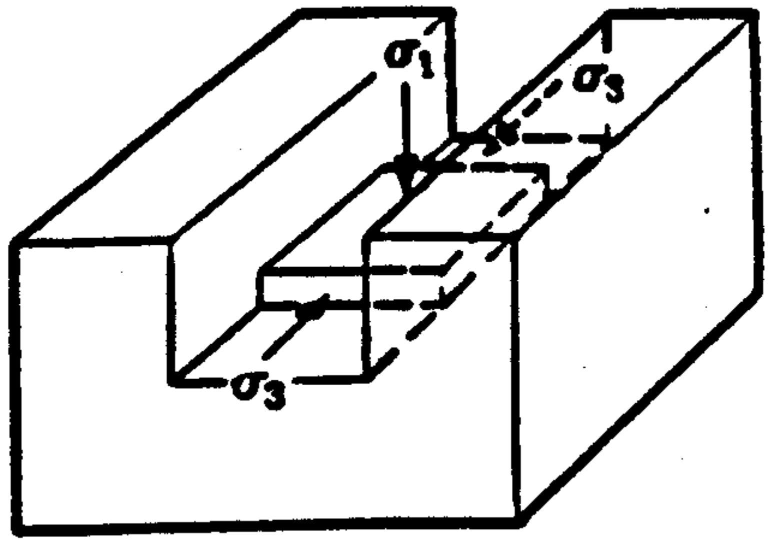

Figure 4: A plane strain forging process. Note that the strain $\epsilon_2 = 0$ and due to incompressibility ($\epsilon_1 + \epsilon_2 + \epsilon_3 = 0$), we can derive that $\epsilon_3 = -\epsilon_1$. Using equation (5), this results in an effective strain of $\bar{\epsilon} = \frac{2}{\sqrt{3}}|\epsilon_1|$. In class, we will show that as a result of $\epsilon_2 = 0$, the stress components are constrained such that $\sigma_2 = \frac{1}{2}(\sigma_3 + \sigma_1)$. Using equ. (4), the equivalent stress can be written as: $\bar{\sigma} = \frac{\sqrt{3}}{2}|\sigma_1 - \sigma_3|$. If $\sigma_3 = 0$ (uniaxial compression), then: $\bar{\sigma} = \frac{\sqrt{3}}{2}|\sigma_1|$.

4 Plane strain forging

Plane strain refers to two-dimensional deformations which take place within parallel planes. In Fig. 4 deformation occurs in the x_1, x_3 plane, while the strain in the x_2 direction (perpendicular to the channels of the die) is zero (like many applications in this course, we work with principal stress/strain axes only, i.e. $1, 2, 3$ are principal stress/strain axes). All stresses and strains are functions of (x_1, x_3) only. Let us here consider the plane strain forging process shown in Fig. 4. We will discuss in class that $\bar{\sigma}$ and $\bar{\epsilon}$ for plane strain conditions are given as follows:

$$\bar{\sigma} = \frac{\sqrt{3}}{2}|\sigma_1 - \sigma_3| \tag{11}$$

and

$$\bar{\epsilon} = \frac{2}{\sqrt{3}}|\epsilon_1| \tag{12}$$

For the plane strain conditions shown in Figure 4, the loading direction is in the 1 direction with the 3 direction being free, hence $\sigma_3 = 0$ and

$$\bar{\sigma} = \frac{\sqrt{3}}{2}|\sigma_1| \tag{13}$$

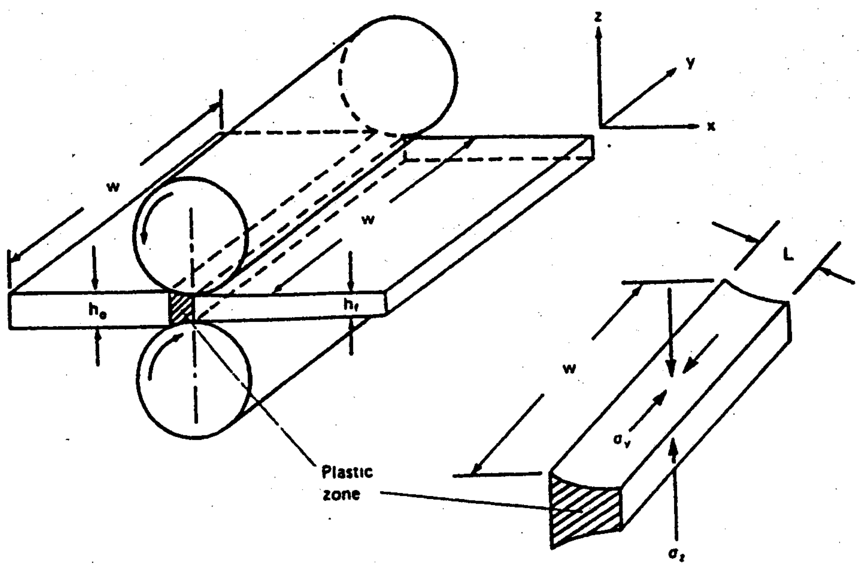

Figure 5: Schematic of flat rolling showing the deformation zone.

5 Flat Rolling

Flat rolling is the process of reducing the thickness of a workpiece by compressive forces applied through a set of rotating rolls. In flat rolling of plates, sheets or strips, we assume plane strain conditions since little widening of the workpiece results.

Important Process Parameters:

- The thickness reduction r, where $r = \frac{h_o - h_e}{h_o}$.
- The friction coefficient μ
- The radius and material properties of the rolls.
- The externally imposed front, σ_f, and back, σ_b, tension stresses.

Maximum possible thickness reduction:

There is an r_{max} that can be obtained in a given rolling process for a given material. More than one pass may be necessary to achieve the desired thickness reduction. This max reduction depends on the process parameters and the workpiece material properties (e.g. K and n for the case of a power law hardening model).

The developments shown in Figures 5–10 will be discussed in class and it it not essential that you understand all the details given here. Your TA will present a simple slab analysis method (see Fig. 7) that allows you to obtain the pressure distribution (shown in Figs. 8–10) as well as to calculate the separating force and torque. The required input variables to this analysis, are the initial and final thicknesses of the strip, the width, the roll diameter, K, n, the coefficient of friction, and the front and back tensions.

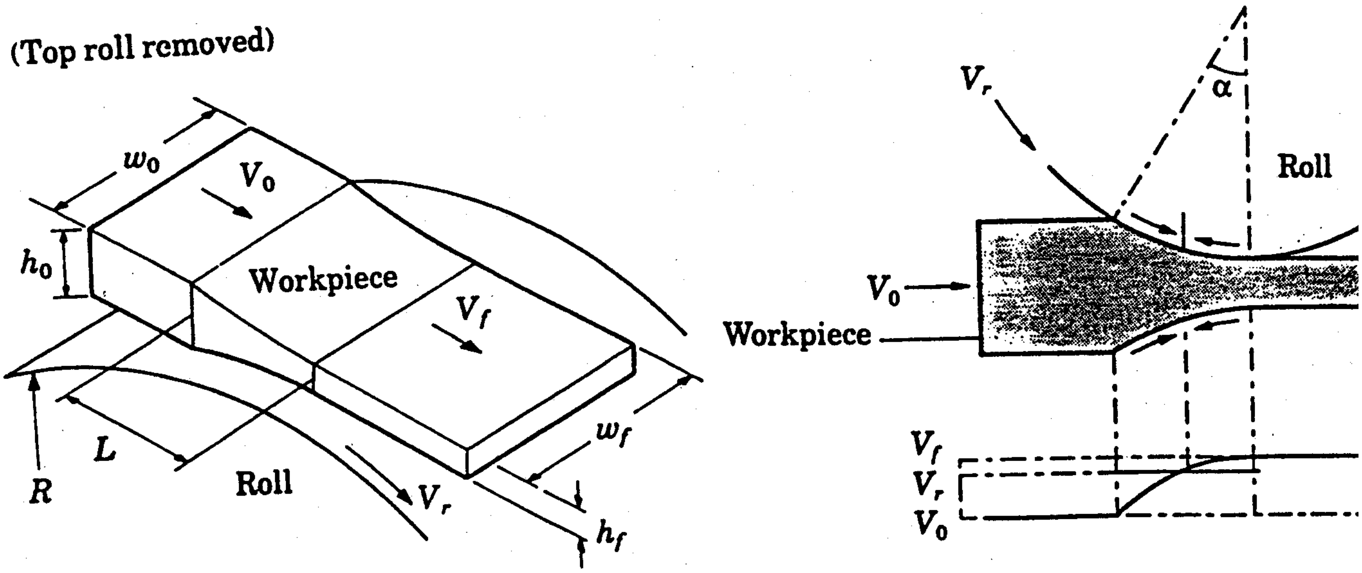

Figure 6: Relative motion between the workpiece and the rolls. At the neutral point N, the relative motion between the rolls and the workpiece is zero.

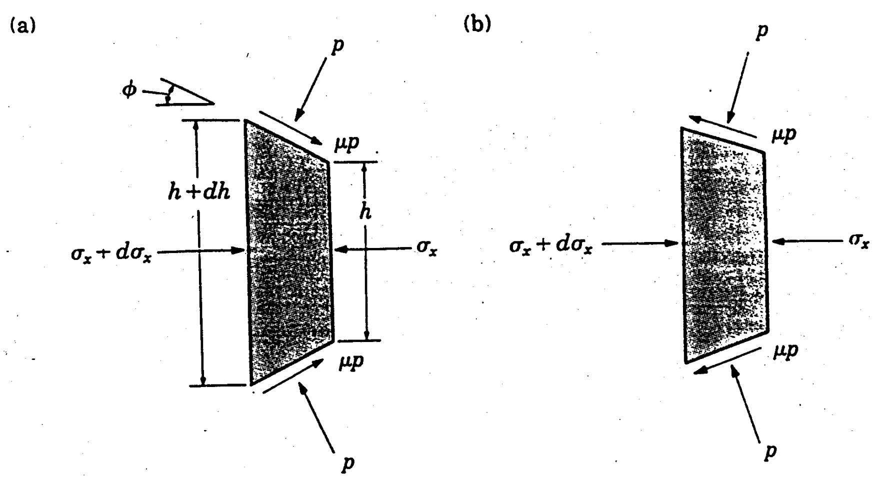

Figure 7: Stresses on an element in rolling: (a) entry zone and (b) exit zone. Explain the direction of the frictional forces based on Fig. 6b.

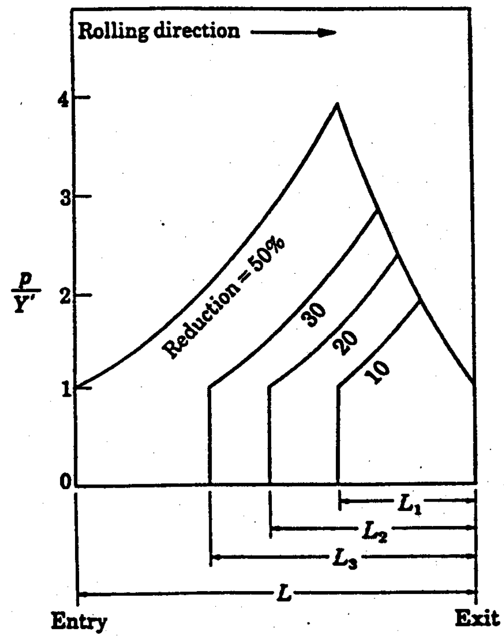

Figure 8: Pressure distribution in the roll gap as a function of reduction in thickness. Note the increase in the area under the curves with increasing reduction in thickness, thus increasing the roll-separating force (here $Y' = 2\frac{average\ flow\ stress}{\sqrt{3}}$).

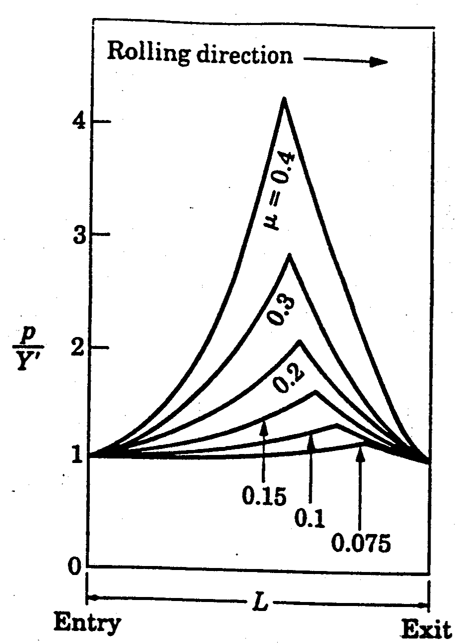

Figure 9: Pressure distribution in the roll gap as a function of the coefficient of friction. Note that, as friction increases, the neutral point shifts toward the entry. Without friction, the rolls slip and the neutral point shifts completely to the exit (here $Y' = 2\frac{average\ flow\ stress}{\sqrt{3}}$).

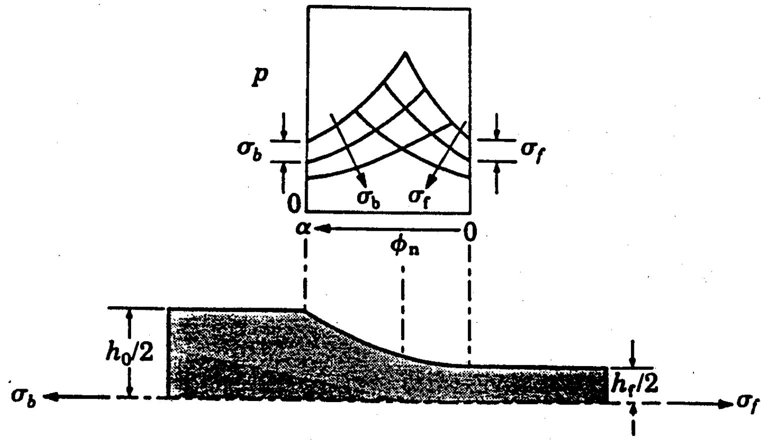

Figure 10: Effect of the front and back tension in the friction hill in rolling. The location of the neutral point as well as the average pressure are being affected by the front tension σ_f and back tension σ_b.

6 Procedure

In this laboratory experiment, we will use the Instron servohydraulic test machine. It consists of:

- A digital controller
- A load frame including grips
- Two test transducers; load cell for load measurements, linear variable differential transformer (LVDT) for displacement measurement
- Hydraulic pump and actuator
- A servovalve

A PC computer is used with the *Labview* software to issue commands to the Instron's controller and to acquire data. The computer and controller communicate via the General Purpose Interface Bus (GPIB) using a GPIB card installed in the PC. Load and displacement data are stored for later analysis. We will use the LVDT to measure displacement in this large strain test. Displacement data are converted to strains via equ. (1). The schedule for this three-week long Module is the following:

- Week I
  - The TA will define the true stress and true strain and discuss the uniaxial stress/strain curve.
  - The TA will introduce the hardening relation $\sigma = K\epsilon^n$ and explain how the coefficients K and n can be obtained from uniaxial load/displacement data.
  - The TA will introduce the Instron machine and demonstrate its use with a frictionless axisymmetric compression test (using a grooved lubricated workpiece). We here lubricate the specimen surfaces to assure a uniform uniaxial stress state

in the material. Throughout the specimen $\bar{\sigma} = \sigma_{Ax}$ and $\bar{\epsilon} = \epsilon_{Ax}$ where σ_{Ax} is the true axial stress, $\frac{P}{A}$, and ϵ_{Ax} is the true axial strain, $\ell n\left(\frac{h}{h_o}\right)$ (h and h_o denote the current and initial heights of the workpiece). We can calculate σ_{Ax} and ϵ_{Ax} from the load-displacement data using equs. (1) and (2). Equation (3) can be re-written as follows:

$$\log \sigma = \log K + n \log \epsilon \tag{14}$$

The measured $\log \sigma$ versus $\log \epsilon$ data can be easily fit to the straight line given by the equation above and the coefficients K and n can then be identified – the strain hardening exponent, n, as the slope and $\log K$ as the y intercept of this line .

The constant true strain rate for this test is selected as $10^{-2}s^{-1}$.[2]

  - The students participating in this Lab will be divided in two groups and each group will repeat this experiment and collect the load/displacement data for further processing.

- Week II
  - The TA will discuss the effects of friction and demonstrate them with an axisymmetric compression test for an ungrooved, unlubricated specimen.
  - The TA will discuss the concepts of equivalent stress and strain and the hardening relation $\bar{\sigma} = K\bar{\epsilon}^n$.
  - To examine the universal nature of the material behavior described by equ. (6), a specimen of the same material as that used in Week I will be subjected to a plane strain forging process using a constant effective strain rate of $10^{-2}s^{-1}$. We are again limited, however, to a constant displacement rate. By equation (12), therefore, we must multiply the displacement rate used in uniaxial compression by $\frac{\sqrt{3}}{2}$.
  - Each group of students will repeat this experiment and collect the appropriate data for further processing.

- Week III
  - The TA will discuss the fundamental aspects of flat rolling including the effects of friction, roll geometry and front and back tensions.
  - The TA will review the basic results of a slab analysis for flat rolling and present the results for the pressure distribution and the separating force. The universal $\bar{\sigma} = K\bar{\epsilon}^n$ curve obtained from the earlier experiments is needed for these calculations.
  - The TA will describe an instrumented rolling mill and flat roll strips of 2024-0 Aluminum. Load washers on the mounting of the rollers detect the separating force and a torque cell mounted on a coupling to the motor senses the torque applied to one of the rolls. A data acquisition board is used to record separating

[2] Since we can only specify a constant displacement rate of the ram $\left(\dot{h}\right)$, we must here settle for a constant engineering strain rate. We must calculate the displacement rate $\left(\dot{h}\right)$ that corresponds to $\dot{e} = 10^{-2}s^{-1}$ as well as the test duration as part of the required setup procedure in *Labview*.

force and torque data. The board is addressed using a *Lab View* virtual instrument. The actual rolling mill is currently not functional and so the TA will describe but not physically run this experiment.

7 Your Written Report

A summary of the experiments performed in this Module should be given together with a brief description of the equipment used.
Your report should also include the following:

- The collected load/displacement data for each experiment with clear, properly labeled and scaled axes.
- The calculated true stress/true strain data for the frictionless axisymmetric compression test plotted as above.
- A calculation of the coefficients K and n from the above data.
- A calculation of the efficiency η for the frictional axisymmetric compression test. The actual work for this process can be evaluated using the area under the load/displacement curve (i.e. by performing an integration!). The ideal work is the area under the load displacement curve for the frictionless axisymmetric compression test. We can now calculate η using equ. (10).
- From the load and displacement data obtained during the plane strain frictionless compression test, calculate $\bar{\epsilon}$ and $\bar{\sigma}$ using equations (12) and (13). Compare this data to the universal stress/strain prediction by plotting Equation (6) on the same graph. Discuss the possible reasons for the discrepancies you observe.
- Describe in brief the flat rolling process. Using the power law model (K and n) determined in your earlier experiments and the expression for the pressure distribution in flat rolling given to you by your TA, plot from the entrance to the exit the pressure distribution for coefficient of friction of $\mu = 0.4$ and various values of the front and back tensions. Confirm with your calculations the results of Fig. 10 regarding the location of the neutral point and calculate the roll separating force. You TA will provide you with values for any needed parameters not given here (initial and final thickness of the strip, roll radius, etc.).