MAE 212: SPRING 2001
HW 9 SOLUTIONS

1. THE ACTUAL WORK IS $w_a = w_i + w_s + w_r = w_i + 0.4 w_i$

$\Rightarrow w_a = 1.4\, w_i$ AND $\eta = \frac{1}{1.4}$

FROM NOTES, $\sigma_d = \frac{K \epsilon_h^{n+1}}{(n+1)\eta}$ AND YIELD STRESS AT THE EXIT (FLOW STRESS) $= K \epsilon_h^{n}$.

SO MAX REDUCTION IS OBTAINED WHEN $\sigma_d = K\epsilon_h^n$

$$\Rightarrow \quad \epsilon_h^* = \eta(n+1) = \frac{1}{1.4}(0.5+1) = 1.07$$

HENCE, $r_{max} = 1 - e^{-\epsilon_h^*} = 1 - e^{-1.07}$ AND $\boxed{r_{max} = 66\%}$

3. AS FOR EXERCISE 1, WE HAVE $w_a = 1.25\, w_i \rightarrow \eta = 0.80$

$$w_i = \int_0^{\epsilon_h} \bar\sigma\, d\bar\epsilon = \int_0^{\epsilon_h} (5000 + 25{,}000\,\bar\epsilon)\, d\bar\epsilon = 5000\,\epsilon_h + 12{,}500\,\epsilon_h^2$$

$$\sigma_d = w_a = 1.25\, w_i = 6{,}250\,\epsilon_h + 15{,}625\,\epsilon_h^2$$

YIELD STRESS AT THE EXIT $= 5{,}000 + 25{,}000\,\epsilon_h$

MAX REDUCTION:

$\sigma_d =$ YIELD STRESS AT EXIT

$$\rightarrow \quad 15{,}625\,\epsilon_h^{*2} - 18{,}750\,\epsilon_h^* - 5000 = 0$$

$$\rightarrow \quad \epsilon_h^* = 1.425$$

AND $\epsilon_h^* = 2 \ln \frac{D_o}{D_f} \rightarrow D_f = D_o\, e^{-\epsilon_h^*/2} = 0.2\, \exp\left(\frac{-1.425}{2}\right)$

$$\rightarrow \boxed{D_f = 0.098 \text{ IN}}$$

Exercise 2

~~We need to reduce the copper wire with $\epsilon_f = 2\ell n\frac{D_o}{D_f} = 2\ell n\frac{0.025}{0.010}$~~ $=$ 1.8326. The efficiency of the process is $\eta = 0.75\,(0.333\epsilon_h + 0.683)$ and we would like $\sigma_d \leq 0.60$Y.S. (yield stress), where Y.S.=Y ($n = 0$, no work hardening). This gives $\sigma_d = \frac{Y\epsilon_h}{\eta}$ and for maximum reduction we are asked to take: $\sigma_d = 0.6Y$, from which we conclude that:

$\epsilon_{h^*} = 0.60\eta$ for each pass.

$$\begin{aligned}\epsilon_{h^*} &= 0.60 \times (0.333\epsilon_{h^*} + 0.683) \times 0.75 \\ \rightarrow \epsilon_{h^*} &= \frac{0.60 \times 0.75 \times 0.683}{1 - 0.60 \times 0.75 \times 0.333} = 0.3615\end{aligned}$$

Each pass is going to have a maximum strain of 0.3615. Thus we will need $\frac{1.8326}{0.3615} = 5.07$, i.e., **6 passes**.

Exercise 4

1. From notes, with $\bar{\sigma} = Y$ (no hardening).

$$P_c = \frac{1}{\eta}\int_0^{\bar{\epsilon}_h} \bar{\sigma}\, d\bar{\epsilon} = \frac{Y\epsilon_h}{\eta} = \frac{Y}{\eta}2\ell n\frac{D_o}{D_f} = \frac{2 \times 10}{0.5}\ell n\frac{4}{1}$$

and

$$\boxed{P_e = 55.45 \text{ ksi}}$$

2. The material is plastically flowing, thus yielding implies that at the die entrance, the following condition must be satisfied:

$$|\sigma_r - \sigma_{axial}| = Y \quad (1)$$

But $\sigma_{axial} = -p_{extrusion}$ and $\sigma_r = -p_{lateral}$, from which

$$Y = |-P_{\text{extrusion}} + P_{\text{lateral}}|$$

i.e., $Y = P_e - P_{\text{lateral}}$ (the other solution with different signs of | | will give a higher lateral pressure)

from which we calculate that $P_{lateral} = P_e - Y = 55.45 - 10 = 45.45\ ksi$.

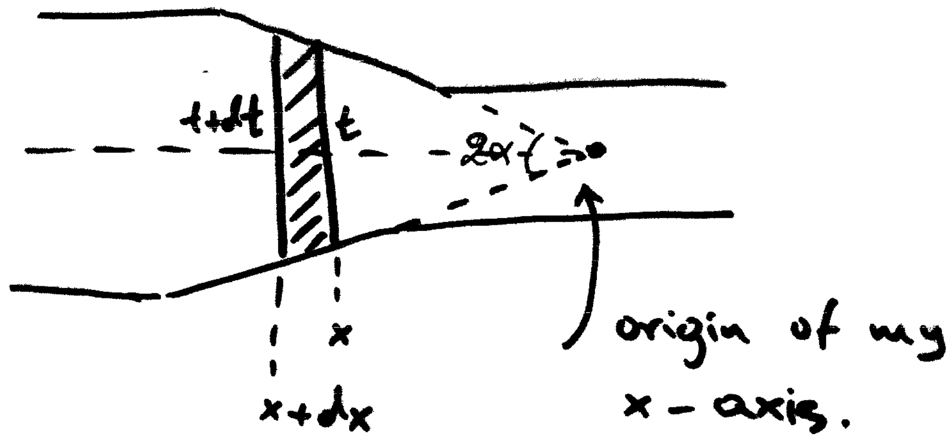


Figure 1: Slab analysis for plane strain drawing (Example 5).

3. Let us assume that the die is a thin-wall tube. For thin tubes under internal pressure (here the lateral pressure), we have: $\sigma_r = 0, \sigma_\theta = \frac{pr}{t}$ and $\sigma_z = \frac{pr}{2t}$. Using von Mises yield criterion, we can write that yield of the die occurs when $\bar{\sigma}_{\text{VM}} = Y_{\text{die}}$:

$$\begin{aligned} \bar{\sigma}_{\text{VM}} &= \frac{1}{\sqrt{2}}\sqrt{(\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_z)^2 + (\sigma_z - \sigma_r)^2} \\ \bar{\sigma}_{\text{VM}} &= \frac{\sqrt{3}}{2}\frac{p_{lateral}\ r}{t} \ \text{and } \bar{\sigma}_{\text{VM}} = Y_{\text{die}} \rightarrow t_{\min} = \frac{\sqrt{3}p_{lateral}\ r}{2Y_{\text{die}}} \end{aligned}$$

and

$$\boxed{t_{\min} = \frac{\sqrt{3}\times 45.45\times 2}{2\times 100} = 0.787 \text{ in}}$$

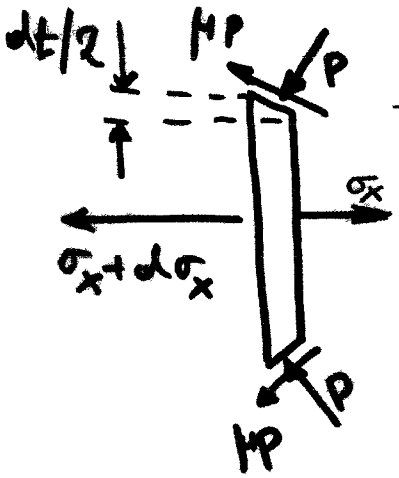
Remark: This is not exactly a thin-wall tube!

Exercise 5

We need to determine the drawing stress to achieve the desired deformation and to do so we have to use the slab analysis method (the derivation is given here as a review of what was done in class):

Balance of forces in the x-direction:

$$(\sigma_x + d\sigma_x)\, w\, (t + dt) - \sigma_x w t + 2\,(P \sin\alpha)\, w dl + 2\,(\mu P \cos\alpha)\, w dl = 0$$

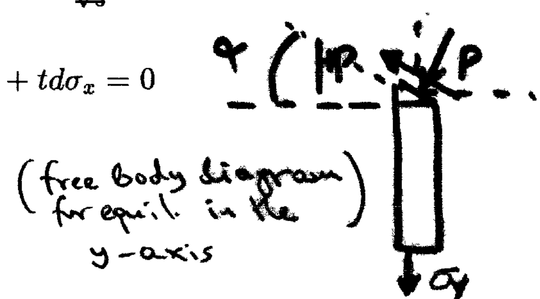


$$\rightarrow \sigma_x dt + t d\sigma_x + P dt + \mu P \frac{\cos\alpha}{\sin\alpha} dt = 0 \,(\text{neglect } d\sigma_x \times dt)$$

$$\rightarrow [\sigma_x + P\,(1 + B)]\, dt + t d\sigma_x = 0\,(1)\,, \text{ with } B = \mu\cot\alpha$$

For plane strain drawing, the von Mises criterion gives $\rightarrow \sigma_x + P = \frac{2Y}{\sqrt{3}}$ (from equilibrium equ. in the y axis we approximate $\sigma_y = -P$). Then, $P = \frac{2Y}{\sqrt{3}} - \sigma_x$ and equ. (1) becomes:

$$\left[-B\sigma_x + \frac{2Y}{\sqrt{3}}(1+B)\right] dt + t d\sigma_x = 0$$



which gives: $\boxed{\frac{d\sigma_x}{B\sigma_x - \frac{2Y}{\sqrt{3}}(1+B)} = \frac{dt}{t}}$

This is the slab equilibrium equation in the x-axis. We have to integrate it to get the drawing stress σ_d:

$$\frac{d\sigma_x}{B\sigma_x - \frac{2Y}{\sqrt{3}}(1+B)} = \frac{1}{B}\frac{d\left(\frac{\sqrt{3}B\sigma_x}{2Y(1+B)} - 1\right)}{\left(\frac{\sqrt{3}B\sigma_x}{2Y(1+B)} - 1\right)} = \frac{1}{B} d\left(\ell n\left(\frac{\sqrt{3}B\sigma_x}{2Y(1+B)} - 1\right)\right) = \frac{dt}{t} = d(\ell n t)$$

Before deformation, $t = t_o, \sigma_x = 0$.

At end of deformation: $t = t_f, \sigma_x = \sigma_d$.

$$\rightarrow \frac{1}{B}\left[\ell n(-1) - \ell n\left(\frac{\sqrt{3}B\sigma_d}{2Y(1+B)} - 1\right)\right] = \ell n\frac{t_o}{t_f} = \epsilon_h$$

So, $\frac{\sqrt{3}B\sigma_d}{2Y(1+B)} - 1 = -e^{-B\epsilon_h}$

Finally, $\boxed{\sigma_d = \frac{2Y}{\sqrt{3}}\frac{(1+B)}{B}\left(1 - e^{-B\epsilon_h}\right)}$

This stress is applied at the exit of the die, so the power can be calculated as follows:

$$\mathcal{P} = (\mathrm{wt_f}\sigma_\mathrm{d}) \times (\mathrm{speed})$$

$$\mathcal{P} = \mathrm{wt_f}\frac{2\mathrm{Y}}{\sqrt{3}}\frac{(1+\mathrm{B})}{\mathrm{B}}\left(1 - \mathrm{e}^{-\mathrm{B}\epsilon_\mathrm{h}}\right) \times \text{ speed}$$

with $B = \mu \cot\alpha = 0.427, \epsilon_h = \ell n\frac{t_o}{t_f} = 0.223$.

So, $\boxed{\mathcal{P} = 876,000 \text{ lbf ft/min } = 175,000 \text{ lbf in/s}}$

Exercise 6

Again we use the slab analysis method for a cylindrical billet in open die forging. Based on the results given in your text, the average pressure in the die/workpiece interface can be approximated as follows:

$$p_{ave} = Y[1 + \frac{\mu D}{3h}] \qquad (2)$$

where, D is the diameter at height h and Y is the yield stress of the material at height h.

From conservation of volume, we can write: $D_o^2 h_o = D^2 h$ or $D^2 h = 1^2\ 1\ in^2 = 1\ in^2$, or $D = \frac{1}{\sqrt{h}} = \frac{1}{\sqrt{1-r}}$, where r is the percentage of height reduction, i.e. $r = \frac{h_o - h}{h_o} = 1 - h$.

For an axisymmetric forging, the equivalent stress is (in absolute value) equal to the axial strain (ϵ_h), while the yield stress for a material obeying $\bar{\sigma} = K\bar{\epsilon}^n$ is given as:

$$\text{Yield stress} = K\epsilon_h^n = K(\ell n \frac{1}{1-r})^n \qquad (3)$$

Finally, the force can be calculated as follows:

$$F = p_{ave}\frac{\pi D^2}{4} = Y[1 + \frac{\mu D}{3h}]\frac{\pi D^2}{4} = K(\ell n \frac{1}{1-r})^n\ [1 + \frac{\mu \frac{1}{\sqrt{1-r}}}{3(1-r)}]\ \frac{\pi \frac{1}{1-r}}{4} \qquad (4)$$

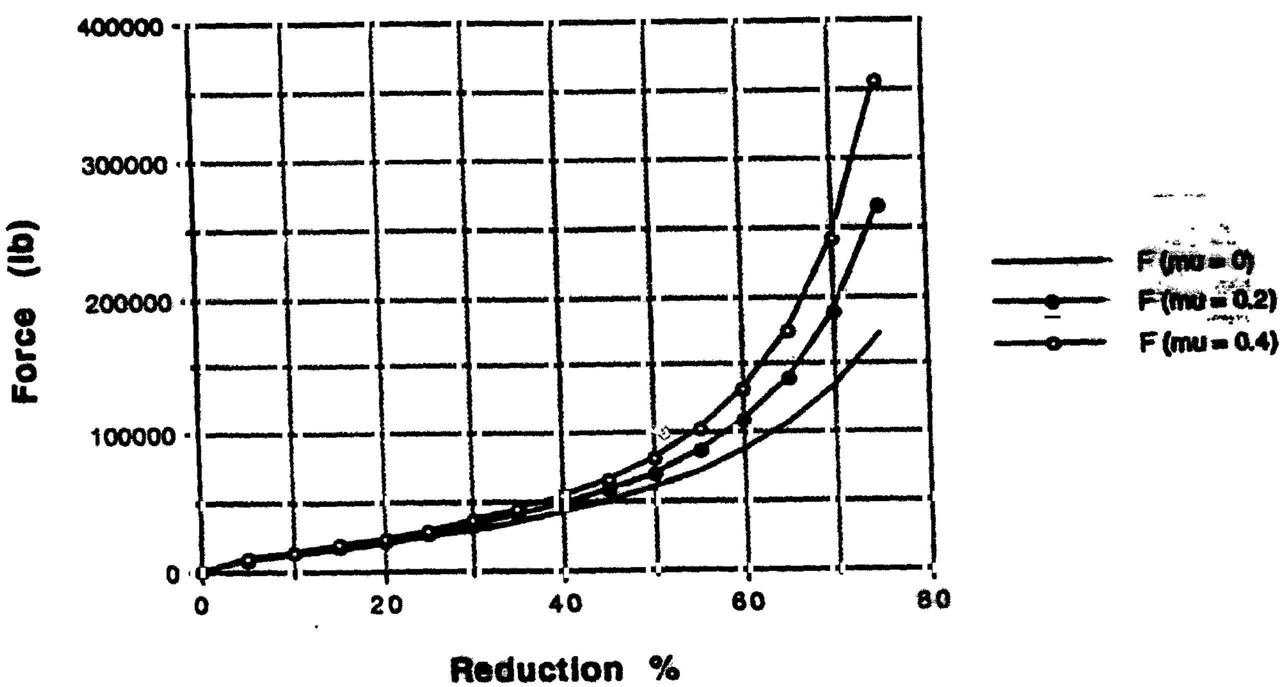
or finally:

$$F = K(\ell n \frac{1}{1-r})^n\ [\frac{1}{1-r} + \frac{\mu}{3(1-r)^{5/2}}]\ \frac{\pi}{4} \qquad (5)$$

Based on the above formula and data and for different μ, one can calculate the reduction/force data and the corresponding plots.

Reduction r %	Force (lb) No friction	Force (lb) mu = 0.2	Force (lb) mu = 0.4
10.00	11908.35787	12838.17352	13767.98918
20.00	20090.64522	21962.48058	23834.31593
30.00	29578.59171	32945.56114	36312.53067
40.00	41895.20514	47904.81373	53914.42231
50.00	59282.16726	70460.51992	81638.87258
60.00	86156.15621	108860.2969	131564.4376
70.00	133125.1403	187136.7273	241148.3142
75.00	172389.2839	264330.2353	356271.1868

Force Vs Reduction in Height



Exercise 7

1. For axially symmetric problems we assume that $\sigma_r = \sigma_\theta$. Here, $\sigma_r = \sigma_\theta = -p$, where p is the pressure in the die/workpiece interface.

   With substitution to the von-Mises criterion, we have:

$$\bar{\sigma}_{\text{VM}} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_x)^2 + (\sigma_x - \sigma_r)^2}$$
$$\bar{\sigma}_{\text{VM}} = \sigma_x + p \quad \rightarrow \sigma_x + p = Y$$

2.

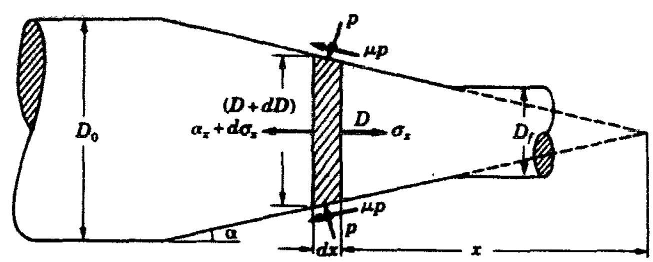


FIGURE Stresses acting on an element in drawing of a solid cylindrical rod or wire through a conical converging die.

In drawing, friction increases the drawing force, because work has to be supplied to overcome that friction. Using the slab method of analysis and on the basis of Fig. , we equate the forces in the horizontal direction as follows:

$$(\sigma_x + d\sigma_x)\frac{\pi}{4}(D + dD)^2 - \sigma_x\frac{\pi}{4}D^2 + p\frac{\pi D\,dx}{\cos\alpha}\sin\alpha + \mu p\frac{\pi D\,dx}{\cos\alpha}\cos\alpha = 0.$$

Simplifying and ignoring the second-order terms, we obtain

$$D\,d\sigma_x + 2\sigma_x dD + 2p\left(1 + \frac{\mu}{\tan\alpha}\right)dD = 0.$$

Note that we have two unknowns (σ_x and p) but only one equation. We obtain the second equation from yield criteria, recognizing that this is an axisymmetric case and that both the maximum shear-stress and the distortion-energy criterion give the relationship

$$\sigma_x + p = Y.$$

Letting $\mu/\tan\alpha = B$ and using the above relationships, we now obtain

$$\frac{dD}{D} = \frac{d\sigma_x}{2B\sigma_x - 2Y(1 + B)}.$$

Integrating this equation between the limits D_f and D_o—and by noting that at $D = D_o$, $\sigma_x = 0$, and at $D = D_f$, $\sigma_x = \sigma_d$—yields

$$\sigma_d = Y\frac{1 + B}{B}\left[1 - \left(\frac{D_f}{D_o}\right)^{2B}\right] \Rightarrow \sigma_d = Y\frac{1 + B}{B}\left[1 - \left(\frac{A_f}{A_o}\right)^{B}\right]$$
$$= Y\left(1 + \frac{\tan\alpha}{\mu}\right)\left[1 - \left(\frac{A_f}{A_o}\right)^{\mu\cot\alpha}\right]. (*)$$

3. Equ. * above can be written in terms of ϵ_h as follows:

$$\sigma_d = Y\frac{1+B}{B}[1 - e^{-B\epsilon_h}] \quad (6)$$

4. The exponential $e^{-B\epsilon_h} = 1 - B\epsilon_h + \frac{B^2\epsilon_h^2}{2} - \ldots$. So we can write:

$$\sigma_d = Y\frac{1+B}{B}[1 - e^{-B\epsilon_h}] = Y\frac{1+B}{B}[1 - (1 - B\epsilon_h + \frac{B^2\epsilon_h^2}{2} - \ldots)] \quad (7)$$

or

$$\sigma_d = Y\frac{1+B}{B}[B\epsilon_h - \frac{B^2\epsilon_h^2}{2} + \ldots)] = Y(1+B)[\epsilon_h - \frac{B\epsilon_h^2}{2} + \ldots)] \quad (8)$$

and for zero friction, $B = \mu cot\alpha = 0$, we have:

$$\sigma_d = Y(1+0)[\epsilon_h - \frac{0\epsilon_h^2}{2} + \ldots)] = Y\epsilon_h \quad (9)$$

which is the result obtained with the ideal work method.

5. Integrating the equilibrium equation from the entry region to the location x gives the following:

$$\int_{D_o}^{D}\frac{dD}{D} = \int_0^{\sigma_x}\frac{d\sigma_x}{2B\sigma_x - 2Y(1+B)} \quad (10)$$

which finally results in the following:

$$\sigma_x = Y\frac{1+B}{B}[1 - (\frac{A_o}{A})^{-B}] \quad (11)$$

where $\frac{A_o}{A} = \frac{D_o^2}{D^2}$.

From the yield condition and the above equation we conclude that:

$$p = Y - \sigma_x = Y - Y\frac{1+B}{B}[1 - (\frac{A_o}{A})^{-B}] = Y[1 - \frac{1+B}{B}(1 - (\frac{A_o}{A})^{-B})] \quad (12)$$

or using l'Hospital's theorem:

$$p = Y[1 - (1+B)\frac{1 - e^{-B\ell n\frac{A_o}{A}}}{B}] = (as\ B \to 0)\ Y[1 - (1+B)\frac{\ell n\frac{A_o}{A}e^{-B\ell n\frac{A_o}{A}}}{-1}] \quad (13)$$

or

$$p = Y[1 - (1+0)\ell n\frac{A_o}{A}e^{-0\ell n\frac{A_o}{A}}] = Y[1 - \ell n\frac{A_o}{A}] \quad (14)$$

Exercise 8

Let us consider a plane strain drawing operation of a power law $\bar{\sigma} = K\bar{\epsilon}^n$ type of material.

The yield condition for plane strain is the following:

$$\sigma_x + p = \frac{2}{\sqrt{3}}\text{Yield Stress} \qquad (15)$$

This equation is valid at any point inside the deformation zone.

Recall that in class to derive the maximum reduction we used the above equation at the exit point ($\sigma_d = \sigma_x$), i.e.

$$\sigma_d + p|_{\text{at the exit point}} = \frac{2}{\sqrt{3}}\text{Yield Stress at the exit} \qquad (16)$$

together with $p \geq 0$ to state that for maximum reuction the following condition should be taken place:

$$\sigma_d \leq \frac{2}{\sqrt{3}}\text{Yield Stress at the exit} \qquad (17)$$

From equation (15) and (16), we now conclude that for conditions of maximum reduction , $p_{\text{at the exit point}} = 0$

Exercise 9

This problem was solved in class!!!