MAE 212 : SP '01

HW 8 SOLUTIONS

1. [redacted]

Looking at the Cu-Ni phase diagram, we have two phases: one liquid richer in Cu and one solid richer in Ni. We need to draw the tie line and use the lever rule.

Liquid: 80% wt Cu and 20% wt Ni.

Solid: 60% wt Cu and 40% wt Ni.

$$\text{Lever rule:} \quad \frac{m_L}{m_L+m_{ss}} = \frac{x_{ss}-x}{x_{ss}-x_L} = \frac{60-70}{60-80} = 0.5$$

$$\text{and} \quad \frac{m_{ss}}{m_L+m_{ss}} = 1 - \frac{m_L}{m_L+m_{ss}} = 0.5$$

So we have **2.5 lbs of liquid and 2.5 lbs of solid**

2. [redacted]

We are slightly above the eutectic temperature (779.2°C).

For an alloy with 20% wt Cu, we have two phases: one solid with 8.8% wt Cu and a liquid phase of eutectic composition with 28.1% wt Cu. Again using the lever rule, we write:[1]

$$X_a = \frac{m_\alpha}{m_\alpha + m_{liquid}} = \frac{x_{\text{eutectic}} - x}{x_{\text{eutectic}} - x_\alpha} = \frac{28.1 - 20}{28.1 - 8.8} = 0.420$$

We have 42.0% wt of the α-solid and 58.0% wt of the liquid.

For an alloy with 60% wt Cu, we again have two phases: the liquid phase at the eutectic composition 28.1% wt Cu and the β-solid at 92% wt Cu. Using the lever rule:

$$X_{liquid} = \frac{m_{\text{liquid}}}{m_{\text{liquid}} + m_\beta} = \frac{x_\beta - x}{x_\beta - x_{\text{eutectic}}} = \frac{92 - 60}{92 - 28.1} = 0.501$$

So we have 50.1% wt of the liquid and 49.9% of the β-solid.

[1] We use X to denote mass fraction of each phase and x to denote composition.

3. [redacted]

We are just under the eutectic temperature (778°C). Let us consider the alloy with 20% Cu composition. The microstructure consists of blocks of proeutectic α with 8.8 % Cu and eutectic $\alpha + \beta$. The proeutectic α is 42 % of the total alloy weight (see Problem 2 where we used the tie line just above the eutectic temperature).[2] The total α is given by using the tie line just below the eutectic temperature:

$$\frac{m_{total\ \alpha}}{m_{alloy}} = \frac{92 - 20}{92 - 8.8} = 86.53\%$$

4. [redacted]

- The eutectic temperature is 183°C and the eutectic composition is 61.9% wt Sn and 38.1% wt Pb.
- At 183.5° C and for an alloy with 25% wt Sn, we have an α-solid (19.2% wt Sn and 80.8% wt Pb) in a liquid matrix (61.9% wt Sn and 38.1% wt Pb).
- At 182.5° C, we now have the α-solid (19.2% wt Sn and 80.8% wt Pb) and the β-solid (97.5% wt Sn and 2.5% wt Pb).
- At 100° with 45% wt Sn, the phases in equilibrium are the α-solid ($\simeq$ 2% wt Sn and 98% wt Pb) and the β-solid, which is at such temperature just pure Sn.
- Lever Rule: $\frac{m_\alpha}{m_\alpha + m_{Sn}} = \frac{x_{Sn} - x}{x_{Sn} - x_\alpha} = \frac{100-45}{100-2} = 0.561$. Thus we have 56.1 % wt of the α-solid and 45.9% wt of pure Sn.

6. [redacted]

For $T = 182$°C, we are just below the eutectic temperature, but with only 15% wt Sn, we are in the α-solid region (only one phase). For $T = 150$°C and $T = 50$°C, we have two solid phases: The α and the β phases. The lever rule is going to be written $\frac{m_\alpha}{m_\alpha + m_\beta} = \frac{x_\beta - x}{x_\beta - x_\alpha}$ and the results are shown in the table below.

[2]The composition of the proeutectic α does not change as we move from slightly above the eutectic temperature to slightly below the eutectic temperature.

Temp	α-solid Comp. % wt Sn	α-solid Comp % wt Pb	β-solid Comp % wt Sn	β-solid Comp. % wt Pb	% wt α-solid	% wt β-solid
182°	15	85	/	/	100	/
150°	10	90	99	1	94.4	5.6
50°	2	98	100	0	86.7	13.3

7. [redacted]

A eutectoid transformation is when a solid decomposes in two different solid phases at a certain temperature (eutectoid temperature). This solid needs to have a specific composition, the eutectoid composition.

In the Fe-C phase diagram (up to 6.67% wt C where one finds Fe_3C, cementite), there is only one eutectoid transformation. It happens for a composition of 0.8% wt C at $T = 723°C$ and is the transformation of austenite (γ-Fe) to ferrite (α-Fe) and cementite (Fe_3C). This is called the pearlite reaction: $\gamma\text{-Fe} \longrightarrow \alpha\text{-Fe} + Fe_3C$

- 1% C, 1000°C: Only one solid phase, fcc austenite (γ-Fe) with 1% wt C (99% wt Fe).
- At 724° C an Fe-1% C alloy is a two phase mixture of γ-iron (composition=0.8 % C) and Fe_3C (6.67 % C). By the lever rule:

$$X_{Fe_3C} = \frac{1 - 0.8}{6.67 - 0.8} = 0.034$$

i.e. 3.4 % Fe_3C and

$$X_\gamma = \frac{\cancel{6.67 - 1}}{6.67 - 0.8} = 0.966$$

i.e. 96.6 % γ-iron.

- At 722° C an Fe-1% C alloy is a two phase mixture of α-iron (ferrite bcc iron composition=0.025 % C) and Fe_3C (6.67 % C). By the lever rule:

$$X_{Fe_3C} = \frac{1 - 0.25}{6.67 - 0.25} = 0.147$$

i.e. 14.7 % Fe_3C and

$$X_\alpha = \frac{6.67 - 1}{6.67 - 0.25} = 0.853$$

i.e. 85.3 % α-iron.

- At 724° C an Fe-0.6% C alloy is a two phase mixture of α-iron (0.025 % C) and γ-iron (0.8 % C). By the lever rule:

$$X_\alpha = \frac{0.8 - 0.6}{0.8 - 0.025} = 0.258$$

i.e. 25.8 % α-iron and

$$X_\gamma = \frac{\cancel{0.6 - 0.025}}{0.8 - 0.025} = 0.742$$

i.e. 74.2 % γ-iron.

- At 722° C an Fe-0.6% C alloy is a two phase mixture of α-iron (0.025 % C) and Fe_3C (6.67 % C). By the lever rule:

$$X_\alpha = \frac{6.77 - 0.6}{6.67 - 0.025} = 0.914$$

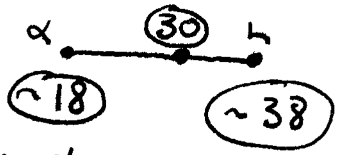
i.e. 91.4 % α-iron and

$$X_{Fe_3C} = \frac{0.6 - 0.025}{6.67 - 0.025} = 0.086$$

i.e. 8.6 % Fe_3C.

5.

a) ANSWER IS APPRX.



$$\text{wt\% } L = \frac{30-18}{38-18} = 60\%$$

$$\text{wt\% } \alpha = \frac{38-30}{38-18} = 40\%$$

b) $$\text{wt\% } L = \frac{30-19.2}{61.9-19.2} = 25\%$$

$$\text{wt\% } \alpha = \frac{61.9-30}{61.9-19.2} = 75\%$$

c) $$\text{wt } \beta = \frac{30-19.2}{97.5-19.2} = 0.14 \text{ kg}$$

$$\text{wt } \alpha = \frac{97.5-30}{97.5-19.2} - \frac{61.9-30}{61.9-19.2} = 0.115 \text{ kg}$$

8.

$$0.091 = \frac{6.67 - X}{6.67-0.025} - \frac{0.8 - X}{0.8-0.025}$$

$$X \cong 0.1$$

13. **10.12** (3) → (2) → (1) → decreasing hardness

(3) will produce > 90 % martensite and high hardness

(2) will produce 100% fine pearlite and lower hardness

(1) will produce 100% coarse pearlite and even lower hardness

10.13. **(a)** A eutectoid steel is cooled at a steady rate from 727 to 200°C in exactly 1 day. Superimpose this cooling curve on the TTT diagram of Figure 10–11 **(b)** From the result of your plot for part (a), determine at what temperature a phase transformation would first be observed. **(c)** What would be the first phase to be observed? (Recall the approximate nature of using an isothermal diagram to represent a continuous cooling process.)

14. **10.13** (a) The cooling rate is:

$$\frac{\Delta T}{\Delta t} = \frac{727°C - 200°C}{24 h \times 3600 s/h} = 6.10 \times 10^{-3} °C/s$$

For various times along the cooling curve,

t (sec)	ΔT(°C) →	T(°C)
1	6.1×10^{-3}	~727
10	0.06	726.94
100	0.6	726.4
1,000	6.1	720.9
2,000	12.2	715
3,000	18.3	709
10,000	61	666

Giving the plot: