MAE 212 : SPRING 2001

HW 5 SOLUTIONS

1.

a) NECKING WILL OCCUR WHEN $\varepsilon = n = 0.5$

THE STRESSES AT NECKING ARE:

$$\sigma_A = 100{,}000\,\varepsilon^{0.5} = 70{,}710 \text{ PSI}$$

$$\sigma_B = 50{,}000\,\varepsilon^{0.5} = 35{,}350 \text{ PSI}$$

THE AREAS AT NECKING ARE:

$$A_A = 0.2\,e^{-0.5} = 0.12 \text{ IN}^2$$

$$A_B = 0.1\,e^{-0.5} = 0.061 \text{ IN}^2$$

TOTAL LOAD:

$$(70{,}710)(0.12) + (35{,}350)(0.061) = \underline{10{,}640 \text{ LBF}}$$

b) IF THE n-VALUES OF THE TWO STRANDS ARE DIFFERENT, THE PROCEDURE WOULD CONSIST OF PLOTTING THE LOAD-ELONGATION CURVES OF THE TWO STRANDS INDIVIDUALLY ON THE SAME CHART, THEN OBTAINING THE MAXIMUM GRAPHICALLY. THE TOTAL LOAD CAN THEN BE OBTAINED EASILY.

3.

Consider a specimen obeying $\sigma = K\epsilon^n$ in a tensile test. Load the specimen in tension up to the point of initiation of necking (i.e. up to true strain $\epsilon = n$). Following our terminology from class, we call this the Day:1 experiment.

After the above experiment, the load is removed. In day:2, a new tensile experiment is then performed using the specimen resulting from the Day:1 experiment. It is obvious that necking will start at zero strain and that the new stress/strain relation measured in the Day:2 experiment will be $\sigma = K\ (n + \epsilon)^n$.

4. SEE LECTURE 10 NOTES

$$\bar{\sigma}_{VM} = \frac{\sqrt{3}}{2}|\sigma_1 - \sigma_3|$$

$$d\bar{\varepsilon} = \frac{2}{\sqrt{3}}|d\varepsilon_1|$$

$$dw = Y\frac{2}{\sqrt{3}}|d\varepsilon_1|$$

5. SEE LECTURE 10 NOTES

$$\bar{\sigma}_{VM} = |\sigma_r - \sigma_z|$$

$$d\bar{\varepsilon} = |d\varepsilon_z|$$

$$dw = Y|d\varepsilon_z|$$

6.

We have shown in class that:

$$\sigma_r = 0, \sigma_\theta = \frac{P\,r}{t}, \sigma_z = \frac{P\,r}{2t} \tag{19}$$

Tresca criterion:

The maximum principal stress is $\sigma_{max} = \sigma_\theta$ and the minimum principal stress is $\sigma_{min} = \sigma_r = 0$. So the maximum shear stress τ_{max} is given as follows:

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{Pr/t - 0}{2} = \frac{Pr}{2t} \tag{20}$$

For yielding according to Tresca: $\tau_{max} = \frac{Y}{2}$:

With Y=40,000 psi $\rightarrow \frac{Pr}{2t} = \frac{40{,}000}{2}$ or $P = \frac{40{,}000\ 0.025}{1.5}$ from which one calculates: $P = 666$ psi.

von Mises criterion:

$$\bar{\sigma}_{VM} = \frac{1}{\sqrt{2}}\left[(\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_z)^2 + (\sigma_z - \sigma_r)^2\right]^{\frac{1}{2}}$$

$$\Rightarrow \bar{\sigma}_{VM} = \frac{1}{\sqrt{2}}\left[\frac{P^2r^2}{t^2} + \frac{P^2r^2}{4t^2} + \frac{P^2r^2}{4t^2}\right]^{\frac{1}{2}} = \frac{\sqrt{3}}{2}\frac{P\,r}{t} \tag{21}$$

For yielding according to Mises: $\bar{\sigma}_{VM} = Y$:

Yield at 40,000 psi $\rightarrow P = \frac{40{,}000\ t}{r} \frac{2}{\sqrt{3}} = \frac{40{,}000\ 0.025}{1.5} \frac{2}{\sqrt{3}} = 770$ psi.

7.

Tresca:
It is here assumed that $\sigma_1 > 0$. For this case, $\sigma_{max} = \sigma_1$ and $\sigma_{min} = -0.5\sigma_1$. Using these values we can calculate τ_{max}:

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{\sigma_1 - (-0.5\,\sigma_1)}{2} = 0.75\sigma_1 \tag{22}$$

For Tresca: $\tau_{max} = \frac{Y}{2}$. With $Y = 300$ MPa $\rightarrow \sigma_1 = \frac{300}{2\,\cancel{0.75}} = 200$ MPa.

von Mises:

$$\begin{aligned} \bar{\sigma}_{VM} &= \frac{1}{\sqrt{2}}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right]^{\frac{1}{2}} \\ \bar{\sigma}_{VM} &= \frac{1}{\sqrt{2}}\left[0.49\sigma_1^2 + 0.64\sigma_1^2 + 2.25\sigma_1^2\right]^{\frac{1}{2}} = 1.3\sigma_1 \end{aligned} \tag{23}$$

Yield = 300 MPa $\Rightarrow \sigma_1 = \frac{300}{1.3} = 231$ MPa.

[redacted] 8.

By definition, $\sigma_m = \frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) = \frac{1}{3}(15 + 10 + 5) = 10$.

The deviatoric stress components are:

$$\begin{pmatrix} 5 & 3 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & -5 \end{pmatrix} \tag{24}$$

The sum of the deviatoric normal stress components is zero $(5 + 0 - 5 = 0!)$. Note that this was not an accident. The deviatoric stress components were defined by subtracting the hydrostatic (pressure) part from the stress state.

$$\frac{d\varepsilon_x}{d\varepsilon_z} = \frac{\sigma_x'}{\sigma_z'} = \frac{5}{-5} = -1$$

2.

$$d\bar{\varepsilon} = \sqrt{\frac{2}{3}\left(d\varepsilon_1^2 + d\varepsilon_2^2 + d\varepsilon_3^2\right)}$$

$$d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_3 = 0$$

$$d\varepsilon_3 = -(d\varepsilon_1 + d\varepsilon_2)$$

$$\varepsilon_1 = \ln\left(\frac{0.325}{0.25}\right) = 0.262$$

$$\varepsilon_2 = \ln\left(\frac{0.275}{0.25}\right) = 0.0953$$

$$\Rightarrow \quad d\bar{\varepsilon} = \sqrt{\frac{2}{3}\left(d\varepsilon_1^2 + d\varepsilon_2^2 + (d\varepsilon_1 + d\varepsilon_2)^2\right)}$$

USING TOTAL STRAINS IN THIS EQUATION

$$\Rightarrow \quad \bar{\varepsilon} = 0.370$$