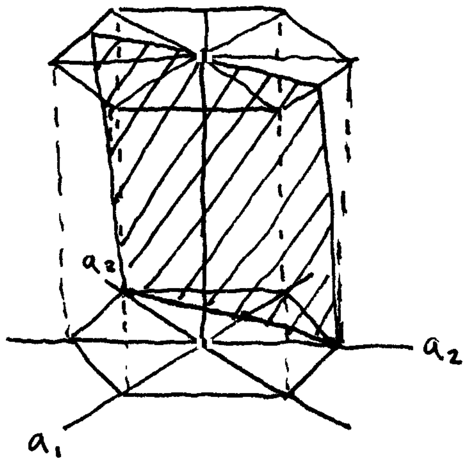
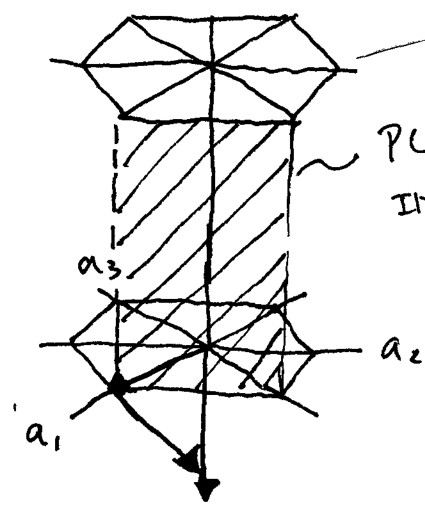
MAE 212 - SPRING 2001
HW 4 SOLUTIONS

1.

PLANE $(\bar{2}\,1\,1\,1)$ DIRECTION $[1\,0\,\bar{1}\,0]$

INTERCEPTS $-\frac{1}{2}, 1, 1, 1$





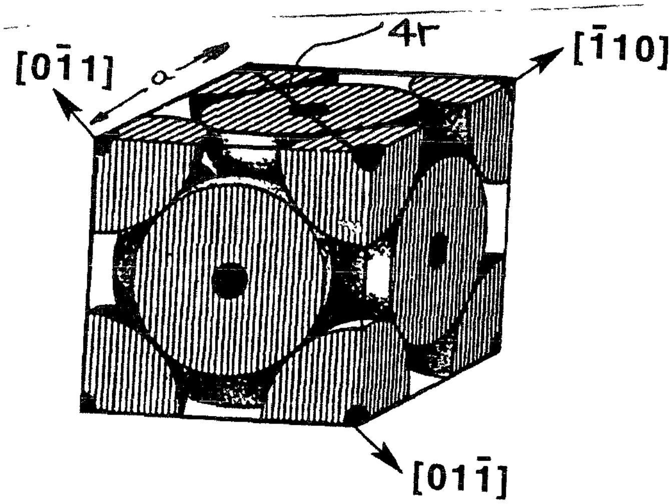
2.

(a) For FCC

$4r = \underbrace{a\sqrt{2}}_{\text{face diagonal}}$

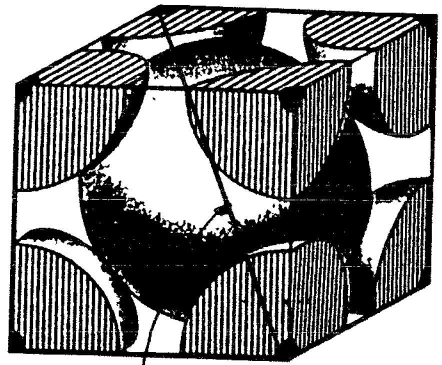
$\Downarrow$

$$\boxed{a = \frac{4}{\sqrt{2}} r}$$



For fcc Nickel $\Rightarrow$ $a = \frac{4}{\sqrt{2}} \frac{0.2492}{2}$ nm $\Rightarrow$ $\boxed{a = 0.3524 \text{ nm}}$

(b) For BCC

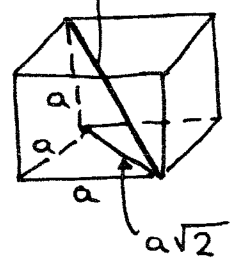


body diagonal

$\ell = 4r$

$\ell^2 = a^2 + (a\sqrt{2})^2 \Rightarrow$

$\ell = a\sqrt{3}$

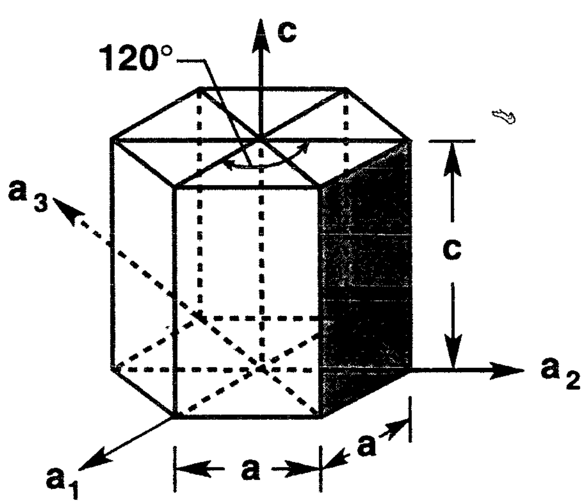


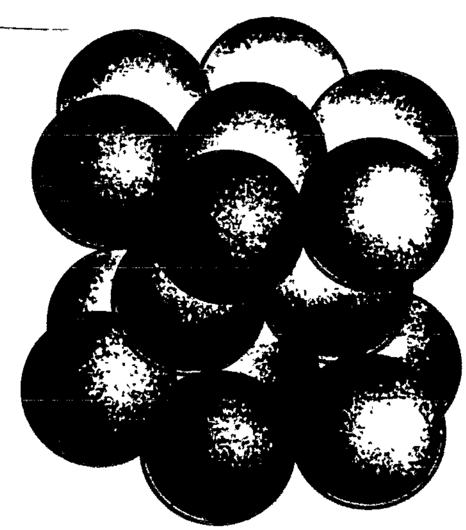
$$\ell = 4r = a\sqrt{3} \implies \boxed{a = \frac{4}{\sqrt{3}} r}$$

For bcc iron $r = 0.2482\text{ nm}/2 = 0.1241\text{ nm} \implies$

$$a = \frac{4}{\sqrt{3}}\, 0.1241\text{ nm} \Rightarrow \boxed{a = 0.2866}$$

(c) For HCP



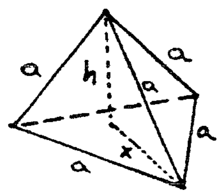


By inspection of these Figures $\Rightarrow$ $\boxed{a = 2r}$

3.

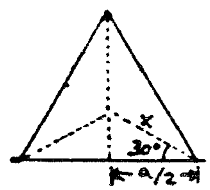
The central atom at $(\frac{2}{3}, \frac{1}{3}, \frac{1}{2})$ rests on the top of 3 basal plane atoms giving a tetrahedral configuration:

(a)



a = length of each side of tetrahedron
h = height of tetrahedron = $c/2$

In basal plane:



$$x = \frac{a/2}{\cos 30°} = 0.5774a$$

$$h^2 = a^2 - x^2 = a^2 - (0.5774a)^2 = 0.6667a^2$$

$$h = 0.8165a$$

$$c = 2h = 1.633a$$

or $\underline{\underline{c/a = 1.633}}$

(b) Rather than perfect spheres, the atoms are effectively ellipsoids (due to some assymmetry in atomic bonding)

4

(a) Packing factor for fcc

There are

8 corner atoms $\times \frac{1}{8}$ + 6 face center atoms $\times \frac{1}{2} \Rightarrow$

4 atoms / unit cell

Then: $APF = \dfrac{4 \times \frac{4}{3}\pi r^3}{a^3} = \dfrac{16}{3}\pi\left(\dfrac{\sqrt{2}}{4}\right)^3 \Rightarrow$

problem 4(a)

$\boxed{APF = 0.74}$

(b) for BCC

There are

8 corner atoms $\times \frac{1}{8}$ + 1 atom on the center $\Rightarrow$

2 atoms/unit cell $\Rightarrow$

Then: $APF = \dfrac{2\,\frac{4}{3}\pi r^3}{a^3} = \dfrac{8}{3}\pi\left(\dfrac{\sqrt{3}}{4}\right)^3 \Rightarrow$

problem 4(b)

$\boxed{APF = 0.68}$

(c) <u>for HCP</u>

From problems (4c) and (5) $\Rightarrow$

$$\boxed{\begin{aligned} a &= 2r \\ c &= 1.633\ a \end{aligned}}$$

$$V_{\text{unit cell}} = c \times \underbrace{a^2 \sin 60^\circ}_{\text{area}} = 1.633\ a^3 \sin 60^\circ =$$

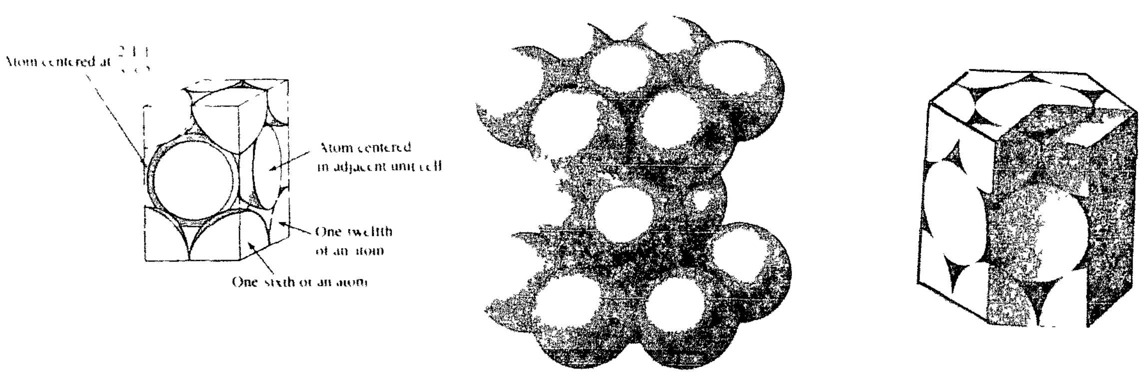
$$= 1.633\ (2r)^3 \sin 60^\circ \Rightarrow$$

$$\boxed{V_{\text{unit cell}} = 11.3138\ r^3}$$

In one unit cell there are (consult figure 3.13 of the Text)

$$\frac{\text{Atoms}}{\text{unit cell}} =$$

$$= 1 + 4 \times \frac{1}{6} + 4 \times \frac{1}{12} = 2$$



$$\text{So } APF = \frac{2\ \frac{4}{3}\pi r^3}{V_{\text{unit cell}}} \Rightarrow APF = \frac{8/3\ \pi\ \cancel{r^3}}{11.3138\ \cancel{r^3}} \Rightarrow$$

$$\boxed{APF = 0.74}$$

5.

We have shown that there are 4 atoms / unit cell in fcc crystals.

Then $\rho = \text{density} = \dfrac{m_{\text{unit cell}}}{V_{\text{unit cell}}} = \dfrac{4 \times m_{\text{atom}}}{V_{\text{unit cell}}}$ (*)

The mass of an atom $m_{\text{atom}} = \dfrac{58.71 \ \text{Kgr/Kmole}}{6.023\ 10^{26} \ \text{atoms/Kmole}} \Rightarrow$

$$\boxed{m_{\text{atom}} = 9.75\ 10^{-26} \ \text{Kgr/atom}}$$

Also $V_{\text{unit cell}} = a^3 = (0.3524 \ \text{nm})^3 = (0.3524\ 10^{-9} \text{m})^3 \Rightarrow$

Problem (4a)

$$\boxed{V_{\text{unit cell}} = 4.38\ 10^{-29} \ \text{m}^3/\text{unit cell}}$$

Finally, (*) $\Rightarrow \rho = \dfrac{4 \times 9.75\ 10^{-26}}{4.38\ 10^{-29}} \Rightarrow \boxed{\rho = 8909 \ \dfrac{\text{Kgr}}{\text{m}^3}}$

6.

Similarly to problem 7 and using the results of Problem 4(b) $\Rightarrow$

$$\rho = \frac{\left(2 \ \dfrac{\text{atoms}}{\text{unit cell}}\right) \times \dfrac{55.85}{6.023\ 10^{26}} \dfrac{\text{Kgr}}{\text{atom}}}{(0.2866\ 10^{-9})^3 \ \dfrac{\text{m}^3}{\text{unit cell}}} \Longrightarrow$$

$$\boxed{\rho = 7878 \ \text{Kgr/m}^3}$$

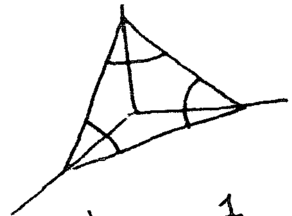
1.

(a) For bcc Fe: Problem 4(b)

$$a = \frac{4}{\sqrt{3}} r_{Fe} = \frac{4}{\sqrt{3}}(0.124\,nm) = 0.286\,nm$$

$$l = \sqrt{2}\,a = \sqrt{2}\,(0.286\,nm) = 0.405\,nm$$

$$A = \frac{\sqrt{3}}{4} l^2 = \frac{\sqrt{3}}{4}(0.405\,nm)^2 = 0.0710\,nm^2$$



$$3\,\text{atoms} \times \frac{1}{6} = \frac{1}{2}$$

$$\text{atomic density} = \frac{0.5\,\text{atom}}{A} = \frac{0.5\,\text{atom}}{0.0710\,nm^2}$$

$$= \underline{\underline{7.04\ \text{atoms}/nm^2}}$$

(b) For fcc Ni:

$$l = \sqrt{2}\,a$$

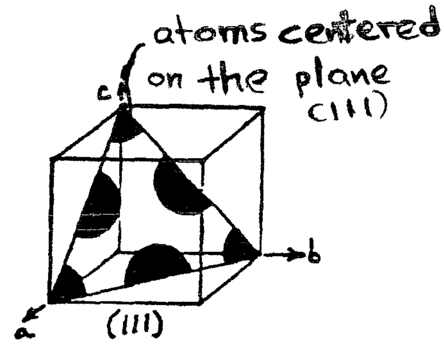
Using $a = 0.354\,nm$ (from P4(a)) gives:

$$l = \sqrt{2}\,(0.354\,nm) = 0.500\,nm$$

$$A = \frac{\sqrt{3}}{4} l^2 = \frac{\sqrt{3}}{4}(0.500\,nm)^2 = 0.108\,nm^2$$

$$\text{atomic density} = \frac{2\,\text{atoms}}{A} = \frac{2\,\text{atoms}}{0.108\,nm^2}$$

$$= \underline{\underline{18.5\ \text{atoms}/nm^2}}$$



$$3 \times \frac{1}{2} + 3\,\frac{1}{6} =$$

$$= 2\ \text{atoms/plane}$$

Note:

(i) planar atomic density =

$$= \frac{\text{equivalent number of atoms whose centers are intersected by selected area}}{\text{selected area}}$$

(ii) linear atomic density =

$$\frac{\text{number of atomic diameters intersected by selected length of line in direction of interest}}{\text{selected length of line}}$$

(Read Sample Problem 3.9 from the Text)

8.

By definition $\epsilon = \ell n \left(\frac{\ell}{\ell_o}\right)$.

For a uniform deformation, the volume is considered:

$$V = V_o \text{ with } V_o = A_o \ell_o = \frac{\pi D_o^2}{4} \ell_o \text{ and } V = A\ell = \frac{\pi D^2}{4} \ell \tag{1}$$

$$A_o\, \ell_o = A\, \ell \Rightarrow \frac{\ell}{\ell_o} = \frac{A_o}{A} \Rightarrow \epsilon = ln \left(\frac{A_o}{A}\right) \tag{2}$$

Also,

$$\frac{\pi D_o^2}{4} \ell_o = \frac{\pi D^2}{4} \ell \Rightarrow \frac{\ell}{\ell_o} = \frac{D_o^2}{D^2} = \left(\frac{D_o}{D}\right)^2 \Rightarrow \epsilon = 2\ell n \left(\frac{D_o}{D}\right) \tag{3}$$

$$r = \frac{A_o - A}{A_o} = 1 - \frac{A}{A_o} \Rightarrow \frac{A}{A_o} = 1 - r \text{ and } \epsilon = \ell n \left(\frac{1}{1-r}\right) \tag{4}$$

So

$$\epsilon = \ell n \left(\frac{\ell}{\ell_o}\right) = \ell n \left(\frac{A_o}{A}\right) = 2\ell n \left(\frac{D_o}{D}\right) = \ell n \left(\frac{1}{1-r}\right) \tag{5}$$

9.

By definition, $\epsilon = \ell n \left(\frac{\ell}{\ell_o}\right)$ and $e = \frac{\ell - \ell_o}{\ell_o}$.

So

$$\frac{\ell}{\ell_o} = \frac{\ell - \ell_o}{\ell_o} + \frac{\ell_o}{\ell_o} = 1 + e \Rightarrow \epsilon = \ell n\,(1+e) \tag{6}$$

By definition, $\sigma = \frac{F}{A}$ and $S = \frac{F}{A_o}$.

So

$$\sigma = \frac{F}{A} = \frac{F}{A_o}\frac{A_o}{A} = S\frac{\ell}{\ell_o} = S\,(1+e) \qquad (7)$$

where we used the volume conservation.

These relations are only good up to necking.

10.

Remember that the tensile strength (UTS) is the maximum engineering stress, and that the elongation given corresponds to engineering strain. We, thus, have to translate that information into true strain and stress. We can use the formulae found in Exercise 2:

$$\epsilon = \ell n\,(1+e) = \ell n\,(1+0.3) = 0.262 \qquad (8)$$

$$\sigma = S\,(1+e) = 340\,(1+0.3) = 442 \text{ MPa} \qquad (9)$$

We are given the UTS, so we are at the point of necking initiation. At that point, $\epsilon = n$. Hence $n = 0.262$ and with $\sigma = K\epsilon^n, K = \frac{\sigma}{\epsilon^n} = \frac{442}{0.262^{0.262}} = 628$ MPa.

11.

This is a very important exercise: we are given results of a test after the specimen has already been cold-worked. In Day:1 some cold work was done, taking the material into its plastic domain. The workpiece is then left to rest, where it contracted a little bit due to elasticity. And finally the material is being tested at Day:2. Since it is always the same material, it has only one representative curve for its plastic domain. The history of actions is reported on the graph.

Yield Load: We are looking for the "new apparent" yield strength of the specimen pre-strained. So it comes directly from the data of Day:2 experiment.

We assume that the elastic line is almost vertical, i. e. the diameter at yielding is equal to the starting diameter for Day:2 experiment.

$$Y = \frac{2,000}{\frac{\pi D'^2}{4}} = 19,980 \text{ psi} \qquad (10)$$

Remark: If we try to account for the elasticity, we get a diameter of $0.3569''$ at yielding for Day:2 experiment.

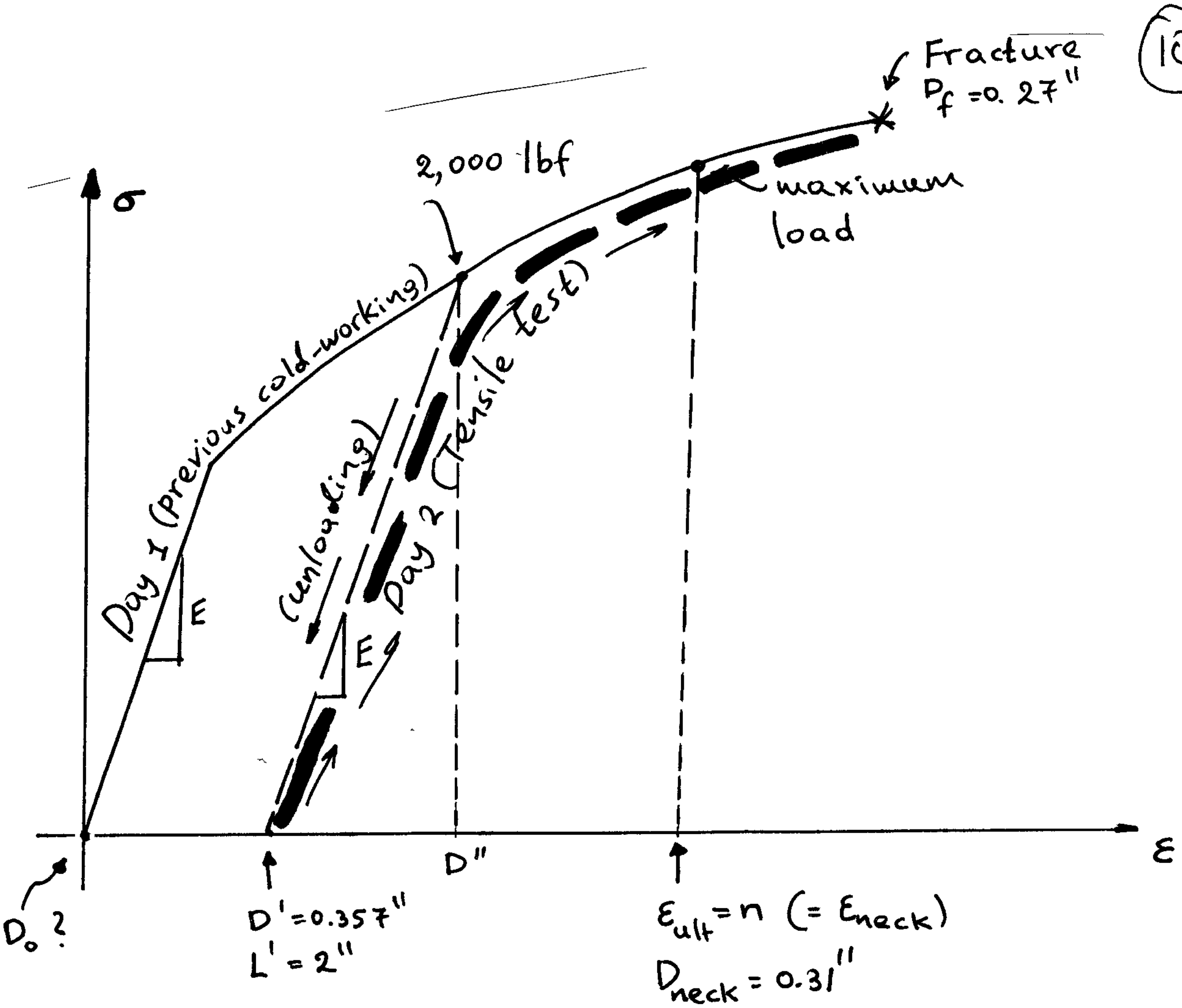


Figure 1: Note that Day:1 is referred to the previous cold-working, while Day:2 is referred to the tensile test. Also note that the elastic deformation has been enlarged to make the presentation clear.

Pre-strain: When considering necking, we must consider the curve representing the material and we can write $\epsilon_{\text{neck}} = n, \epsilon$ starting without pre-strain. So

$$\epsilon_{\text{neck}} = n = 0.5 = 2\ell n \frac{D_o}{D_{\text{neck}}} \tag{11}$$

$$\Rightarrow D_o = D_{\text{neck}} \exp\left(\frac{\epsilon_{\text{neck}}}{2}\right) = 0.398 - in \tag{12}$$

Pre-strain:

$$\epsilon' = 2\ell n \frac{D_o}{D'} = 0.217 \tag{13}$$

Maximum load: Let us first determine K from yielding point of Day:2 experiment:

$$\sigma' = Y = K\epsilon'^n \Rightarrow K = \frac{Y}{\epsilon'^n} = 42,892 \text{ psi} \tag{14}$$

Let us use K and n at necking where the maximum load is applied.

$$\epsilon_{\text{neck}} = n, \sigma_{\text{neck}} = K\epsilon_{\text{neck}}^n = Kn^n = 30,329 \text{ psi} \tag{15}$$

And

$$F_u = \frac{\pi D_{\text{neck}}^2 \sigma_{\text{neck}}}{4} = 2289 \text{ lbf} \tag{16}$$

Summary

- $Y = 19,980$ psi (yielding for Day 2 experiment).
- Cold work induced strain $\epsilon' = 0.217$.
- Maximum load $= F_u = 2,289$ lbf.

12. Exercise

From the first tensile specimen, we can infer K and n. Area reduction $= 40\% \Rightarrow \epsilon_{\text{neck}} = \ell n\left(\frac{1}{1-r}\right) = \ell n\left(\frac{1}{1-0.4}\right) = 0.51$ at necking. So, $n = 0.51$.

Starting diameter: $0.505'' \Rightarrow D_{\text{neck}} = D_o \exp\left(-\frac{\epsilon_{\text{neck}}}{2}\right) = 0.391''$.

$$\text{UTS} = 120,000 \text{ lbf} \Rightarrow \sigma_{\text{neck}} = \frac{4 \times \text{ UTS}}{\pi D_{\text{neck}}^2} = 999 \times 10^3 \text{ psi} \tag{17}$$

$$K = \frac{\sigma_{\text{neck}}}{n^n} = 1409 \times 10^3 \text{ psi} \tag{18}$$

Second specimen: $\epsilon = \frac{n}{2} = 0.255$.

So $\sigma = K\epsilon^n = 702 \times 10^3$ psi, $D = D_o \exp\left(-\frac{\epsilon}{2}\right) = 0.445''$.

And load $= \frac{\pi D^2 \sigma}{4} = 110,000$ lbf.