MAE 212: SPRING 2001
HW 1 SOLUTION

EXERCISE 1

THE IDEA HERE IS TO DETERMINE THE PRINCIPLE STRESSES. WE CAN IMMEDIATELY NOTICE THAT THE Z-AXIS IS A PRINCIPLE AXIS. SO NOW WE HAVE TO DETERMINE THE OTHER TWO PRINCIPLE AXES WHICH ARE IN THE X-Y PLANE (THE PROBLEM BECOMES A PLANE STRESS ONE). WE WILL USE MOHR'S CIRCLE.

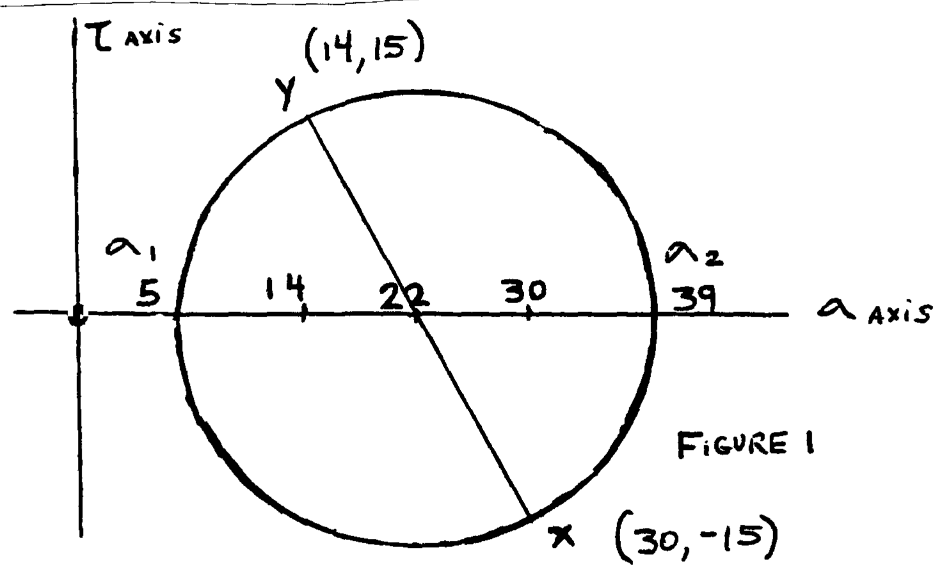

FIGURE 1

$$\sigma_{AVE} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(30+14) = 22$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{8^2 + 15^2} = 17$$

$$\sigma_1 = 39 \text{ ksI} \qquad \sigma_2 = 5 \text{ KSI}$$

1. (a) WHEN $\sigma_z = 0$, WE HAVE A SIMPLE PLANE STRESS PROBLE

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = \frac{1}{2}(39-0) = 19.5 \text{ ks}$$

(b) WHEN $\sigma_z \neq 0$, WE CAN STILL USE THE VALUES FOUND ABO

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = \frac{1}{2}(39-5) = 17 \text{ ks}$$

(c) SIMILARLY:

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = \frac{1}{2}(39-(-10)) = 24.5 \text{ ksi}$$

EXERCISE 2

WE NEED TO DETERMINE THE SHEAR STRESS AND THE NORMAL STRESS IN A NEW DIRECTION. USING THE EQUILIBRIUM OF FORCES (NOTES "THE CONCEPT OF STRESS" P.7), WITH $\sigma_x = -4$ MPa, $\tau_{xy} = 0$, $\sigma_y = -1.6$ MPa AND $\phi = -15°$, WE GET:

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\phi + \tau_{xy}\cos 2\phi \Rightarrow \tau_{x'y'} = -0.6 \text{ MPa}$$

$$\sigma_{x'x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\phi + \tau_{xy}\sin 2\phi$$

$$\Rightarrow \sigma_{x'x'} = -3.84 \text{ MPa}$$

EXERCISE 3

LET'S CONSIDER THE WEDGE:

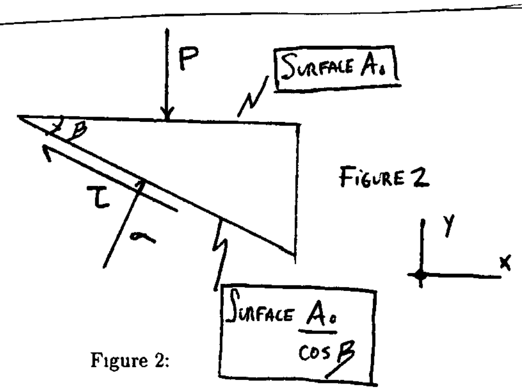

Figure 2:

LET'S WRITE THE EQUILIBRIUM OF FORCES:

X-DIR

$$0 - \frac{\tau A_0}{\cos\beta}\cos\beta + \frac{\sigma A_0}{\cos\beta}\sin\beta = 0$$

$$\Rightarrow \tan\beta = \frac{\tau}{\sigma} = \frac{4}{20} = 0.20 \Rightarrow \beta = 11.31°$$

Y-DIR

$$-P + \frac{\tau A_0}{\cos\beta}\sin\beta + \frac{\sigma A_0}{\cos\beta}\cos\beta = 0$$

$$\Rightarrow -P + \frac{\tau^2 A_0}{\sigma} + \sigma A_0 = 0 \Rightarrow \frac{P}{A_0} = \sigma + \frac{\tau^2}{\sigma} = 20.8 \text{ ksi}$$

THE MAX COMPRESSIVE STRESS IS IN THE Y-DIR AND IT'S VALUE IS −20.8 KSI.

EXERCISE 4

WE USE THE GENERALIZED HOOKE'S LAW.

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_z + \sigma_x)\right]$$

$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

HERE, $\sigma_z = 0 \Rightarrow E\epsilon_x = \sigma_x - \nu\sigma_y$

$$E\epsilon_y = \sigma_y - \nu\sigma_x$$

AND WITH ADDITION OF THE ABOVE EQUATIONS.

$$E(\epsilon_x + \epsilon_y) = (\sigma_x + \sigma_y)(1-\nu)$$

BUT $\epsilon_z = \frac{-\nu}{E}(\sigma_x + \sigma_y)$ SO $\epsilon_z = \frac{-\nu}{1-\nu}(\epsilon_x + \epsilon_y)$

$$\epsilon_z = \frac{-0.3}{1-0.3}(0.002 + 0.001) = -0.0015$$

EXERCISE 5

$$\boxed{1 \text{ KSI} = 68.94 \times 10^{-4} \text{ GPa}}$$

1. (a) AGAIN, WE USE THE GENERALIZED HOOKE'S LAW:

$$\epsilon_2 = 0 \Rightarrow \sigma_2 = \nu(\sigma_3 + \sigma_1) = 0.4(0+80) = 32 \text{KSI}$$

HENCE

$$\epsilon_1 = \frac{1}{E}\left[\sigma_1 - \nu(\sigma_2 + \sigma_3)\right]$$

$$= \frac{1}{300}\left[80 - 0.4(32+0)\right] \times 68.94\times10^{-4} = 0.00154$$

$$\epsilon_2 = 0$$

$$\epsilon_3 = \frac{1}{E}\left[\sigma_3 - \nu(\sigma_1 + \sigma_2)\right]$$

$$= \frac{1}{300}\left[0 - 0.4(80+32)\right] \times 68.94\times10^{-4} = -0.0010$$

(b) THE STRAIN ENERGY PER UNIT VOLUME IS GIVEN BY (IN TERMS OF PRINCIPLE STRESSES AND STRAINS):

$$W = \frac{1}{2}(\sigma_1 \epsilon_1 + \sigma_2 \epsilon_2 + \sigma_3 \epsilon_3)$$
$$= \frac{1}{2}(80 \times 0.001544 + 0 + 0) = 0.0618 \text{ KSI}$$

REMARK: W IS A WORK PER UNIT VOLUME, SO IT HAS THE SAME UNITS AS STRESS.

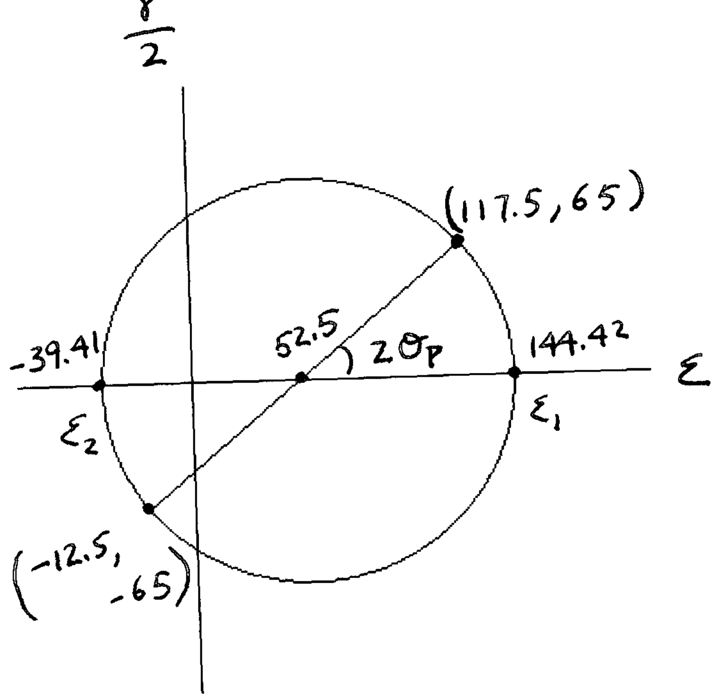

FIGURE 3

EXERCISE 6

1. (a) USING THE GENERALIZED HOOKE'S LAW FIRST:

$$\epsilon_x = \frac{1}{E}(\sigma_x - \nu(\sigma_y + \sigma_z)) = \frac{1}{200 \times 10^6}(5 \times 10^3 - 0.3(25 \times 10^3))$$
$$= -1.25 \times 10^{-5}$$

$$\epsilon_y = \frac{1}{E}(\sigma_y - \nu(\sigma_z + \sigma_x)) = \frac{1}{200 \times 10^6}(25 \times 10^3 - 0.3(5 \times 10^3))$$
$$= 1.175 \times 10^{-4}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{2(1+\nu)\tau_{xy}}{E} = \frac{2(1+0.3)\,10 \times 10^3}{200 \times 10^6}$$
$$= 1.3 \times 10^{-4}$$

NOW USE MOHR'S CIRCLE FOR STRAIN:

$$\epsilon_{AVE} = \frac{1}{2}(-1.25 \times 10^{-5} + 1.175 \times 10^{-4}) = 5.25 \times 10^{-5}$$

$$R = \sqrt{(1.175 \times 10^{-4} - 5.25 \times 10^{-5})^2 + (6.5 \times 10^{-5})^2}$$

$$= 9.192 \times 10^{-5}$$

$$2\theta_p = \tan^{-1}(1) \Rightarrow \theta_p = 22.5^\circ$$

$$\epsilon_1 = \epsilon_{AVE} + R = 5.25 \times 10^{-5} + 9.192 \times 10^{-5} = 1.444 \times 10^{-4}$$

$$\epsilon_2 = \epsilon_{AVE} - R = 5.25 \times 10^{-5} - 9.192 \times 10^{-5} = -3.941 \times 10^{-5}$$

(b) USING MOHR'S CIRCLE FIRST:

$$\sigma_{AVE} = \frac{1}{2}(25+5) = 15 \text{ KSI}$$

$$R = \sqrt{10^2 + 10^2} = 14.14 \text{ KSI}$$

$$2\theta_p = \tan^{-1}\left(\frac{10}{10}\right) \Rightarrow \theta_p = 22.5^\circ$$

$$\sigma_1 = \sigma_{AVE} + R = 29.14 \text{ KSI}$$

$$\sigma_2 = \sigma_{AVE} - R = 0.86 \text{ KSI}$$

USING HOOKE'S LAW:

$$\epsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2)$$

$$= \frac{1}{200 \times 10^6}\left(29.14 \times 10^3 - 0.3(0.86 \times 10^3)\right) = 1.444 \times 10^{-4}$$

$$\epsilon_2 = \frac{1}{E}(\sigma_2 - \nu\sigma_1)$$

$$= \frac{1}{200 \times 10^6}\left(0.86 \times 10^3 - 0.3(29.14 \times 10^3)\right)$$

$$= -3.941 \times 10^{-5}$$

WE OF COURSE GET THE SAME RESULTS.

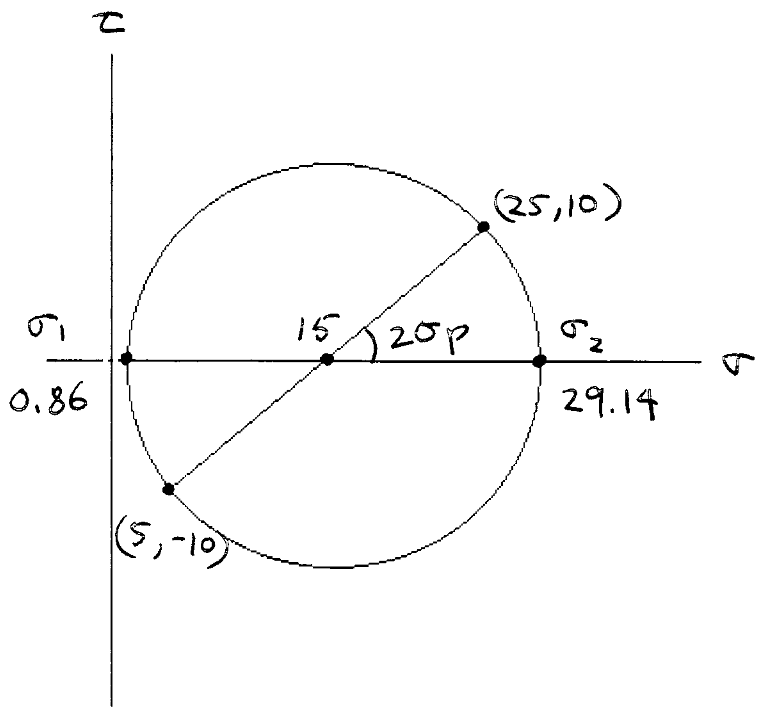

FIGURE 4