MAE 212 : SPRING 2001
HW 11 SOLUTIONS

1.

(a) — F

(b) — F

(c) — F

(d) — F

(e) — T

(f) — F

(g) — F

(h) — F

(i) — F

(j) — F

(k) — F

(l) — F

(m) — F

(n) — F

(o) — T

(p) — T

(q) — F

(r) — F

2.

AT ULTIMATE POINT $dF = 0$

$$F = \sigma A$$

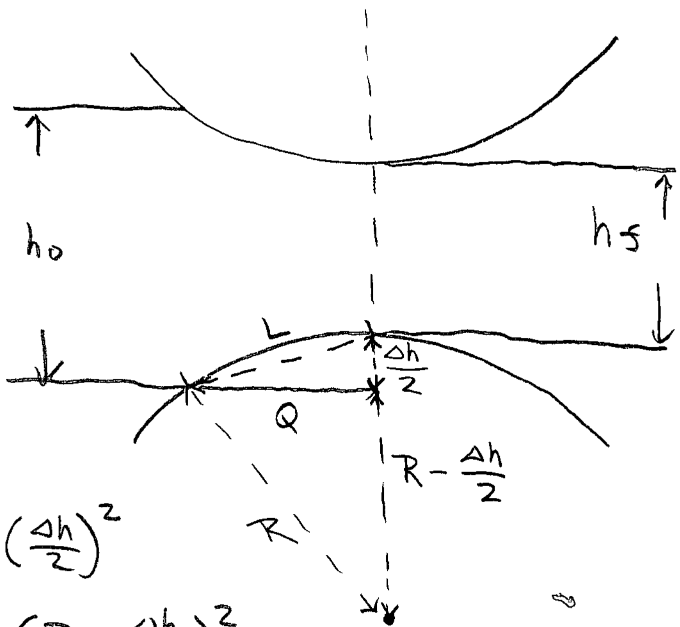
$$dF = d(\sigma A) = \sigma dA + A d\sigma = 0$$

$$\frac{d\sigma}{\sigma} = -\frac{dA}{A} = d\varepsilon$$

$$\boxed{\frac{d\sigma}{d\varepsilon} = \sigma}$$

3.

a)



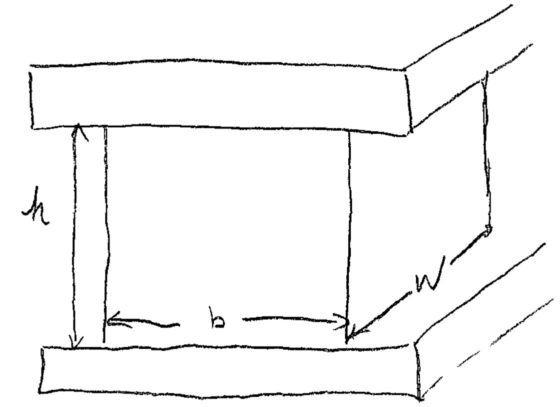
$$L^2 = Q^2 + \left(\frac{\Delta h}{2}\right)^2$$

$$Q^2 = R^2 - \left(R - \frac{\Delta h}{2}\right)^2$$

$$= R\Delta h - \left(\frac{\Delta h}{2}\right)^2$$

$$\boxed{L = \sqrt{R\Delta h}}$$

b)



$$Y = \frac{Y_o + Y_f}{2} \qquad P_{AVE} = \frac{\frac{Y_o + Y_f}{\sqrt{3}}}{\frac{2\mu\sqrt{R(h_f - h_o)}}{h_o + h_f}}\left(e^{\frac{2\mu\sqrt{R(h_f - h_o)}}{h_o + h_f}} - 1\right)$$

$$h = \frac{h_o + h_f}{2}$$

$$b = \sqrt{R\Delta h}$$

4.

a)

$$\bar{\epsilon} = \frac{2}{\sqrt{3}} \ln\left(\frac{h_o}{h}\right)$$
$$= \frac{2}{\sqrt{3}} \ln(2)$$

b)

$$Y = K(0.8)^n$$

c)

$$F = Y A_o$$

$$\boxed{F = \left(\frac{\pi d_o^2}{4}\right) K(0.8)^n}$$

5. SEE HW10 #3

6.

$$\frac{1}{r} - \frac{1}{r'} = \frac{3\sigma_o}{tE'}$$

$$Y = \frac{\sqrt{3}}{2}\sigma_o$$

$$\sigma_o = \frac{2}{\sqrt{3}}(38{,}971)$$

$$\frac{1}{r} - \frac{1}{10} = \frac{\frac{6}{\sqrt{3}}(38{,}971)}{0.03\left(\frac{30\times10^6}{1-0.33^2}\right)}$$

$$r = 4.2 \text{ IN}$$

7.

1. The elastic strain at yielding is $\epsilon_x = \sigma_o/E'$, where E' is the plane-strain modulus, $E/\left(1-\nu^2\right)$. The limit of the elastic core will be at $z = r\epsilon_x = r\sigma_o/E'$. Taking E' as 33×10^6 psi, $z = 5 \times 33 \times 10^3/33 \times 10^6 = 0.005$in. The elastic fraction is $2 \times 0.005/0.036 = 0.28$ or 28%.

2. To calculate the bending moment, for the elastic portion $(0 \le z \le 0.005)$, $\sigma_x = \epsilon_x E' = zE'/r$, and for the plastic portion $(0.005 \le z \le 0.018)$, $\sigma_x = \sigma_o$.

$$\begin{aligned} M &= 2\int_0^{0.005} w\frac{E'}{r}z^2dz + 2\int_{0.005}^{0.018} w\sigma_o z dz \\ &= \frac{2}{3}\frac{33\times10^6}{5}(0.005)^3 w + 33\times10^3\left(0.018^2 - 0.005^2\right)w = 10.42w \end{aligned} \tag{7}$$

Using the equation which neglects the elastic core,

$$M = \left(33\times10^3\right)\frac{\left(0.036^2\right)w}{4} = 10.96w \tag{8}$$

The error is $(10.69 - 10.42)/10.42 = 0.026$ or 2.6%.