MAE 212 : SPRING 2001

HW 10 SOLUTIONS

Problem 1

1.

$$P_{av} = [h/(\mu L)]\left[\exp(\mu L/h) - 1\right]\frac{2\cancel{Y}}{\sqrt{3}} \text{ where } h = (h_o + h_f)/2 = 0.0625 \quad (1)$$

$$\Delta h = h_o - h_f = 0.025 \text{ so } L = \sqrt{R\Delta h} = \sqrt{4(.025)} = 0.316, Y = 20,000 \quad (2)$$

Substituting, $P_{av} = [0.0625/(0.1 \times 0.316)]\left[\exp(0.1 \times 0.316/0.0625) - 1\right] \times \frac{2\times 20,000}{\sqrt{3}}$ $P_{av} =$ 30, 054 psi

2. Here we should use R' instead of R. We must calculate simultaneously F_s and R'.

Substituting $E' = 33 \times 10^6, R = 4$, and $\Delta h = 0.025$, in the equ. given in class for roll flattening gives:

$$\begin{aligned} F_s &= (R'/R - 1)\,\pi \Delta h E'/16 \\ &= (R'/R - 1)\,\pi\,(0.025)\left(33 \times 10^6\right)/16 \\ F_s &= 1.62 \times 10^5\,(R'/4 - 1) \end{aligned} \quad (3)$$

and the equ. for the separating force becomes

$$F_s = (h/\mu)\left\{\exp\left[(\mu/h)(R'\Delta h)^{\frac{1}{2}}\right] - 1\right\} \times \frac{2\cancel{\times 20,000}}{\sqrt{3}} \quad (4)$$

or

$$F_s = (0.0625/0.1)\left\{\exp\left[(.1/0.0625)(0.025)^{\frac{1}{2}}\sqrt{R'}\right] - 1\right\} \times \frac{2 \times 20,000}{\sqrt{3}} \quad (5)$$

or

$$F_s = 14,433\left[\exp\left(0.253\sqrt{R'}\right) - 1\right] \quad (6)$$

Solving together equs. (3) and (6) gives the following:

$R' = 4.24$ and $F_s = 9,872$ and $P_{av} = F_s/\sqrt{R'\Delta h} = 30,323$ psi.

3. Using the equ. given in class with $C = 7.5$,

$$h_{min} = 7.5\,(0.1)\,(4)\left(\frac{2 \times 20,000}{\sqrt{3}}\right)/\cancel{\left(33 \times 10^6\right) = 0.0021} \text{ in} \quad (7)$$

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2. [redacted]

Using the approximation for plane-strain compression with sticking friction, one can write:

$$P_{av} = 2\kappa \left(1 + \frac{b}{(4h)}\right] \text{ where } \kappa = \frac{Y}{\sqrt{3}} \tag{10}$$

and initially $b = h = 1$ and at the end of compression $b = 2, h = \frac{1}{2}$.

1. Substituting final values, $F = -20P_{av} = -20 \times 2\left(\frac{2500}{\sqrt{3}}\right)\left[1 + \frac{2}{4 \times .5}\right] = -115,500$ pounds (the minus sign for compression).

2.

$$\begin{aligned} W &= \int F dh; \text{ substituting } F = -P_{av} \times 10b \text{ and } P_{av} = 2\kappa\left[1 + b/(4h)\right] \\ W &= \int F dh = -2\kappa \int \left[10b\left[1 + b/(4h)\right]\right] dh, \text{ but } b = 1/h \text{ (since } bh = 1) \text{ so} \\ W &= -2\kappa \int \left[10\left(1/h\right)\left[1 + \left(1/h\right)/\left(4h\right)\right]\right] dh = -20\kappa \int \left[1/h + \left(1/\left(4h^3\right)\right)\right] dh = \\ &= -20\kappa\left[ln\left(h_f/h_o\right) + \left(-1/8\right)\left(h_f^{-2} - h_o^{-2}\right)\right] \end{aligned} \tag{11}$$

Evaluating at $h_o = 1$ and $h_f = \frac{1}{2}$,

$$\begin{aligned} W &= -20\kappa\left[ln\left(\frac{1}{2}\right) - \frac{1}{8}(4-1)\right] = -20\kappa\left[ln\left(\frac{1}{2}\right) - \frac{3}{8}\right] = -20\kappa\,(-1.068) \\ &= 20\left(\frac{2500}{\sqrt{3}}\right)(1.068) = 30,800 \text{ in} - \text{lbs. } (2,570 \text{ ft} - \text{pounds}) \end{aligned}$$

3. Ideal work, $W_i = \text{volume} \cdot Y\bar{\epsilon} = 10 \text{ in.}^3 \cdot 2,500 \text{ psi} \cdot \frac{2}{\sqrt{3}} \cdot ln(2) = 20,000 \text{ in} - \text{lbs}$

   $\eta = \frac{W_i}{W_a} = 20,000 \text{ in} - \text{lbs}/30,800 \text{ in} - \text{lbs} = 65\%$.

3. a) $h_{min} = \dfrac{7.5 \times 0.3 \times 5}{\dfrac{30\times10^6}{1-0.33^2}} \left(\dfrac{2}{\sqrt{3}} 31{,}000\right)$

$$\boxed{h_{min} = 0.012 \text{ in}}$$

b) CONSERVATION OF VOLUME

$$\underbrace{h_{min}}_{0.012\text{ IN}} \underbrace{V_*}_{\text{EXIT VELOCITY}} = 0.02 \times 37 \times 12 \ \text{IN/s}$$

$$\boxed{V_* = 740 \ \text{IN/s} = 61.7 \ \text{ft/s}}$$

c) $P_{AVG} = \dfrac{h}{\mu L}\left(e^{\frac{\mu L}{h}} - 1\right)\dfrac{2Y}{\sqrt{3}}$

$$= \frac{\dfrac{0.02+0.012}{2}}{0.3\sqrt{5(0.02-0.012)}} \left(e^{\frac{0.3\sqrt{5(0.02-0.012)}}{\frac{0.02+0.012}{2}}} - 1\right) \frac{2}{\sqrt{3}} \times (31{,}000)$$

$$P_{AVG} = \frac{0.016}{0.06}\left(e^{\frac{0.06}{0.016}} - 1\right)(35{,}796)$$

$$= 396{,}344 \text{ PSI}$$

$$F = P_{AVG} L W$$

$$= (396{,}344)\sqrt{5(0.02-0.012)}\ (13)$$

$$\boxed{F = 1.03 \times 10^6 \text{ LBF}}$$

d) SEE CLASS NOTES

4.

$$P_{AVG} = Y + \frac{2KR}{3h}$$

$$K = Y/\sqrt{3}$$

$$P_{AVG} = Y + \frac{2YR}{3\sqrt{3}\,h}$$

$$= 25{,}000 + \frac{\cancel{2}(25{,}000)(0.95)}{3\sqrt{3}\,(0.06)\,\cancel{2}}$$

$$= 101{,}178 \text{ PSI}$$

$$F = P_{AVG} \cdot \pi R^2$$

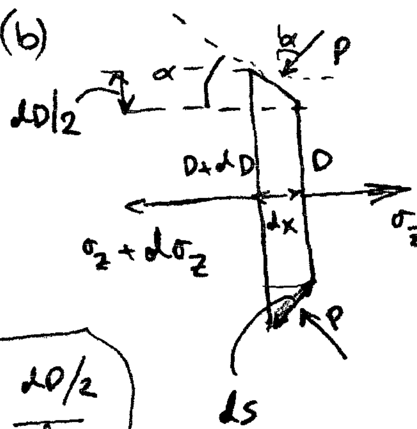
$$\underline{\underline{F = 71{,}717 \text{ LBF}}}$$

5. (a) $\begin{cases} \sigma_r = \sigma_\theta & \text{(axisymmetric assumption)} \\ \sigma_z & \end{cases}$

$$\bar{\sigma} = Y \Rightarrow \frac{1}{\sqrt{2}}\sqrt{\cancel{(\sigma_r-\sigma_\theta)^2} + (\underset{\sigma_r = -p}{\underset{\|}{\sigma_\theta}} - \sigma_z)^2 + (\sigma_z - \underset{-p}{\underset{\|}{\sigma_r}})^2} = Y \Rightarrow$$

$$\frac{1}{\sqrt{2}}\sqrt{(\sigma_z+p)^2 + (\sigma_z+p)^2} = Y \Rightarrow \boxed{\sigma_z + p = Y}$$

(b)



$$\sin a = \frac{dD/2}{ds}$$

$$\Sigma F_z = 0 \rightarrow$$

$$(\sigma_z + d\sigma_z)\frac{\pi}{4}(D+dD)^2 - \sigma_z \frac{\pi}{4} D^2 + \boxed{p \sin a (\pi D\, ds)} = 0$$

attention with the area

NO FRICTION HERE (Compare with problem 7/HW10)

2nd order

$$\Rightarrow \frac{\pi}{4} d\sigma_z D^2 + \cancel{\sigma_z \frac{\pi}{4}(dD)^2}$$

$$+ \cancel{d\sigma_z \frac{\pi}{4} 2D\, dD} + \sigma_z \frac{\pi}{4} 2D\, dD + p \cancel{\sin a}\, \pi D \left(\frac{dD}{2}\frac{1}{\cancel{\sin a}}\right) = 0$$

$$\Rightarrow \frac{\pi}{4} d\sigma_z D^2 + \sigma_z \frac{\pi}{4} 2D\, dD + p\pi \frac{D}{2} dD = 0 \rightsquigarrow$$

$$D\, d\sigma_z + 2\sigma_z\, dD + 2p\, dD = 0 \Rightarrow$$

Substitute from (a) $\sigma_z = -p + Y \Rightarrow$

$$-D\,dp + 2(-p + Y)\,dD + 2p\,dD = 0 \Rightarrow$$

$$D\,dp = +2Y\,dD \Rightarrow \boxed{\frac{dp}{2Y} = \frac{dD}{D}}$$

(c) $$\int_{Y}^{p} \frac{dp}{2Y} = \int_{D_0}^{D} \frac{dD}{D}$$

note at $D = D_0$, $\sigma_x = 0 \Rightarrow \cancel{\sigma_x}^{\,0} + p = Y \Rightarrow$

$$p\Big|_{D=D_0} = Y$$

$$\Rightarrow \frac{1}{2Y}(p - Y) = -\ln\frac{D_0}{D} \Rightarrow$$

$$p = Y - Y\ln\left(\frac{D_0}{D}\right)^2 \rightarrow \boxed{p = Y\left[1 - \ln\left(\frac{D_0}{D}\right)^2\right]}$$

6.

$$W_a = W_c + W_s + W_r$$

$$W_r = 0.4\,W_c$$

$$W_s = 0.25\,W_a$$

$$\Rightarrow \quad 0.75\,W_a = 1.4\,W_c$$

$$\frac{W_c}{W_a} = \eta = \frac{0.75}{1.4} = 0.536$$

$$P_e = \frac{\int_0^{\epsilon_s} \bar{\sigma}\, d\bar{\epsilon}}{\eta} = \frac{K \epsilon_h^{n+1}}{(n+1)\eta}$$

$$\epsilon_h = 2 \ln\left(\frac{D_o}{D_s}\right)$$

$$= 2 \ln\left(\frac{6}{2}\right)$$

$$= 2.2$$

$$P_e = \frac{(26{,}000)(2.2)^{1.2}}{1.2\,(0.536)} \approx 104{,}120 \text{ PSI}$$

$$F_e = P_e \left(\frac{\pi D_o^2}{4}\right)$$

$$F_e = 2.94 \times 10^6 \text{ LBF}$$