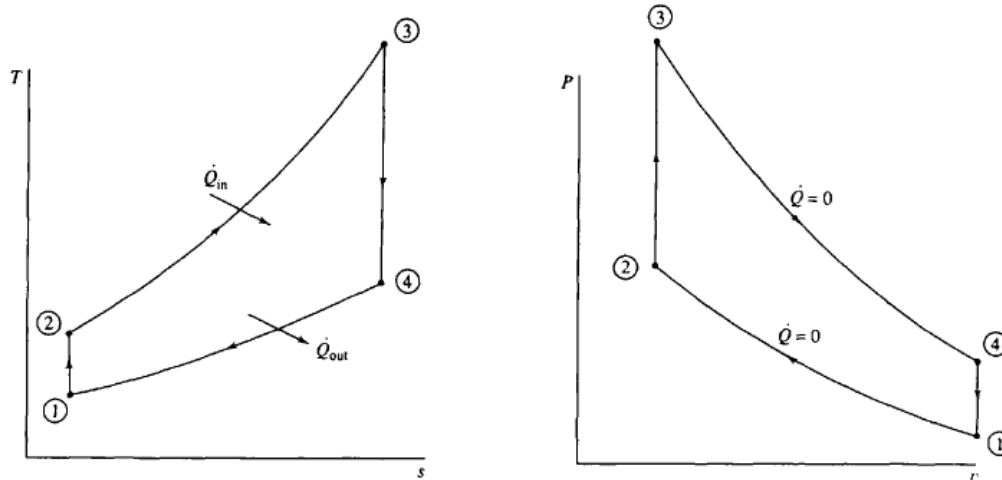


AGENDA

1. Air-Standard Otto, Diesel, Dual Cycle
2. Thermodynamic Relations

1. Otto cycle



The thermal efficiency of the Otto cycle is found from

$$\eta = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}} = \frac{\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}} = 1 - \frac{\dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}} \quad (9.14)$$

Noting that the two heat transfer processes occur during constant-volume processes, for which the work is zero, there results

$$\dot{Q}_{\text{in}} = \dot{m}c_v(T_3 - T_2) \quad \dot{Q}_{\text{out}} = \dot{m}c_v(T_4 - T_1) \quad (9.15)$$

where we have assumed each quantity to be positive. Then

$$\eta = 1 - \frac{T_4 - T_1}{T_3 - T_2} \quad (9.16)$$

This can be written as

$$\eta = 1 - \frac{T_1}{T_2} \frac{T_4/T_1 - 1}{T_3/T_2 - 1} \quad (9.17)$$

For the isentropic processes we have

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1} \quad \text{and} \quad \frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{k-1} \quad (9.18)$$

But, using $V_1 = V_4$ and $V_3 = V_2$, we see that

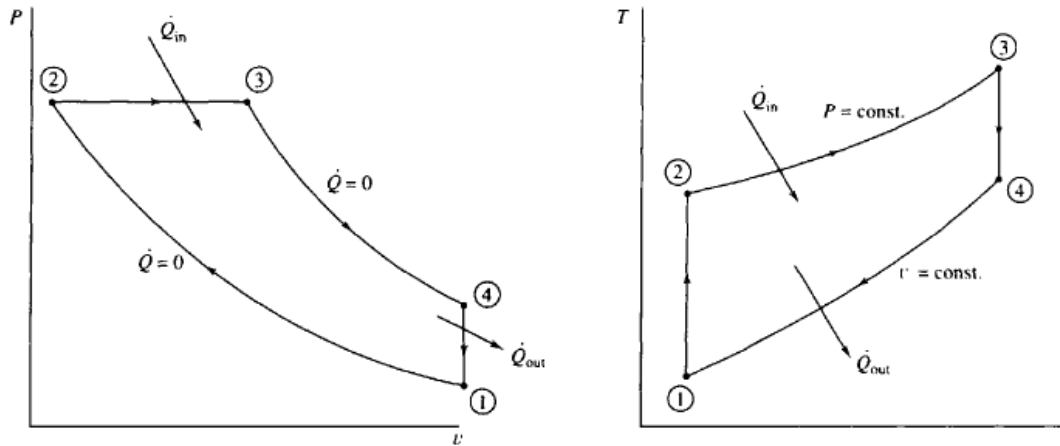
$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \quad (9.19)$$

Thus, (9.17) gives the thermal efficiency as

$$\eta = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{V_2}{V_1}\right)^{k-1} = 1 - \frac{1}{r^{k-1}} \quad (9.20)$$

We see, then, that the thermal efficiency in this idealized cycle is dependent only on the compression ratio r : the higher the compression ratio, the higher the thermal efficiency.

2. Diesel cycle



The thermal efficiency of the diesel cycle is expressed as

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}} \quad (9.21)$$

For the constant-volume process and the constant-pressure process

$$\dot{Q}_{out} = \dot{m}c_v(T_4 - T_1) \quad \dot{Q}_{in} = \dot{m}c_p(T_3 - T_2) \quad (9.22)$$

The efficiency is then

$$\eta = 1 - \frac{c_v(T_4 - T_1)}{c_p(T_3 - T_2)} = 1 - \frac{T_4 - T_1}{k(T_3 - T_2)} \quad (9.23)$$

This can be put in the form

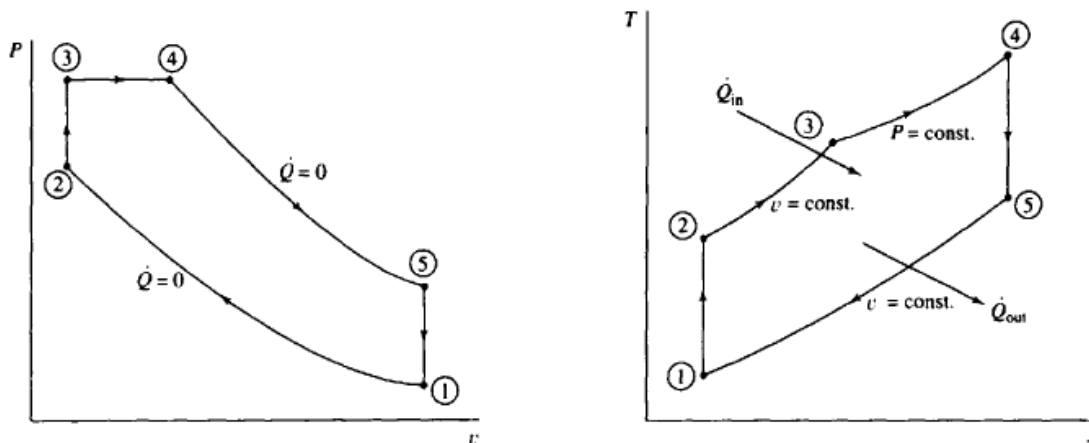
$$\eta = 1 - \frac{T_1}{kT_2} \frac{T_4/T_1 - 1}{T_3/T_2 - 1} \quad (9.24)$$

This expression for the thermal efficiency is often written in terms of the compression ratio r and the cutoff ratio r_c which is defined as V_3/V_2 ; there results

$$\eta = 1 - \frac{1}{r^{k-1}} \frac{r_c^k - 1}{k(r_c - 1)} \quad (9.25)$$

From this expression we see that, for a given compression ratio r , the efficiency of the diesel cycle is less than that of an Otto cycle. For example, if $r = 10$ and $r_c = 2$, the Otto cycle efficiency is 60.2 percent and the diesel cycle efficiency is 53.4 percent. As r_c increases, the diesel cycle efficiency decreases. In practice, however, a compression ratio of 20 or so can be achieved in a diesel engine; using $r = 20$ and $r_c = 2$, we would find $\eta = 64.7$ percent. Thus, because of the higher compression ratios, a diesel engine typically operates at a higher efficiency than a gasoline engine.

3. Dual cycle



An ideal cycle that better approximates the actual performance of a compression-ignition engine is the *dual cycle*, in which the combustion process is modeled by two heat-addition processes: a constant-volume process and a constant-pressure process, as shown in Fig. 9-11. The thermal efficiency is found from

$$\eta = 1 - \frac{\dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}} \quad (9.26)$$

where

$$\dot{Q}_{\text{out}} = \dot{m}c_v(T_5 - T_1) \quad \dot{Q}_{\text{in}} = \dot{m}c_v(T_3 - T_2) + \dot{m}c_p(T_4 - T_3) \quad (9.27)$$

Hence, we have

$$\eta = 1 - \frac{T_5 - T_1}{T_3 - T_2 + k(T_4 - T_3)} \quad (9.28)$$

If we define the *pressure ratio* $r_p = P_3/P_2$, the thermal efficiency can be expressed as

$$\eta = 1 - \frac{1}{r^{k-1}} \frac{r_p r_c^k - 1}{kr_p(r_c - 1) + r_p - 1} \quad (9.29)$$

If we let $r_p = 1$, the diesel cycle efficiency results; if we let $r_c = 1$, the Otto cycle efficiency results. If $r_p > 1$, the thermal efficiency will be less than the Otto cycle efficiency but greater than the diesel cycle efficiency.

4. Thermodynamic relations

- Legendre Transformations replace independent variables of a function with their conjugate pair. The resulting differential has the term $-X_i dC_i$ in place of $C_i dX_i$.
- Legendre Transformations applied to the combined statement of the first and second laws results in 3 new energy functions; the enthalpy, the Helmholtz free energy and the Gibbs free energy.
- The differential forms of the energy functions are:

$$\begin{aligned}dU &= TdS - PdV \\dH &= TdS + VdP \\dF &= -SdT - PdV \\dG &= -SdT + VdP\end{aligned}$$

- The eight **coefficient relationships** are:

$$\begin{aligned}T &= \left(\frac{\partial U}{\partial S}\right)_V = \left(\frac{\partial H}{\partial S}\right)_P \\P &= -\left(\frac{\partial U}{\partial V}\right)_S = -\left(\frac{\partial F}{\partial V}\right)_T \\S &= -\left(\frac{\partial F}{\partial T}\right)_V = -\left(\frac{\partial G}{\partial T}\right)_P \\V &= \left(\frac{\partial H}{\partial P}\right)_S = \left(\frac{\partial G}{\partial P}\right)_T\end{aligned}$$

- The four **Maxwell relations** are:

$$\begin{aligned}\left(\frac{\partial T}{\partial V}\right)_S &= -\left(\frac{\partial P}{\partial S}\right)_V \\ \left(\frac{\partial T}{\partial P}\right)_S &= \left(\frac{\partial V}{\partial S}\right)_P \\ \left(\frac{\partial S}{\partial V}\right)_T &= \left(\frac{\partial P}{\partial T}\right)_V \\ \left(\frac{\partial S}{\partial P}\right)_T &= -\left(\frac{\partial V}{\partial T}\right)_P\end{aligned}$$

- The **volume coefficient of thermal expansion** α is the differential change in the volume of a material for a differential change in temperature under constant pressure per unit volume. That is;

$$\alpha \equiv \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_P$$

- The **volume coefficient of compressibility** β is the negative differential change in volume of a material for a differential change in pressure under constant temperature per unit volume. That is;

$$\beta \equiv -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_T$$

- The set of six thermodynamic state functions S , V , U , H , F , and G can be expressed as functions of T and P as:

$$\begin{aligned} dS &= \left(\frac{C_P}{T} dT - V\alpha dP \right) \\ dV &= V\alpha dT - V\beta dP \\ dU &= (C_P - PV\alpha) dT + V(P\beta - T\alpha) dP \\ dH &= C_P dT + V(1 - T\alpha) dP \\ dF &= -(S + PV\alpha) dT + PV\beta dP \\ dG &= -S dT + V dP \end{aligned}$$

- The General Procedure can be used to derive the thermodynamic relationship between any three thermodynamic variables. The process can be followed step by step, or by using the Jacobian expressions for the necessary partial derivatives.
- Often times the partial derivatives that can be derived using the General Procedure are more easily obtained by using the Coefficient Relations or Maxwell Relations.
- In cases where the dependent variable is either T or P , rewrite the function so that neither T or P are the independent variable, then use the General Procedure and finally rearrange the resulting expression for dT or dP , whichever is the dependent variable.

Part B: Questions and Answers

Question: What is the mean effective pressure (mep)?

Answer: The mean effective is a theoretical constant pressure that, if acted on the piston during the power stroke, would produce the same net work as actually developed by one cycle.

$$\text{mep} = \frac{\text{net work for one cycle}}{\text{displacement volume}}$$

Question: There is a trick for problem 6;

$$\left(\frac{\partial V}{\partial U}\right)_T = \frac{\left(\frac{\partial V}{\partial P}\right)_T}{\left(\frac{\partial U}{\partial P}\right)_T}$$

It will be nice if you can resolve this mystery --- why is the equation (trick) above valid?

Answer: Since in both terms on the right side of the equation above T is constant, you 'may want to think' the numerator and denominator as total derivatives with respect to P, and as you know in that case, you can cancel dP (i.e. if T is constant, think of everything is a function of only P).

If you are not happy with this, here is a formal proof using the general procedure discussed in the 2nd handout (Chapter 6 *** BE SURE YOU BECOME AN EXPERT IN THESE TYPE OF DERIVATIONS AS WE WILL ASK YOU TO DEMONSTRATE YOUR KNOWLEDGE ON THESE TYPE OF CALCULATIONS!):

$$\left(\frac{\partial V}{\partial U}\right)_T = \frac{\begin{vmatrix} V_T & T_T \\ V_P & T_P \end{vmatrix}}{\begin{vmatrix} U_T & T_T \\ U_P & T_P \end{vmatrix}} = \frac{V_T T_P - V_P T_T}{U_T T_P - U_P T_T} = \frac{V_T 0 - V_P 1}{U_T 0 - U_P 1} = \frac{\left(\frac{\partial V}{\partial P}\right)_T}{\left(\frac{\partial U}{\partial P}\right)_T}$$

Note the trivial substitutions above: $T_T = \left(\frac{\partial T}{\partial T}\right)_P = 1$ and $T_P = \left(\frac{\partial T}{\partial P}\right)_T = 0$