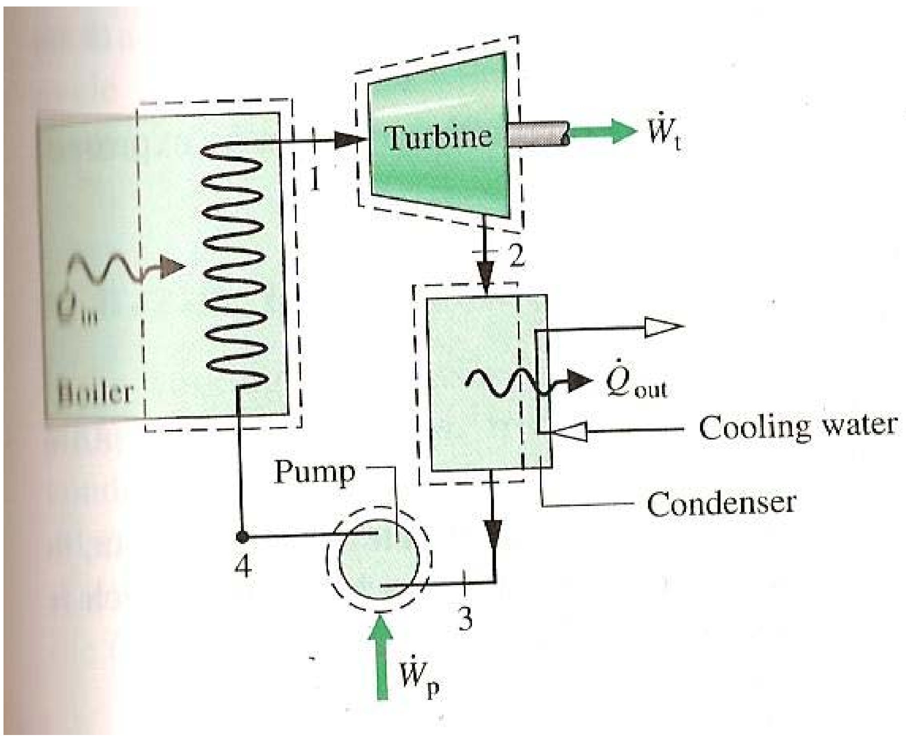
Recitation Handout 7

10/15/2007 – 10/29/2007

AGENDA

1. **Rankine cycles**
2. **Gas power cycles**

RANKINE CYCLE



Perform energy balance on the following subsystems to obtain these relations.

Turbine

$$\frac{\dot{W}_t}{\dot{m}} = h_1 - h_2$$

Condenser

$$\frac{\dot{Q}_{out}}{\dot{m}} = h_2 - h_3$$

Pump

$$\frac{\dot{W}_p}{\dot{m}} = h_4 - h_3$$

Boiler

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_1 - h_4$$

Thermal Efficiency

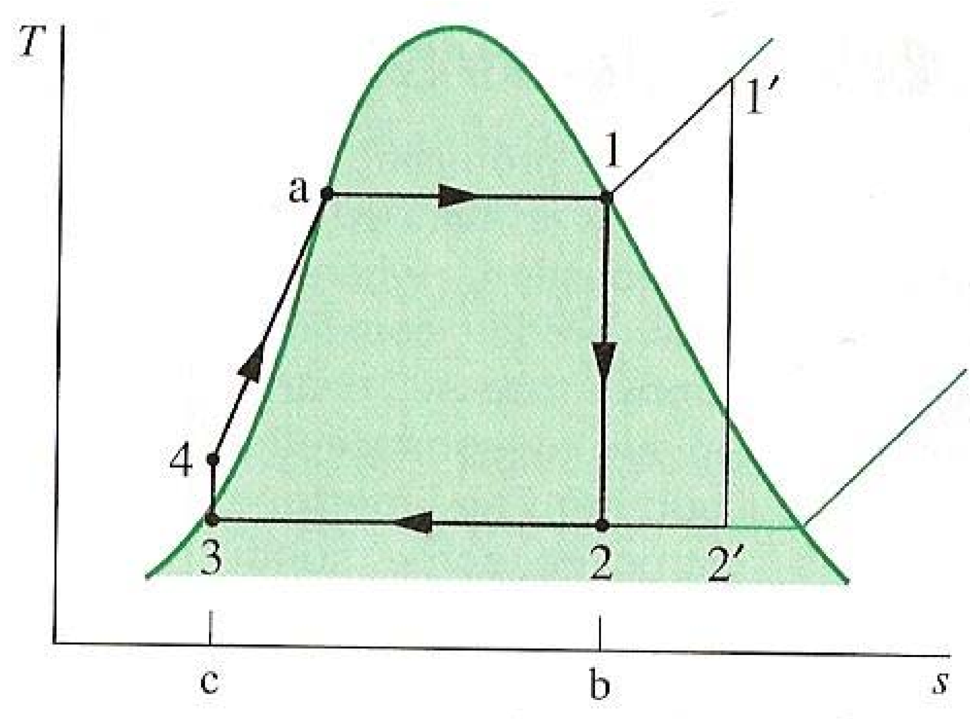
$$\eta = \frac{\dot{W}_t/\dot{m} - \dot{W}_p/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{(h_1 - h_2) - (h_4 - h_3)}{h_1 - h_4}$$

Alternatively,

$$\eta = \frac{\dot{Q}_{in}/\dot{m} - \dot{Q}_{out}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = 1 - \frac{\dot{Q}_{out}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = 1 - \frac{(h_2 - h_3)}{h_1 - h_4}$$

Back Work Ratio

$$bwr = \frac{\dot{W}_p/\dot{m}}{\dot{W}_t/\dot{m}} = \frac{(h_4 - h_3)}{(h_1 - h_2)}$$



To obtain the enthalpy h values:

Start at **state 1:**
Saturated vapor with pressure given.
h_1 = look up table.

State 2:
Fixed by a pressure; notice that the working fluid is in the saturation region.
The turbine is adiabatic and reversible in this case.
$s_2 = s_1$**,** with s2, look at the saturated liquid and saturated vapor data, solve for quality factor x_2.
$x_2 = s_2 – s_f / s_g - s_f$

with x_2, solve for
$h_2 = h_f + x_2 h_{fg}$

State 3:
Saturated liquid at a set pressure.
h_3 = look up table.

State 4:
Fixed by boiler pressure p4.
We can approximate the enthalpy change from 3 to 4 with the following
$h_4 = h_3 + \dot{W}_p / \dot{m} = h_3 + v_3(p_4 - p_3)$

With all the h values solved, we can simply solve these kind of problems with the net power, bwr, thermal efficiency using the previously introduced relations.

Irreversibilities and Losses
With an actual Rankine cycle, the pump and turbine are never reversible. To take into account the irreversibilities present, we have to use isentropic turbine efficiencies for these two subsystems.

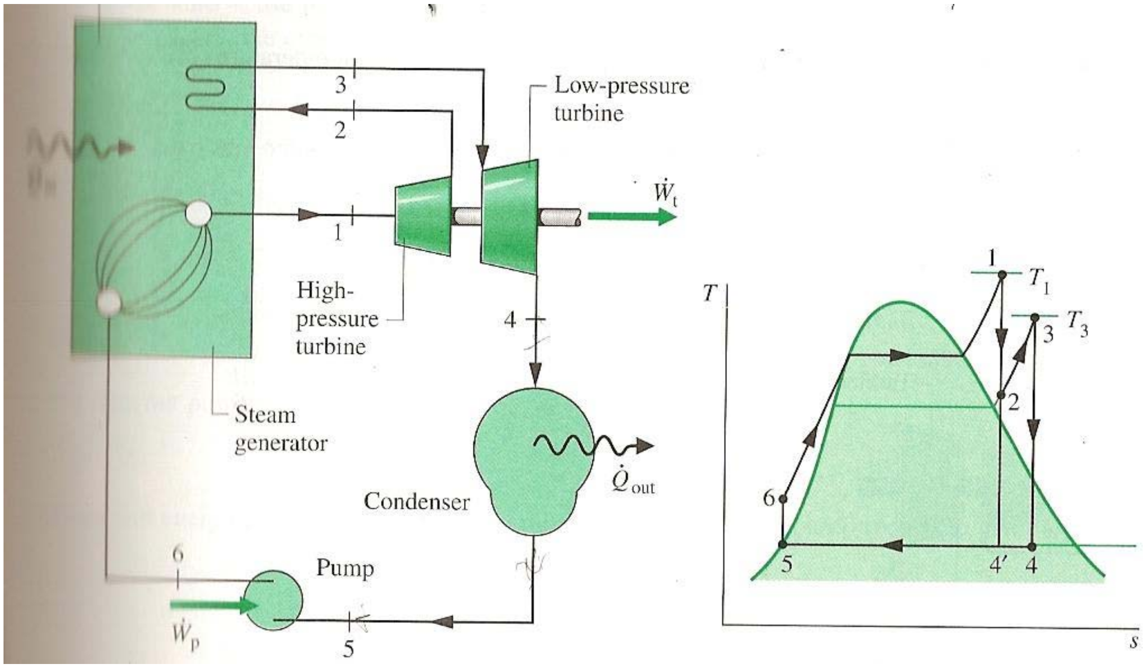
Turbine

$$\eta_t = \frac{(\dot{W}_t / \dot{m})}{(\dot{W}_t / \dot{m})_s} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

Pump

$$\eta_t = \frac{(\dot{W}_p / \dot{m})_s}{(\dot{W}_p / \dot{m})} = \frac{h_{4s} - h_3}{h_4 - h_3}$$

SUPERHEAT & REHEAT



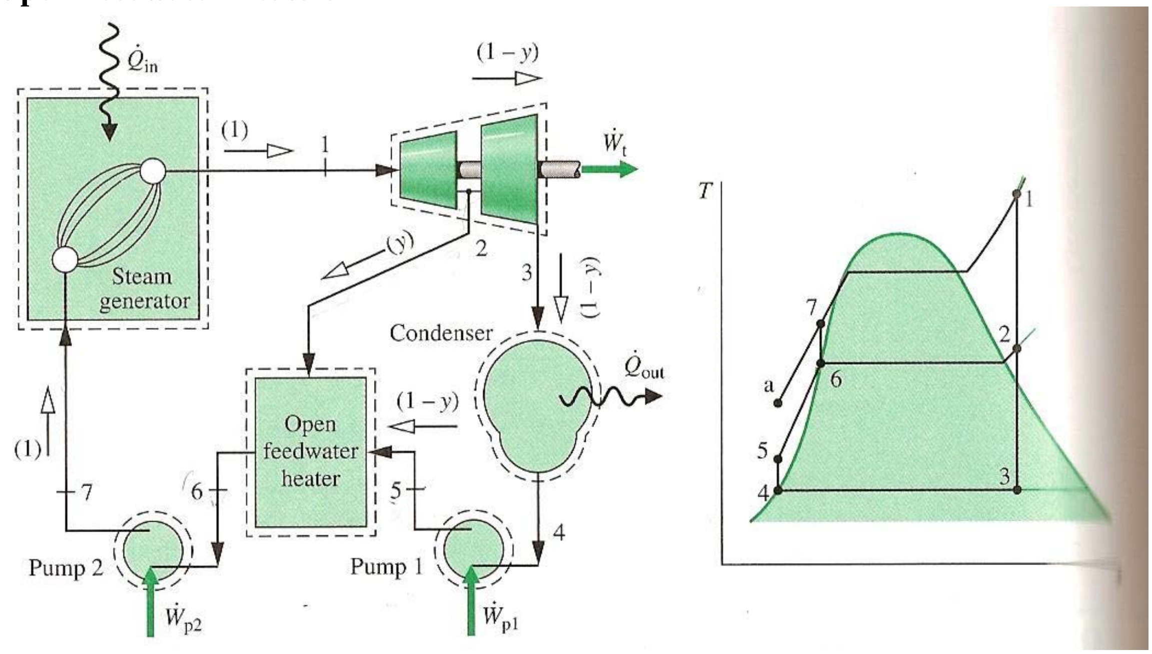
Note that the pressure from state 2 to state 3 is not increased.

$$\eta = \frac{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)}{(h_1 - h_6) + (h_3 - h_2)}$$

REGENERATIVE VAPOR POWER CYCLES

For generative vapor power cycles, perform an energy balance on feedwater heaters to obtain the fraction of flow in each line.

Open Feedwater Heaters



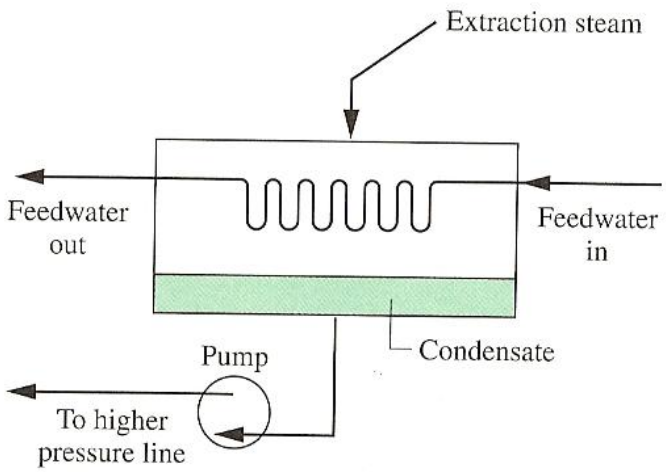
Mass and energy balance on the open feedwater heater comes out to

$0 = yh_2 + (1-y)h_5 - h_6$

Rearrange to solve for y,

$$y = \frac{h_6 - h_5}{h_2 - h_5}$$

Closed Feedwater Heaters



For example, a steam trap takes the condensate at the bottom and sends it to the condenser

Mass and energy balance on the open feedwater heater comes out to
$0 = y(h_2-h_7) + (h_5-h_6)$

Rearrange to solve for y,

$$y = \frac{h_6 - h_5}{h_2 - h_7}$$

PART B: QUESTIONS AND ANSWERS

For all problems, we highly recommend that you always start by drawing a T-s or P-V diagram indicating all states as well as what you know and what you don't know.

Hint 1: Eq. 6.53b states that $\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{\text{int}\\ rev}} = -\int_1^2 v\,dp$ for $(\Delta ke = \Delta pe = 0)$

This is an essential equation for calculating the work per unit mass for REVERSIBLE and STEADY-STATE CONTROL VOLUMES (in addition you assume KE=PE=0). You apply this equation for turbines, compressors, etc.—not for closed systems!
Note for many problems, you couple the above equation with the classical work equation in terms of enthalpies to compute desired unknowns. For e.g., for the pump (state 3 to state 4), you can arrive at $\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{\text{int}\\ \text{rev}}} = -\int_3^4 v\,dp = v_3(p_4-p_3) = h_4-h_3$, from which you can compute h_4.

Hint 2: The back work ratio is defined in the text as follows:

$$bwr = \frac{(\dot{W}_p/\dot{m})}{(\dot{W}_t/\dot{m})}$$

i.e. as the ratio of the pump work to turbine work.

Hint 3: Pay attention to all isentropic processes. Lets say you go from state 1 (s_1) to state 2 at a given pressure (P_2) (that's the case in some of the homework problems).
Be sure you compute s_g at state 2 (from the given pressure P_2) and compare it with $s_2 = s_1$ to decide if you have steam ($s_2 > s_g$) or mixture!!

Hint 4: Review all equations for isentropic processes of ideal gases. For example,

$$v_{r2} = \frac{V_2}{V_1} v_{r1}$$

and $\frac{p_2}{p_1} = \frac{p_{r2}(T_2)}{p_{r1}(T_1)}$

All of these equations are discussed in the Q&A handout 6. They can be useful for gas power cycles as in problem 7.

Q: **What is *relative pressure* and how is it related to pressure of an ideal gas in an isentropic process?**

A. In an isentropic process for an ideal gas, the change in entropy is expressed as

$$s_2 - s_1 = s^o(T_2) - s^o(T_1) - R\ln\left(\frac{p_2}{p_1}\right) = 0 \quad \Rightarrow \quad s^o(T_2) = s^o(T_1) + R\ln\left(\frac{p_2}{p_1}\right) \quad \ldots(1)$$

From (1), we have $p_2 = p_1 \exp\left(\frac{s^o(T_2) - s^o(T_1)}{R}\right) \quad \Rightarrow \quad \frac{p_2}{p_1} = \exp\left(\frac{s^o(T_2)/R}{s^o(T_1)/R}\right) = \frac{p_{r2}}{p_{r1}}$

where p_{r1} and p_{r2} are non-dimensional quantities dependent on temperatures alone. The function p_r is called the *relative pressure*. Once p_r is known, all other states can be fixed from data in tables.
