Recitation Handout 6

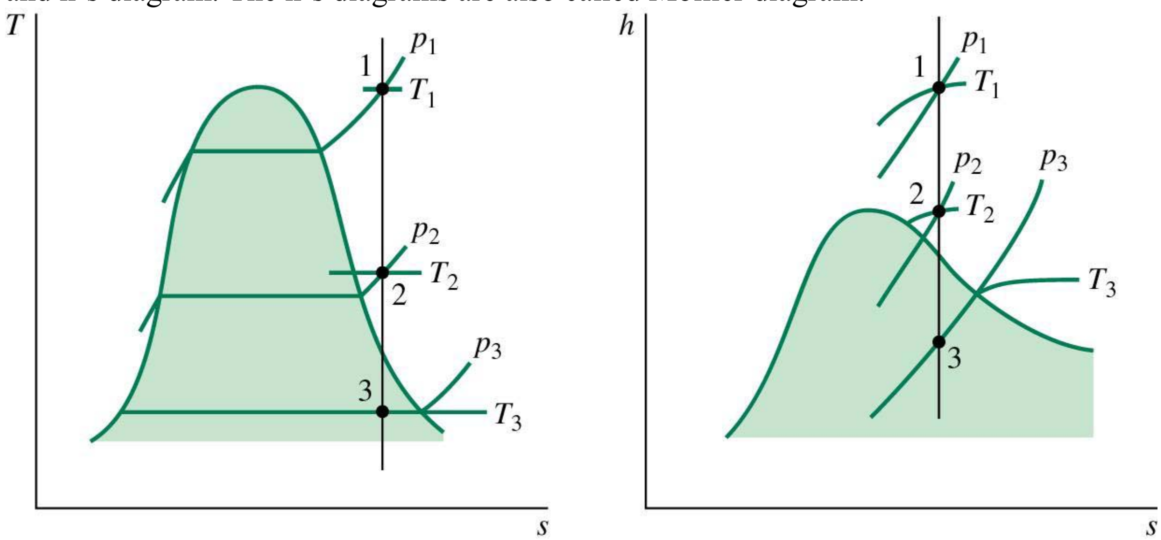
10/01/2006 – 10/10/2007

AGENDA

1. **Isentropic processes**
2. **Relative pressure for AIR**
3. **Isentropic efficiencies for turbines and compressors**
4. **Ideal Valve - Throttling device**
5. **Work for steady-state internally reversible pumps**

1: Isentropic processes

As the term indicates, isentropic processes are processes which have a constant entropy. An example of such processes is shown in the figure, representing the process on a T-s and h-s diagram. The h-s diagrams are also called Mollier diagram.



From equation (6.20a) in text and previous recitation handout, the change in specific entropy **FOR AN IDEAL GAS** between states 1 & 2 is

$$0 = s(T_2, p_2) - s(T_1, p_1) = s^0(T_2) - s^0(T_1) - R\ln\left(\frac{p_2}{p_1}\right)$$

$$\Rightarrow s^0(T_2) = s^0(T_1) + R\ln\left(\frac{p_2}{p_1}\right) \qquad \rightarrow 1$$

Therefore, given any 3 quantities, the fourth quantity can be evaluated using the Tables provided in the text.

2: Relative pressure for AIR

Consider Equation 1 above, the pressure ratio p_2/p_1 can be evaluated for an isentropic process as (Look section 6.11.2 in the book for details)

$$\frac{p_2}{p_1} = \exp\left[\frac{s^0(T_2) - s^0(T_1)}{R}\right] = \frac{\exp\left[s^0(T_2)/R\right]}{\exp\left[s^0(T_1)/R\right]} = \frac{p_{r2}(T_2)}{p_{r1}(T_1)}$$

$p_{r2}(T_2)$ and $p_{r1}(T_1)$ are called the relative pressures and are functions of Temperature ONLY. **IT IS IMPORTANT TO NOTE THAT THIS IS VALID ONLY FOR AIR. The values of p_r are tabulated in Table A-22 for air.**

Look at solved example 6.9 on page 294 of text for a problem using the reduced pressure.

3: Isentropic efficiencies

Isentropic efficiencies involve a comparison between the actual performance of a device and the performance that would be achieved under idealized circumstances (isentropic process) **for the SAME INLET STATE and THE SAME EXIT PRESSURE as the actual process.**

3a. Isentropic efficiency of a turbine: ASSUME a turbine with negligible heat transfer with the surroundings. The inlet state (normally, the temperature T_1 and pressure p_1) and exit states (pressure p_2) are fixed. Such a process is shown on the h-s diagram shown below.

From mass and energy balances, we get

$$\frac{\dot{W}}{\dot{m}} = h_1 - h_2$$

From the second law, we get

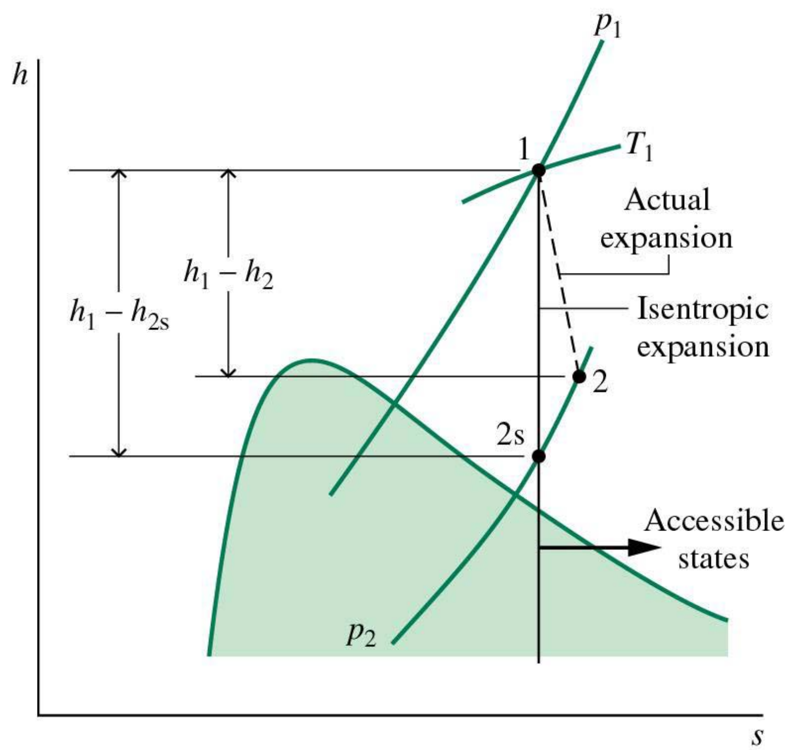
$$\frac{\dot{\sigma}}{\dot{m}} = s_2 - s_1 \geq 0$$

⇨ $s_2 \geq s_{2s} = s_1$

The isentropic turbine efficiency is then defined as

$$\eta = \frac{\left(\frac{\dot{W}}{\dot{m}}\right)}{\left(\frac{\dot{W}}{\dot{m}}\right)_s}$$

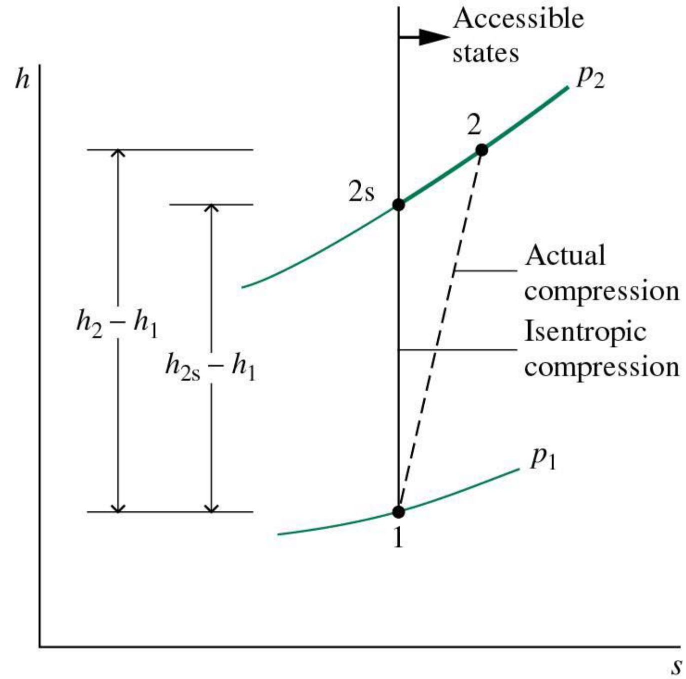
The isentropic efficiency is strictly ≤ 1.0



3b. Isentropic efficiency for a compressor: Following a similar analysis as that of a turbine, the isentropic compressor efficiency is defined as

$$\eta = \frac{\left(\frac{-\dot{W}}{\dot{m}}\right)_s}{\left(\frac{-\dot{W}}{\dot{m}}\right)}$$

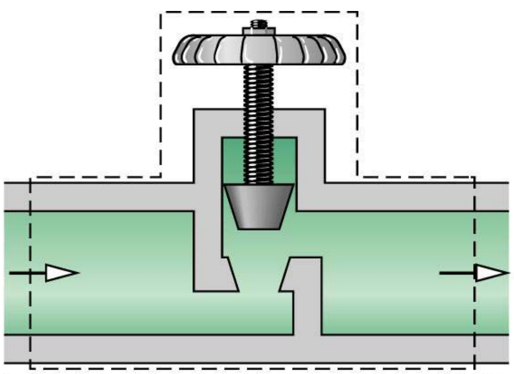
, where $\dot{W}$ is the power input to the compressor. The h-s diagram for a compressor is shown in the figure below.



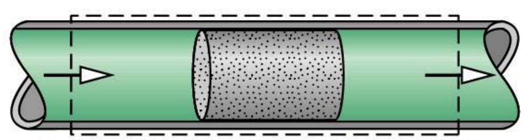
NOTE:
The isentropic efficiency of a compressor is also strictly ≤ 1.0, UNLIKE the COP (which is always > 1).

4: Ideal Valve – throttling device

A valve is generally inserted in a flow to achieve a reduction in pressure. Such devices are called throttling devices. Shown in the picture below are a couple of throttling devices. Look up section 4.3.3 in text for more details.



Partially open valve



Porous plug

Consider the valve to have negligible heat interaction with the surroundings, then from mass and energy balance, we get $h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$. If the kinetic energies at inlet and exit are same (ideal assumption), then $h_1 = h_2$. Such an idealized process is called throttling process and UNLESS OTHERWISE MENTIONED ALL VALVES ARE IDEAL VALVES.

NOTE: IDEAL VALVES DOES NOT MEAN REVERSIBLE PROCESSES. You will need to calculate the entropy production from second law.

5. Work for steady-state internally reversible devices (mostly pumps)

Eq. 6.51c states that $\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{\text{int}\\ rev}} = -\int_1^2 v dp$ for $(\Delta ke = \Delta pe = 0)$

This is an essential equation for calculating the work per unit mass for REVERSIBLE and STEADY-STATE CONTROL VOLUMES (in addition you assume KE=PE=0). You can apply this equation for turbines, compressors, etc.—not for closed systems! – most problems in the book use this equation for pumps where you specify the pressures at the inlet and outlet and you assume you know the specific volume at the inlet.

For example, for a pump going from p_1 to p_2, you can compute:

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{\text{int}\\ rev}} = -\int_1^2 v dp = -v_1(p_2-p_1) .$$

Many times as shown in class, you couple the above equation with the first law of thermodynamics in terms of enthalpies to compute desired unknowns (e.g. enthalpy at the exit of the pump). For e.g., for the pump (state 1 to state 2), you can arrive at $\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{\text{int}\\ rev}} = -\int_1^2 v dp = -v_1(p_2-p_1) = h_2-h_1$, from which you can compute h_2 (assuming you know h_1).

Finally note that if the pump (or other device) are not operating reversibly, you need to account for the isentropic efficiency. Thus the reversible work is (take state 2s and not 2)

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{\text{int}\\ \text{rev}}} = -\int_1^{2s} v dp = -v_1(p_{2s}-p_1)$$

and the actual work is then

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right) = \left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{\text{int}\\ \text{rev}}} / \eta$$

Also note in this case: $\left(\frac{\dot{W}_{cv}}{\dot{m}}\right) = h_2-h_1$ (you use the actual work and not the reversible work).

PART B: QUESTIONS AND ANSWERS

Q: When do I use c_v and c_p in calculations related to the first law?

A: From the definition of c_v, you can write: $du = c_v\, dT$. This is generally true for constant v (remember that $c_v = \frac{\partial u}{\partial T}|_v$). However, remember that for ideal gases u is only a function of T thus you can always write $du = c_v\, dT$ and integrate as done in class.

Similarly, from the definition of c_P, you can write: $dh = c_p\, dT$. This is generally true for constant P (remember that $c_p = \frac{\partial h}{\partial T}|_p$). However, remember that for ideal gases h is only a function of T thus you can always write $dh = c_P\, dT$ and integrate as done in class.

Don't mix c_v and c_p. There are two different things. Also note $c_p - c_v = R$ ONLY for ideal gases (can you prove it?).

For incompressible materials (liquids for examples), we consider that u depends only on T (see Section 3.10.2). In this section, it is shown that for such incompressible materials, $c_v = c_p = c$ (they are the same thing).

Q: In the equations given above, why there is a negative sign for the isentropic efficiency of the compressor? Why not for turbine?

A: In a compressor, work is done on a gas to raise the pressure in a system, whereas for a turbine, work is developed and pressure is decreased. By putting a negative sign, we are expressing the work INPUT (which is negative according to the first law) as opposed to the turbine's work OUTPUT (which it is positive). Just use common sense in these definitions. There is nothing to memorize.

HINTS TO HOMEWORK 6

Problem 1

(a) Use thermal efficiency equation

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_H} = 1 - \frac{\dot{Q}_C}{\dot{Q}_H}$$

Do mass and energy balances for the boiler and condenser

(b) For a Carnot Cycle,

$$\left|\frac{\dot{Q}_C}{\dot{Q}_H}\right| = \frac{T_C}{T_H}$$

Problem 2

Apply Eq. 6.28,

$$\frac{dS}{dt} = \frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2} + \frac{\dot{Q}_3}{T_3} + \dot{\sigma} = 0$$

Problem 3

(a) Energy balance

$$\Delta U = 0$$

For constant specific heat c, $\Delta u = c\Delta T$

Entropy balance

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \sigma = 0 + \sigma \Rightarrow \Delta S = \sigma$$

(b) $\sigma \geq 0$ using expression in part a

Problem 4

Part a and b in previou HW. Do differentiation on part b and set it to zero for maximum value.

Problem 5

(a) Do mass and energy rate balances for the condenser

(b)

$$\eta = \frac{\dot{W}_t - \left|\dot{W}_p\right|}{\dot{Q}_{in}}$$

(c) Entropy balance

(d) Compare the values obtained in part c.

Problem 6

(a) Using Eq. 2.47 expressed in a rate basis,

$$\gamma = \frac{\dot{Q}_{out}}{\dot{W}_c}$$

(b) Consider an ideal turbine: one whose isentropic turbine efficiency is 100%.

Problem 7 & 8

Use mass balance, energy balance and isentropic efficiencies equations.