Recitation Handout 5

09/24/2007 – 10/1/2007

Topics covered in Class: Chapter 6 – Using Entropy

1. Entropy Production:

σ_{cycle} is defined as the entropy produced.

- If $\sigma_{cycle} = 0$; the process is reversible, there are no irreversibilities present in the cycle
- If $\sigma_{cycle} > 0$; there is an increase in entropy, there are irreversibilities present.
- $\sigma_{cycle} < 0$ is an invalid statement. This violates the second law of thermodynamics.

The entropy produced is related to the ratio of Q/T over the boundary of the control volume that is defined. If you have several Q_{in} and Q_{out}, you can use this as a summation of the ratios between Q and T:

$$\oint\left(\frac{dQ}{T}\right)_b = -\sigma_{cycle} \quad \text{or} \quad \sum\frac{\dot{Q}_{in}}{T} - \sum\frac{\dot{Q}_{out}}{T} = -\sigma_{cycle}$$

$$\left(\int_1^2 \frac{\delta Q}{T}\right)_{rev} = S_2 - S_1$$

2. Retrieving Entropy Data - Saturation

For saturation states, the values of sf and sg are tabulated as a function of either saturation pressure of saturation temperature. The entropy of a saturation state has relations to s_f and s_g identical in form to those of v, u, and h.

$$s = (1-x)s_f + xs_g = s_f + x(s_g - s_f)$$

Also, for a compressed liquid data, $s(T, p) \approx s_f(T)$

3. TdS Equations

The TdS equations allow entropy changes to be evaluated from other more readily determined property data.

$$(\delta Q)_{rev} = dU + (\delta W)_{rev}$$

$$\underline{TdS = dU + pdV}$$

$$H = U + pV$$

$$\mathrm{dH = dU + d(pV) = dU + pdV + vdp}$$

$$\mathrm{dU + pdV = dH - Vdp}$$

$$\underline{\mathrm{TdS = dH - Vdp}}$$

4. Solving for Entropy Change for an ideal gas:
Equations 6.17 and 6.18 are the two general statements for change in entropy with respect to the specific heat c_v, c_p where (for an ideal gas) $c_p(T) = c_v(T) + R$:

From previous TdS equations, using

$du = c_v(T)dT, dh = c_p(T)dT, pv = RT$

$$s(T_2, v_2) - s(T_1, v_1) = \int_{T_1}^{T_2} c_v(T)\frac{dT}{T} + R\ln\frac{v_2}{v_1} \qquad 6.17$$

$$s(T_2, v_2) - s(T_1, v_1) = \int_{T_1}^{T_2} c_p(T)\frac{dT}{T} - R\ln\frac{p_2}{p_1} \qquad 6.18$$

Choose which one to use depending on the known information. It is recommended to use the respective ratios if one of them is 1. Since the ln(1) is zero that term will cancel out.

5. Looking up values of Entropy for ideal gases and the introduction of s^o (when the specific heats are functions of temperature)

The term $s^o(T)$ is the specific entropy at temperature T and 1 atm – it allows you to compute changes in entropy for ideal gases when the c_v and c_p are functions of temperature. The values of s^o can be computed directly for a given T – in principle s^o is nothing else but the computed values for a given T of the integrals that appear in Equations 6.17 and 6.18.

Use equations 6.20a, (or 6.20b for a per molar basis) for variable c_v and c_p:

$$s(T_2, v_2) - s(T_1, v_1) = s^o(T_2) - s^o(T_1) - R\ln\frac{p_2}{p_1} \qquad 6.20a$$

6. Solving for change in entropy of an ideal gas when the specific heats are constant:

$$s(T_2, v_2) - s(T_1, v_1) = c_v \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1} \qquad 6.21$$

$$s(T_2, v_2) - s(T_1, v_1) = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1} \qquad 6.22$$

This is just a rewritten version of equations 6.17 and 6.18 to be used when ***specific heat is a constant***.

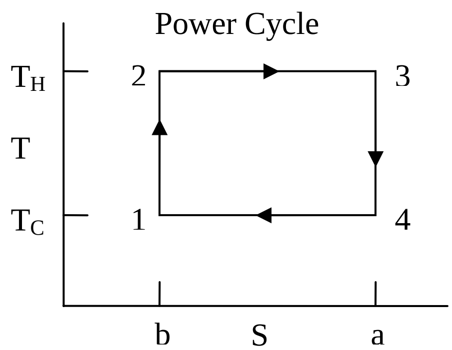
7. Incompressible Substance
This formula is applicable to solids and liquids that are modeled as incompressible.

$$s_2 - s_1 = c \ln \frac{T_2}{T_1}$$

8. Thermal efficiency:
Carnot Power Cycle

$$\eta = \frac{W_{cycle}}{Q_{23}} = \frac{area(1-2-3-4-1)}{area(2-3-1-b-2)}$$

$$= \frac{(T_H - T_c)(S_3 - S_2)}{T_H(S_3 - S_2)} = 1 - \frac{T_C}{T_H}$$



9. Entropy Production:

$$S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \sigma$$

The left term, ΔS is the change in entropy between state one and two. The integral term is the entropy transfer, the entropy entering or leaving, typically in the form of heat. The final σ is the entropy production within the system over the process 1-2.

To extend this equation to a control volume we add terms for mass transfer and the entropy of the entering and leaving masses, in a rate balance:

$$\frac{dS_{cv}}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_{cv}$$

Just as in the previous equation, ΔS is the change in entropy within the control volume. The summation terms are the influx and outflux of entropy in the form of heat and in the specific entropy of the entering and exiting masses. σ is the entropy production within the control volume.

HW Hints

1. **Problem 1:** Assume an ideal gas. Part (a) only needs to use the efficiency of the Carnot cycle. The temperature at the end of the isothermal expansion can be computed. That should give you the maximum temperature of the cycle. For part (b) use conservation of energy for closed system: $m(u_2 - u_1) = Q_{12} - W_{12}$ -- this is an isothermal process (what does that mean for u?) – can you compute the work for an isothermal process of an ideal gas? (c) Recall that $\frac{|Q_{34}|}{T_C} = \frac{|Q_{12}|}{T_H}$ (proved in class. Can you check and repeat this proof?)

2. **Problem 2:** This was solved in class… be sure you repeat the calculations here line by line… this is an important derivation!

3. **Problem 3:** (a) Use an energy and entropy balance for the cycle. (b) Heat transfer happens from hot to cold! (c) You need to figure out from your derivations in part s (a) and (b) where are the internal and external to the system irreversibilities.

4. **Problem 4:** The final temperature can be determined using an energy balance for a closed system. What is Q and W here? Also for incompressible materials can use for each little piece dz of the rod: $du = c(T_f - T(z))dm$ where T_f is the final temperature. Then for the whole rod you can write: $\Delta U = \int_0^L du$ ….. That together with the 1st law should get you a nice result for the final temperature.To find the entropy production, an entropy balance reduces to $\Delta S = \underbrace{\int_1^2 \frac{\delta Q}{T}} + \sigma$ -- what is dQ here? Also check the dS calculation for incompressible materials (see Section 3.10.2 for a review of what is an incompressible material). For each little piece of the rod…. $dS = dm \times c \ln \frac{T_f}{T(z)} = \rho A c \ln \frac{T_f}{T(z)} dz$. Can you integrate that?

5. **Problem 5:** Done in recitation!

6. **Problem 6:** Use equations 6.20-6.22 (pages 264 and 265 from the text – also given above). $s^o(T)$ data from Table A22 and constant specific heat values from Table A-20. Compare the two methods and understand what $s^o(T)$ is all about.

7. **Problem 7:** Use Eq.(6.23) $Q = \int_1^2 TdS = mT(s_2 - s_1)$ and get s data from Tables A-2 and A-4. Once you obtain Q, apply an energy balance (1st law) $W = Q - m(u_2 - u_1)$ and again use u-data from Tables A-2 and A-4.

8. **Problem 8:** Use $\Delta S = \underbrace{\int_1^2 \frac{\delta Q}{T}} + \sigma$ and figure out if ΔS is positive, zero or negative. What does the 2nd law tells you? Does it control the sign of ΔS or the sign of σ? You may not be able to figure out the sign of ΔS (indeterminate solution). For the last part of the problem use Eq. 6.20(a) that is also given earlier in this handout.