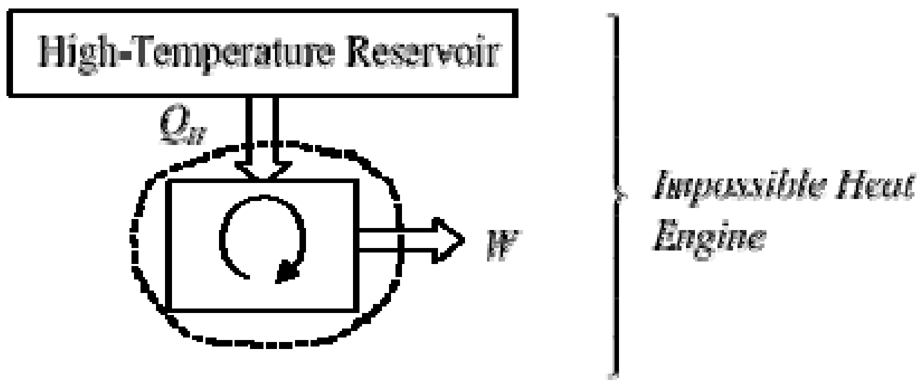
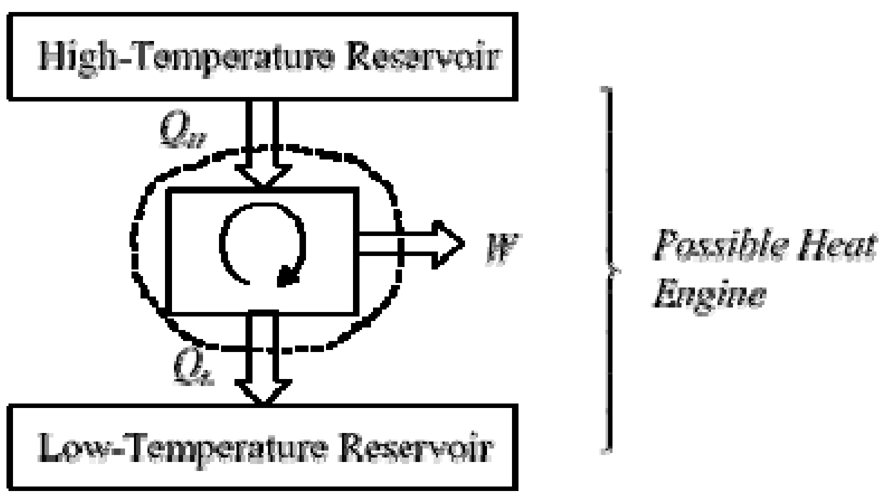
Recitation Handout 4

09/17/2007 – 09/24/2007

KELVIN-PLANCK STATEMENT OF SECOND LAW

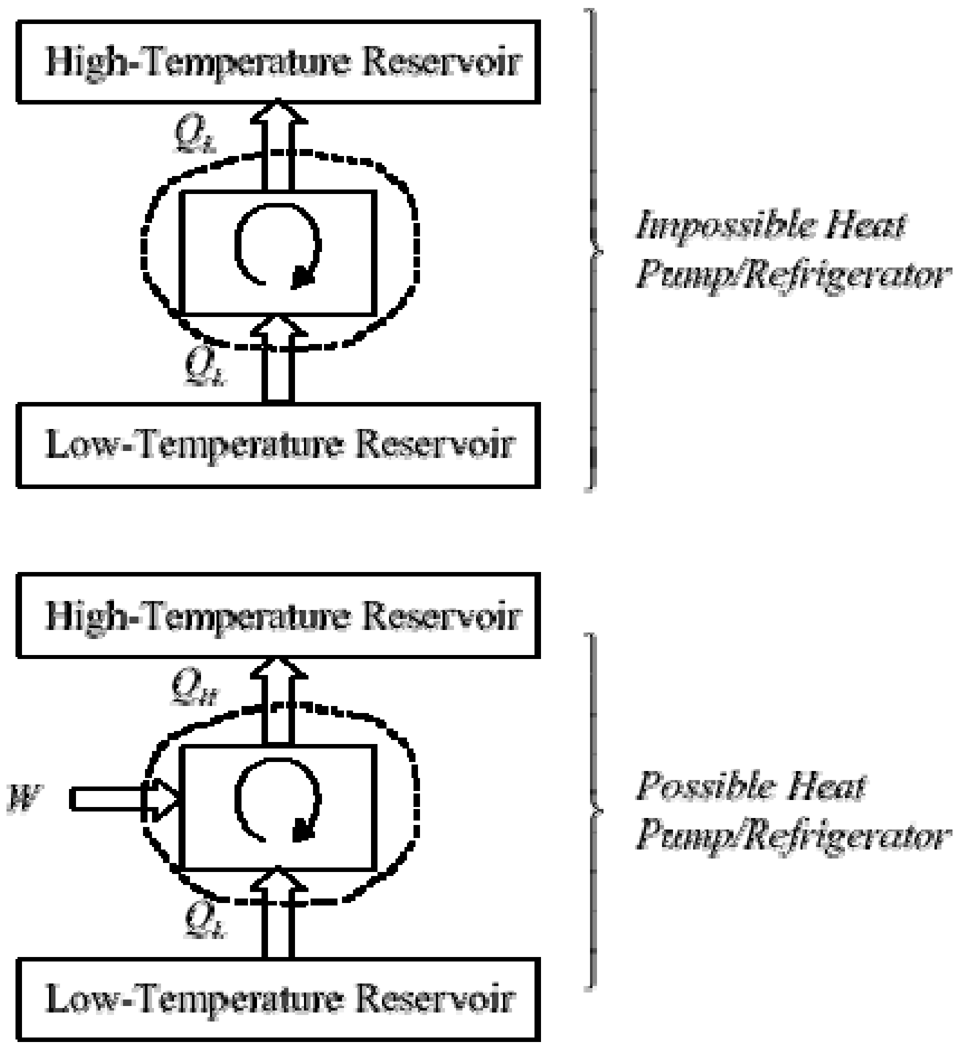
It is impossible to construct a heat engine that will produce nothing but work and the enchange of heat with a single reservoir.





$$\eta_{thermal} = \frac{W}{Q_H} = \frac{Q_H - Q_L}{Q_H}$$

It is impossible to construct a refrigerator that will transfer heat from a low temperature body to a high temperature body without any other effect on the surroundings.



For refrigerator: $\beta = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L}$ For heat pump: $\beta = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_L}$

A truly Reversible process is ONLY theoretical; a list of what makes a process irreversible is on pg 220.

IN A CARNOT CYCLE

Ratio of Q_L and Q_H is proportional to the ratios of the cold and hot reservoirs.

$\frac{Q_L}{Q_H} = \frac{T_L}{T_H}$ where T_L and T_H are the temperatures of the cold and hot reservoirs.
Look at Equations (5.7) and (5.9 on pages 230 through 234or more details on efficiency of Carnot cycles. These efficiencies of the Carnot cycles are ideal – the best possible when working between two reservoirs of given temperatures T_L and T_H.

REVISITING ONCE MORE EXPRESSIONS FOR WORK DONE IN DIFFERENT PROCESSES

FOR AN IDEAL GAS

The derivation of the efficiency of the Carnot cycles given in class required some calculations of the work in isothermal and adiabatic (reversible) processes. These developments are summarized here in case you did not have time to copy them from the board. We will make extensive use of these results in follow up lectures and homework.

a) Isothermal process at temperature T;

$$W_{1\text{-}2} = \int_{V1}^{V2} Pdv = RT \int_{V1}^{V2} dv/v = RT \ln(v_2 / v_1)$$

b) Adiabatic process (dQ = 0)

Energy balance: $dQ = dU + dW = dU + PdV = 0$ for an adiabatic process

$\rightarrow dU = -PdV$

$\rightarrow m\, C_v\, dT = -P\, dv$. For an ideal gas => $P = mRT/V$

$\rightarrow m\, C_v\, dT/T = -m\, R\, dV/V$.

Integrating between states 1&2, $\int_{T1}^{T2} C_v\, dT/T = -R \int_{v1}^{v2} dV/V$

$C_v \ln(T_2 / T_1) = -R \ln(V_2 / V_1)$

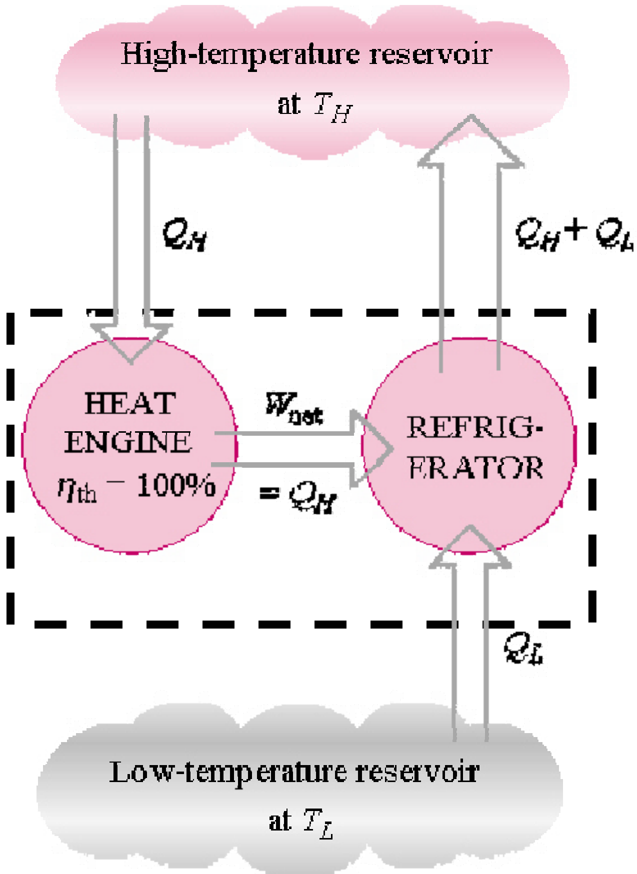
=> $T_2 / T_1 = (V_1 / V_2)^{(R/C_v)} = (V_1/V_2)^{(k-1)}$; $R/C_v = (C_p - C_v)/C_v = C_p/C_v - 1 = k-1$ for an ideal gas.

=> $(P_2 V_2) / (P_1 V_1) = (V_1/V_2)^{(k-1)}$ (because $T_2/T_1 = P_2 V_2 / P_1 V_1$ for an ideal gas)
=> $P_2 / P_1 = (V_1/V_2)^k$ => $P V^k$ = constant.

As discussed earlier in class, this is a polytropic process with polytropic process constant n = k ! So an adiabatic process of an ideal gas is a polytropic process.

PROOF THAT VIOLATION OF KELVIN-PLANCK STATEMENT IS A VIOLATION OF CLAUSIUS STATEMENT

Consider a heat engine and a refrigerator. Assume that the heat engine violates the Kelvin-Planck statement. Therefore, $W_{net} = Q_H$ for the heat engine cycle. Consider the refrigerator cycle, which takes heat from the cold reservoir (Q_L) and dumps the heat into the hot reservoir ($Q_H + Q_L$). Consider the system shown in dotted lines (also undergoing a thermodynamic cycle). The net heat transfer is a heat transfer from the cold reservoir to the hot reservoir with no external work. Thus it is a violation of the Clausius statement. Hence proved.



Various Questions and Answers

Question 1: How does one use the quality when solving a problem? What information can one get using the tables and the quality?

Answer: The quality is a measure of the ratio of vapor present to the total mass of the mixture (mixture because it the fluid/gas is in the two phase region). It is defined as:

$$x = \frac{m_{vapor}}{m_{liquid} + m_{vapor}}$$

See page 81 of the text for more information about quality. The quality can be used to find a variety of properties, including the specific volume, internal energy, enthalpy, and entropy. In solving homework problems one can use the quality in conjunction with the tables to determine values for the properties mentioned above. This is done with the following formula:

$$A = A_f + x(A_g - A_f)$$

where A is the property of interest, A_f is the value of the property as a saturated liquid, and A_g is the value of the property as a saturated vapor. A_f and A_g are given in the tables.

Question 2: How does one tell if a cycle is operating reversibly? Irreversibly? Is impossible?

Answer: One can determine the feasibility of a cycle by looking at the thermal efficiency. The thermal efficiency is given by:

$$\eta_{\max} = \frac{W_{cycle}}{Q_H} = 1 - \frac{Q_C}{Q_H}$$

The best possible efficiency a cycle can have is when it is operating reversibly. For a reversible proves, equations for thermal efficiency, and coefficient of performance are simplified to depend only on the temperatures of the hot and cold reservoirs the cycle is interacting with. By comparing the efficiency or coefficient of performance of the cycle to the best possible , one can determine the feasibility of the cycle….

If $\eta_{calculated} = \eta_{\max}$, the cycle is reversible,

If $\eta_{calculated} < \eta_{\max}$, the cycle is irreversible, and

If $\eta_{calculated} > \eta_{\max}$, the cycle is impossible.

Question 3: What is the difference between internal and external irreversibilities?

Answer: The only difference between the two types is only in the location of the system boundary wit respect to the location of the irreversibility. An internal irreversibility is located within the system bounds and an external irreversibility is located outside the system bounds.

For a piston cylinder system, in which there is friction acting on the piston, if the system is defined as only the gas inside of the cylinder, the friction is an external irreversibility. If the system is defined to include any of the piston, the friction is then an internal irreversibility. In both cases, it is the same irreversibility, and effects the system in the same way.

Question 4: What is the COP β for refrigeration cycles? What values this takes for reversible refrigeration cycles?

Answer: β is defined as Q_{in}/W_{cycle} or alternately as $Q_{in}/(Q_{out} - Q_{in})$. This definition is on page 67 of Moran & Shapiro (5th ed.). The maximum theoretical beta is an important quantity, and is defined in chapter 5, on pages 234 -- 237.

Question 5:When can I assume that a gas is an ideal gas and use the ideal gas equation?

A: Unless the problem clearly states that the gas is ideal, use the compressibility charts. This will never be a problem in the exams where it is stated how to treat the gas. However, in some problems in the text you suppose to use an ideal gas assumption even though it is not clearly stated to do so.

Hints for HW 4

- Problem 1:Remember that h is the same before and after the valve. So you can compute h_2. With a given h_2 and P_2, you can compute from the tables T_2. An energy balance for the turbine will give you what you want.
- Problem 2: As always define efficiency of cycles as the ratio of what you get (here work rate) divided by what you give (here $\dot{Q}_{in}$). However, be sure that the work you use here is the work of the turbine MINUS the work needed to operate the pump. Use absolute values in these calculations. The two work rates and the heat rate can be calculated EASILY in terms of the enthalpies of states 1,2…. (basically from energy balance of the different devices). To compute the mass rate of the condenser cooling water, do an energy balance there – energy flowing in = energy flowing out…. That should do it (we can only hope!)
- Problem 3: This was done in your recitation ….. no hints…. (but also don't copy it from others if you did not attend your recitation!)
- Problem 4: Life is hard (and indeed will get harder!). No hints can help here but lets try. To get the two required masses use the constant volume & pressure and the temperatures T_1 and T_2. The ideal gas equation should take care of the masses. For the second part of the problem, recall the transient problem from the book that Prof. Z did in class? (read the example from the book). Start the same way here but note that h_e is not constant here: $h_e = u + pv = u + RT$ (a function of the temperature T that varies from T_1 to T_2. Also use $U_{cv} = m\,u$ and $dU_{cv}/dt = m\,du/dt + u\,dm/dt$. Recall that $du = c_v\,dT$ and the relation of c_v and c_p for ideal gases. You should get a nice looking answer!!! Good luck..... Don't just copy these formulas in your HW just try to solve the problem……
- Problem 5: You know the best possible efficiency for a cycle between two reservoirs (it's the reversible Carnot cycle efficiency). Nothing can be better. If it is better, there is a problem. If less, it is irreversible.
- Problem 6 & 7: I don't think you need any more help. Just use the efficiency for a reversible cycle in terms of T_H and T_C (in your formulas for efficiency and coefficient of performance substitute the Q's with the T's to get the reversible cycle results). This is the best you can do efficiency wise.