

Recitation Handout 3

09/10/2007 – 09/16/2007

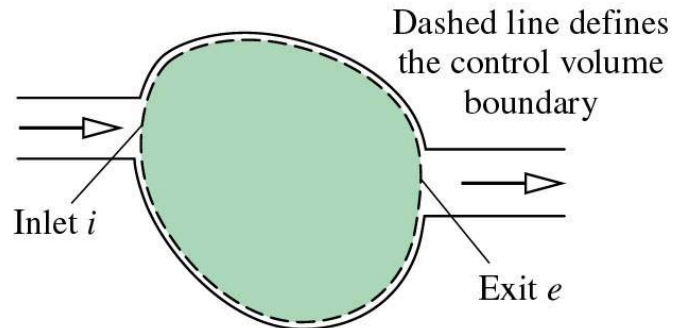
Part A: Recitation Notes

CONSERVATION OF MASS

Conservation of mass states that the time rate of change of mass in a control volume given by

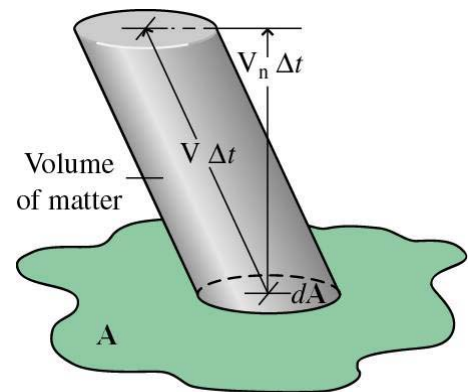
$$\frac{dm_{cv}}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e$$

where \dot{m}_i and \dot{m}_e are the mass flow rates into and out of the control volume.



EVALUATING THE MASS FLOW RATE

Instantaneous mass flow rate = $\dot{m} = \rho V_n A$; V_n is the velocity normal to the cross-section area A shown in the picture to the right. It is easier to use $1/v$ instead of ρ as v is available from the tables. v here is the specific volume.



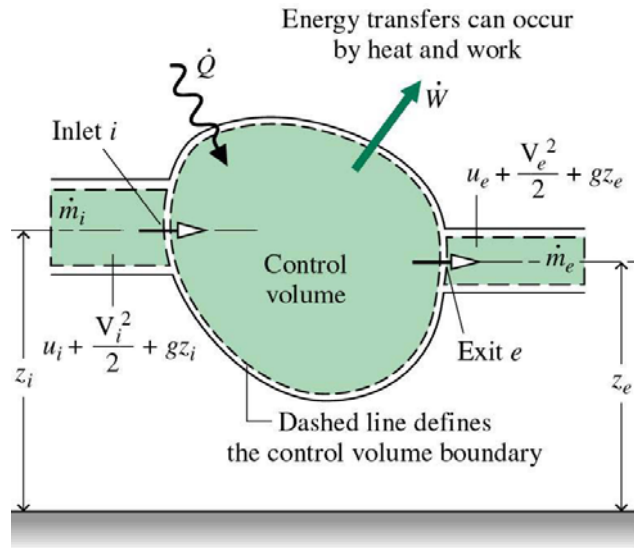
ONE-DIMENSIONAL FLOW FORM

The flow is said to be one-dimensional if

1. The flow is normal to the boundary at locations where the mass enters or exits
2. All intensive properties, including the velocity and density are uniform throughout the cross-section.

CONSERVATION OF ENERGY AT STEADY STATE

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right)$$



THERMODYNAMIC DEVICES

Nozzles and Diffusers

Nozzle – Velocity of a gas or liquid increases in the direction of flow.

Diffuser – The gas or liquid decelerates in the direction of flow.

$$\dot{W} = 0$$

Turbine

Work output develops as a result of a gas or liquid passing through a set of blades inside the turbine.

Compressor

Work is done on a gas passing through the compressor in order to raise the pressure.

Pump

The work input to a pump is used to change the state of a liquid passing through.

Heat Exchanger

A Device that transfers energy between fluids at different temperatures by heat transfer modes.

Throttling Device (Valve, Plug, etc...)

A restriction in a line through which a gas or liquid flows causes reduction in pressure.

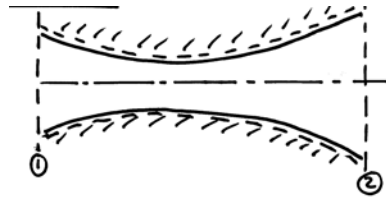
$$h_1 = h_2$$

EXAMPLE PROBLEM

Air enters an insulated diffuser operating at steady state with a pressure of 1 bar, a temperature of 300 K, and a velocity of 250 m/s. At the exit, the pressure is 1.2 bar and

the velocity is 150 m/s. Potential energy effects can be neglected. Using the ideal gas model, determine

- the ratio of the exit flow area to the inlet flow area.
- the exit temperature, in K.



Known: Data is provided for a diffuser at steady state, through which air is flowing.

Find: Determine the ratio of the exit flow area to the inlet flow area, and the exit temperature.

Assumptions:

- The control volume shown in the schematic is at steady state.
- For the control volume, heat transfer, work, and potential energy effects can be ignored.
- Air is modeled as an idea gas with a constant c_p .

Analysis: The mass rate balance is $\dot{m}_1 = \dot{m}_2$, or

$$\frac{(AV)_2}{v_2} = \frac{(AV)_1}{v_1} \Rightarrow \frac{(AV)_2 P_2}{RT_2} = \frac{(AV)_1 P_1}{RT_1} \Rightarrow \frac{A_2}{A_1} = \left(\frac{P_1}{P_2}\right) \left(\frac{T_2}{T_1}\right) \left(\frac{V_1}{V_2}\right) \quad (1)$$

The exit temperature, T_2 , can be obtained using an energy rate balance:

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right] \Rightarrow h_2 = h_1 - \frac{V_2^2 - V_1^2}{2}$$

Using data from table A-22

$$h_2 = 300.19 \text{ kJ/kg} + \left(\frac{250^2 - 150^2 \text{ m}^2}{2 \text{ s}^2} \right) \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{1000 \text{ N} \cdot \text{m}} \right| = 320.19 \text{ kJ/kg}$$

Then, interpolating in table A-22 for h_2 we find that

$$T_2 = \underline{\underline{319.9 \text{ K}}}$$

Returning to equation (1)

$$\frac{A_2}{A_1} = \left(\frac{1 \text{ bar}}{1.2 \text{ bar}} \right) \left(\frac{319.9 \text{ K}}{300 \text{ K}} \right) \left(\frac{250 \text{ m/s}}{150 \text{ m/s}} \right) = \underline{\underline{1.481}}$$

Hints for the problems in HW 3
(your thanks should go to Iris for carefully preparing this)

1. a) Remember the formula for work.

$$W_{12} = \int_1^2 P dV \rightarrow W_{12} = mP(v_1 - v_2)$$

b) Use the energy balance formula. In this problem, del KE and del PE are negligible
You can easily calculate u1 (liquid-vapor mixture → use the given “quality” value) and u2 from the table

2. a) Use the ideal gas equation of state, which is $m = PV/RT$. Be careful with R so that the equation is dimensionally correct (Units)

b) Use compressibility chart. Use equations for P_R and T_R and if you divide a volume by a specific volume, you can calculate the mass!

3. a)

$$P_1 V_1 = mRT_1$$

Combine these two equations to get a single equation in terms of P, V and

$$P_2 V_2 = mRT_2$$

T

b) Energy Balance Equation again. (del KE and del PE are negligible)

Use the ideal gas equation of state to find the mass that you need it to put in the energy balance equation.

$$4. \quad m_1 = \frac{(AV)_1}{v_1} = \frac{P_1(AV)_1}{RT_1}$$

5. Energy rate balance and $dm_{out}/dt = dm_{in}/dt$

$$0 = -\dot{W}_{cv} + \dot{m}_1 \left(h_1 + \frac{V_1^2}{2} \right) - \dot{m}_2 \left(h_2 + \frac{V_2^2}{2} \right) - \dot{m}_3 \left(h_3 + \frac{V_3^2}{2} \right)$$

6. $\dot{W}_{cv} = \dot{Q}_{cv} + \dot{m}(h_1 - h_2)$ ← This is a reduced energy rate balance equation for this problem.

7. Use Conservation of Mass → $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}_R$,
 $\dot{m}_3 = \dot{m}_4 \equiv \dot{m}_A$ and energy rate balance equation.