

## Recitation Handout 2

### ENRGD 221: Engineering Thermodynamics – Prof. Zabararas

Topics covered in class: Chapters 2 and 3

### RETRIEVING THERMODYNAMIC PROPERTIES

The emphasis in this class is to retrieve thermodynamic properties using *tables* commonly available for *pure simple compressible substances*.

Page 815 of your text gives a list of all tables for appropriate quantities in S.I. units.

Tables A-2 through A-6 (pages 817-826 in the text) give the properties of water – These tables are frequently referred to as *steam tables*.

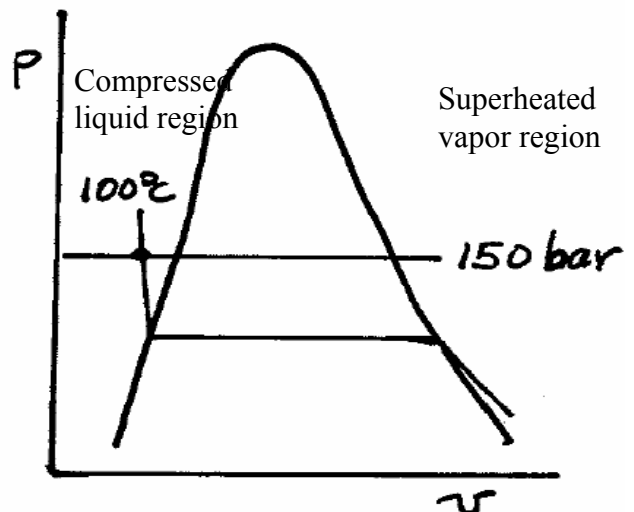
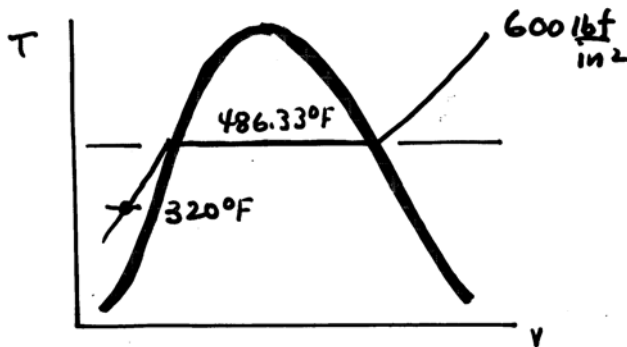
Table A-4 – water vapor table – superheated vapor;

Table A-5 – liquid water table – compressed liquid;

Tables A-2E through A-6E are the steam tables for water in English units.

#### Compressed liquid and Superheated vapor region

Pressure and temperature are independent properties in the single-phase liquid and vapor regions – we need both of them to fix a state in these regions



Some examples follow:

1. At a pressure of 10.0MPa, from Table A-4 (page 823 in text), the saturation temperature is 311.06°C.
2. For superheated water vapor at a pressure of 10.0MPa and temperature of 600 °C, the specific volume, from Table A-4 – look up the value for  $v$ , is seen as 0.03837 m<sup>3</sup>/kg.
3. At 10 MPa and 100 °C, the specific

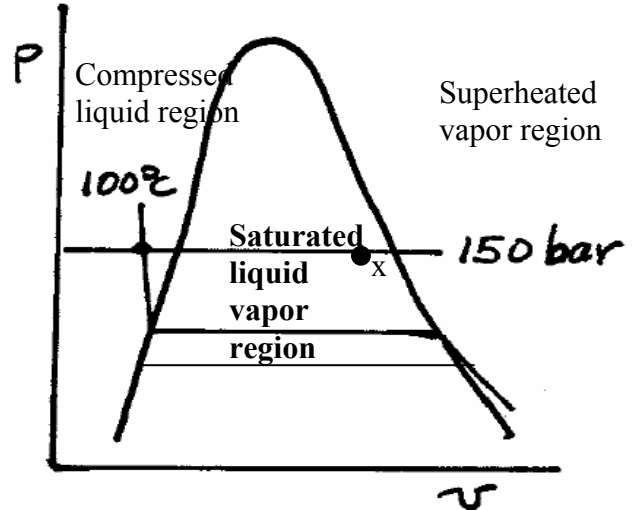
**volume of compressed water is – from Table A-5 (page 825 in text) –  $1.0385 \times 10^{-3} \text{ m}^3/\text{kg}$ . (Note that  $v \times 10^3$  is given)**

**Saturated liquid and vapor region**

The saturation tables A-2 and A-3 (pages 817-820 in text) list the properties of saturated liquid and vapor states

The properties are denoted by subscripts *f* and *g* to denote the fluid and gas, respectively.

In the saturated liquid or vapor region, only one property is necessary to fix the state – This property is normally either the pressure or the temperature - Table A-2 is called the temperature table as the temperature is listed in the first column while Table A-3 is called the pressure table because pressures are listed in convenient increments.



What is quality? Quality is the ratio of the mass of vapor to the total mass of the closed system. It is commonly denoted by *x*.

$$x = m_{gas} / (m_{fluid} + m_{gas})$$

If the quality is known, then the average specific volume of the saturated liquid – vapor mixture is calculated as

$$v = (1-x) v_f + x v_g = v_f + x(v_g - v_f)$$

For example:

**Consider a system with a 2 – phase liquid-vapor mixture of water at 100 °C and a quality of 0.9. From Table A-2, page 817 in text, looking up the column with temperature=100 °C; we see that –**

Specific volume of liquid ( <i>v<sub>f</sub></i> )	Specific volume of vapor ( <i>v<sub>g</sub></i> )	Internal energy of liquid ( <i>u<sub>f</sub></i> )	Internal energy of vapor ( <i>u<sub>g</sub></i> )	Enthalpy of liquid ( <i>h<sub>f</sub></i> )	Enthalpy of vapor ( <i>h<sub>g</sub></i> )
$1.0435 \times 10^{-3} \text{ m}^3/\text{kg}$	$1.673 \text{ m}^3/\text{kg}$	418.94 kJ/kg	2506.5 kJ/kg	419.04 kJ/kg	2676.1 kJ/kg

Therefore the specific volume of the mixture =

$$\begin{aligned} v &= v_f + x(v_g - v_f) \\ &= 1.0435 \times 10^{-3} + 0.9 * (1.673 - 1.0435 \times 10^{-3}) \\ &= 1.506 \text{ m}^3/\text{kg} \end{aligned}$$

Similarly, the average internal energy of the mixture is

$$\begin{aligned} u &= u_f + x(u_g - u_f) \\ &= 418.94 + 0.9*(2506.5 - 418.94) \\ &= 2297.75 \text{ kJ/kg} \end{aligned}$$

Enthalpy is defined - in chapter 3, page 89 in text – as

$$h = u + pv$$

and can be evaluated for the mixture with a given quality as

$$\begin{aligned} h &= h_f + x(h_g - h_f) \\ &= 419.04 + 0.9*(2676.1 - 419.04) \\ &= 2450.4 \text{ kJ/kg} \end{aligned}$$

### Approximation for liquids using the saturated liquid data

There are methods that approximate the properties of the compressed liquids – they are easier than using the compressed liquid tables (for example the compressed water table – Table A-5).

Approximate the values of  $v$ ,  $u$  at a given pressure,  $p$  and temperature,  $T$  by the corresponding values of the saturated liquid at temperature  $T$  (these approximations account for the fact of little pressure dependence of  $v$  and  $u$ );

$$\begin{aligned} v(p, T) &= v_f(T) \\ u(p, T) &= u_f(T) \end{aligned}$$

The approximation of enthalpy is not the enthalpy at the saturated liquid but is a correction defined as

$$h(p, T) = h_f(T) + v_f(T)*(p - p_{saturation})$$

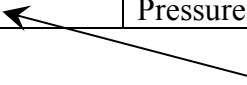
See Section 3.10.1 for more details.

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## TYPICAL EXAMPLES OF POSSIBLE INPUTS AND OUTPUTS TO TABLES

### 1. Compressed liquid and Superheated vapor region

	INPUT 1	INPUT 2	OUTPUT
Case 1	Pressure	Specific volume	Temperature
Case 2	Specific volume	Temperature	Pressure
Case 3	Pressure	Temperature	Specific volume


**Most common case – Table A-4 for superheated water vapor and Table A-5 for compressed water**

### 2. Saturated liquid and vapor region

	INPUT	OUTPUT 1	OUTPUT 2
Case 1	Specific volume of saturated liquid and/or vapor. <b>Not the specific volume of the mixture</b>	Pressure	Temperature
Case 2	Pressure	Temperature	Specific volumes of the saturated liquid and vapor
Case 3	Temperature	Pressure	Specific volumes of the saturated liquid and vapor


**Most common cases – Table A-3 for case 2 and Table A-2 for case 3**

Note: Many of the thermodynamic tables in your text list several other properties not referred above. As we introduce these properties during the semester, you should be able to think of many other input/output use of these tables.

**IDEAL GAS MODEL**

$$pv = RT \quad \text{Ideal gas equation of state}$$

Alternative forms:

$$pV = mRT \quad (\text{Substitute in } v = V/m)$$

$$p\bar{v} = \bar{R}T \quad \begin{array}{l} v = \bar{v} / M \\ R = \bar{R} / M \end{array} \quad M = \text{Molecular Weight}$$

$$pv = n\bar{R}T \quad \bar{v} = V / n$$

**Enthalpy:**

$$h = u + pv$$

$$pv = RT$$

$$u = u(T)$$

$$h = h(T) = u(T) + RT \quad (\text{Ideal Gas}) \quad (3.37)$$

**Specific Internal Energy:**

$u$  depends only on temperature.

$$c_v(T) = \frac{du}{dT} \quad (\text{Ideal Gas}) \quad (3.38)$$

$$u(T_2) - u(T_1) = \int_{T_1}^{T_2} c_v(T) dT \quad (\text{Ideal Gas})$$

**Specific Enthalpy:**

$h$  depends only on temperature.

$$c_p(T) = \frac{dh}{dT} \quad (\text{Ideal Gas}) \quad (3.41)$$

$$h(T_2) - h(T_1) = \int_{T_1}^{T_2} c_p(T) dT \quad (\text{Ideal Gas})$$

By differentiating Eq 3.37 with respect to temperature, and plugging in Eq 3.38 & 3.41, we can obtain

$$c_p(T) = c_v(T) + R \quad (\text{Ideal Gas}) \quad (3.44)$$

Specific heat ratio  $k$  is a function of temperature only

$$k = \frac{c_p(T)}{c_v(T)} \quad (\text{Ideal Gas})$$

Combining with Eq 3.44 results in

$$c_p(T) = \frac{kR}{k-1} \quad (\text{Ideal Gas})$$

$$c_v(T) = \frac{R}{k-1} \quad (\text{Ideal Gas})$$

### GENERALIZED COMPRESSIBILITY CHARTS

$\lim_{p \rightarrow 0} \frac{p\bar{v}}{T} = \bar{R} \rightarrow (1)$ , where  $\bar{R}$  is a common limit for all ideal gases as  $p \rightarrow 0$ .

We define compressibility factor as,  $Z = \frac{p\bar{v}}{RT} = \frac{pv}{RT}$ , where  $Z$  is a dimensionless ratio.

Equation (1) is expressed in terms of the compressibility factor  $Z$  as  $\lim_{p \rightarrow 0} Z = 1$

When the pressure of a gas is relatively small compared to the critical pressure,  $P_c$ ,  $Z$  is approximately 1.

When coordinates of the compressibility plots of  $Z$  with  $p$  at different  $T$  are modified, we obtain *quantitatively* similar plots for different gases.

When  $Z$  is plotted with reduced variables,  $T_R$  and  $p_R$ , defined as

$$p_R = \frac{p}{p_c} \quad \text{and} \quad T_R = \frac{T}{T_c},$$

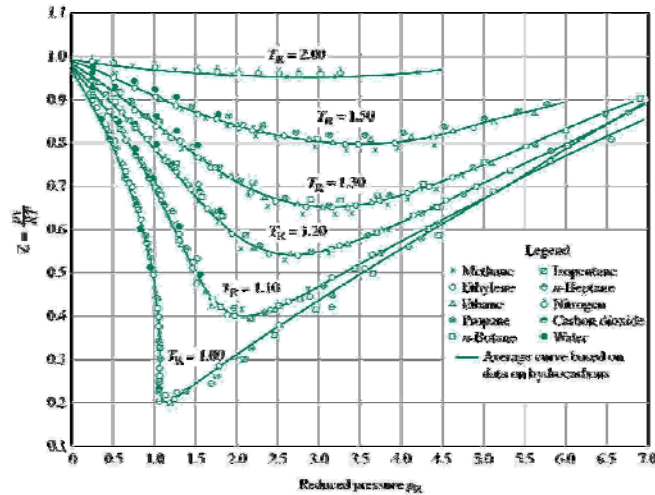
generalized compressibility charts are obtained.  $Z = f(T_R, p_R)$ .

Figures A1 – A3 (page 911-912) in text provide generalized charts where  $v'_R = \frac{\bar{v}p_c}{RT_c}$

where  $v'_R$  is called the *pseudo specific volume*.

**Note: Absolute temperature (K) and absolute pressures should be used in compressibility charts**

Values of either  $T_R$  or  $p_R$  or  $v'_R$  can be obtained from the plots if any two of them are known.



**Some examples computing states from the thermodynamic tables:**

**Exercise:**

1. Determine the specific volume of water at a state where  $p = 10 \text{ bar}$  and  $T = 215^\circ \text{C}$ . (ans.  $0.2141 \text{ m}^3/\text{kg}$ )
2. Determine the temperature of water at a state of  $P = 0.5 \text{ MPa}$  and  $h = 2890 \text{ kJ/kg}$ . (ans.  $216.4^\circ \text{C}$ )
3. Using the tables for water, at  $p = 3 \text{ bar}$  and  $v = 0.5 \text{ m}^3/\text{kg}$ , find  $T$  in  $^\circ \text{C}$  and  $u$  in  $\text{kJ/kg}$ . (ans.  $133.6^\circ \text{C}$ ,  $2196.7 \text{ kJ/kg}$ )

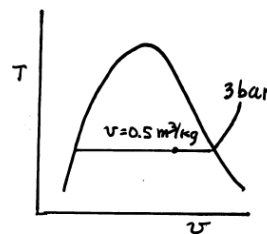
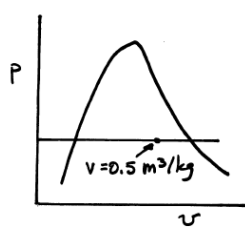
(b)  $p = 3 \text{ bar}$ ,  $v = 0.5 \text{ m}^3/\text{kg}$

Table A-3;  $v_g < v < v_f$

$\Rightarrow T = 133.6^\circ \text{C}$

$$x = \frac{v - v_f}{v_g - v_f} = \frac{0.5 - 1.0732 \times 10^{-3}}{0.6058 - 1.0732 \times 10^{-3}} = 0.825$$

$$\begin{aligned} \therefore u &= u_f + x(u_g - u_f) \\ &= 561.15 + (0.825)(2543.6 - 561.5) \\ &= 2196.7 \text{ kJ/kg} \end{aligned}$$



IT Results

$x = 0.825$   
 $2196 \text{ kJ/kg}$

4. Using the tables for water, at  $p = 4 \text{ MPa}$  and  $T = 160^\circ \text{C}$ , find  $v$  in  $\text{m}^3/\text{kg}$  and  $u$  in  $\text{kJ/kg}$ . (ans.  $1.1011 \times 10^{-3} \text{ m}^3/\text{kg}$ ,  $673.71 \text{ kJ/kg}$ )

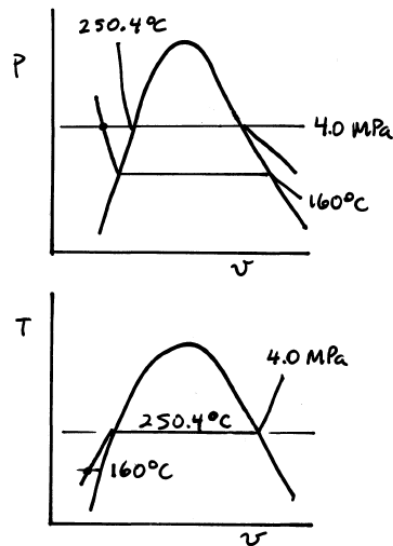
(h)  $p = 4 \text{ MPa} = 40 \text{ bar}$ ,  $T = 160^\circ\text{C}$   
 Table A-3 at 40 bar;  $T_{\text{sat}} = 250.4^\circ\text{C}$   
 $\Rightarrow$  liquid state ( $T < T_{\text{sat}}$ )  
 Table A-5; double interpolation

	$p = 2.5 \text{ MPa}$	$p = 5.0 \text{ MPa}$
$T = 140^\circ\text{C}$	$v = 1.0784 \times 10^{-3}$ $u = 587.82$	$1.0768 \times 10^{-3}$ $586.76$
$T = 180^\circ\text{C}$	$v = 1.1261 \times 10^{-3}$ $u = 761.16$	$1.1240 \times 10^{-3}$ $759.63$
at $p = 4.0 \text{ MPa}$		
$140^\circ\text{C}$	$v = 1.0774 \times 10^{-3}$ $u = 587.18$	
$180^\circ\text{C}$	$v = 1.1248 \times 10^{-3}$ $u = 760.24$	

Then, at  $4.0 \text{ MPa}$ ,  $160^\circ\text{C}$

$$v = 1.1011 \times 10^{-3} \text{ m}^3/\text{kg}$$

$$u = 673.71 \text{ m}^3/\text{kg}$$



### Some hints related to the HW

#### Problem 1:

- $\Delta U = U_2 - U_1$
- $W_{\text{tot}} = W_{\text{pw}} + W_{\text{piston}}$

#### Problem 2:

- UNITS!!! -> get everything in the same units right away!
- Adiabatic!
- Sign Convention

#### Problem 3:

- UNITS!
- $\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}}$

#### Problem 4:

- Quasi-static---equilibrium...
- Yet again, unit conversion is a must

#### Problem 5:

- Use correct tables
- Interpolate
- Isothermal

**Problem 6:**

- $p_R = \frac{P}{p_c}$
- $v'_R = \frac{\bar{v}p_c}{RT_c}$
- $T_R = \frac{T}{T_c}$
- $R = \frac{8314 Nm}{32 kgK}$
- Table A-1, fig A-2

**Problem 7:**

- $C_v = C_p - R$
- A-21E get  $C_v(T)$
- $u(T_2) - u(T_1) = \int_{T_1}^{T_2} c_v(T) dT$

**Problem 8:** To be discussed in class.