Thursday November 16, 7:30 pm – 9:30 pm

Closed books and notes. You are allowed to have only whatever information fits in the two sides of the `index card' that was given to you in class.

Wireless or any other form of internet/phone connection is strongly prohibited during the exam.

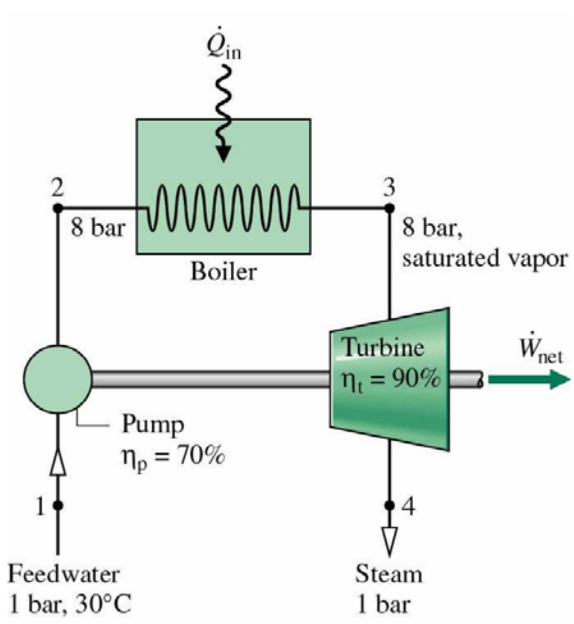
Answer all questions. Make sure your answers are legible. Circle your final answer.

The TAs and instructor will not respond to any questions during the exam. If you think that something is wrong with one of the problems below, please state your concern in your exam books.

Problem 1 (20 points)

The figure shows three devices operating at steady state: a pump, a boiler, and a turbine. The turbine provides the power required to drive the pump and also supplies power to other devices. For adiabatic operation of the pump and turbine, and ignoring kinetic and potential energy effects, determine, in kJ per kg of steam flowing:

a) (5 points) the work required by the pump
b) (5 points) the heat transfer to the boiler
c) (10 points) the net work developed by the turbine



Useful data:

h_1=125.74 KJ/Kgr, v_1=1.0043 x 10^{-3} m^3/Kgr.

h_3=2769.1 KJ/Kgr,

s_3=6.6628 KJ/Kgr K

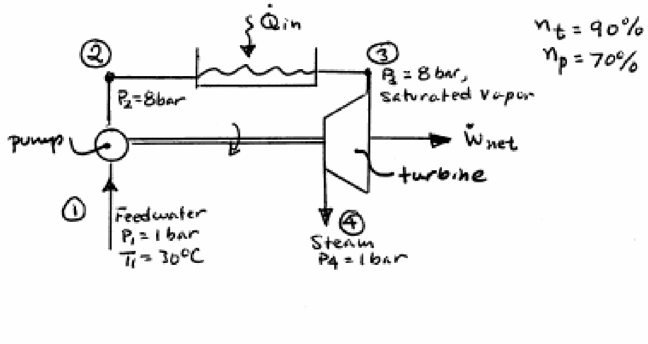
At 1 bar, s_f=1.3026 KJ/Kgr K, s_g=7.3594 KJ/Kgr K, h_f=417.46 KJ/Kgr, h_g=2675.5 KJ/Kgr.

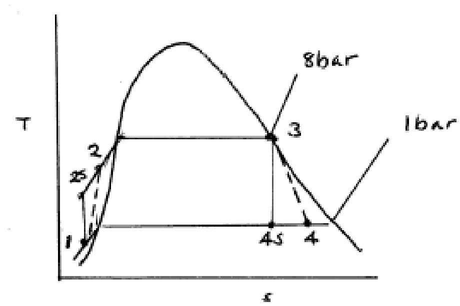
Solution:

Known: Steady state operating data are provided for a pump, a boiler and a turbine in series.

Find: Determine in KJ per Kg of steam flowing (a) the pump work, (b) the net work developed by the turbine and (c) the heat transfer to the boiler.

Schematic and given data:





Find: (1) Control volumes enclosing the pump, the boiler and the turbine are at steady-state. For the pump and turbine we neglect all heat transfers to or from the surroundings $\dot{Q}_{cv} = 0$. (2) All kinetic and potential energy effects are negligible.

Analysis:

For the pump, Eq. 6.53b from the text can be invoked to evaluate the work in the internally reversible process. With data from Table A-2, $h_1 \approx = h_f(T_1)$=125.74 KJ/Kgr, $v_1 = v_f(T_1) = 1.0043 \times 10^{-3}$ m^3/Kgr. Then

$$\left(\frac{\dot{W}_p}{\dot{m}}\right)_{\substack{int\\rev}} \approx -v_1 \Delta p = \left(\frac{1.0043}{10^3}\ \frac{m^3}{kg}\right)(8-1)\times 10^5\ \frac{N}{m^2}\ \left|\frac{1\ kJ}{10^3\ N.m}\right| = -0.7\ \frac{kJ}{kg}$$

Then using the isentropic pump efficiency, we can compute the following:

$$\eta_p = \frac{(\dot{W}_p/\dot{m})_{s=c}}{(-\dot{W}_p/\dot{m})} \Rightarrow \frac{\dot{W}_p}{\dot{m}} = \frac{-0.7\ kJ/kg}{0.7} = -1\ \frac{kJ}{kg}$$

← PUMP

Mass and energy rate balances reduce to give:

$$0 = \cancel{\dot{Q}_{cv}}^{\,0} - \dot{W}_p + \dot{m}(h_1 - h_2)$$

Thus

$$h_2 = (-\dot{W}_P/\dot{m}) + h_1 = 126.79 \text{ KJ/Kg}$$

For the boiler, mass and energy rate balances reduce to give:

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_2 = 2769.1 - 126.79 = 2642.3 \text{ KJ/Kg} \quad \longleftarrow \text{BOILER}$$

where h_3 is from Table A-3.

For the turbine, mass and energy rate balances reduce to give:

$$\frac{\dot{W}_t}{\dot{m}} = h_3 - h_4$$

or on introducing the isentropic turbine efficiency:

$$\frac{\dot{W}_t}{\dot{m}} = \eta_t (h_3 - h_{4s})$$

From Table A-3, $s_3 = s_{4s} = 6.6628$ KJ/Kgr K. The quality at 4_s is:

$$x_{4s} = \frac{6.6628 - 1.3026}{7.3594 - 1.3026} = 0.885$$

Then, $h_{4s} = 417.46 + 0.885\ (2258) = 2415.8$ KJ/Kgr. Accordingly,

$$\frac{\dot{W}_t}{\dot{m}} = 0.9\ (2769.1 - 2415.8) = 318 \text{ KJ/Kg}$$

Of this amount, 1 KJ/Kgr is required by the pump leaving

$$\frac{\dot{W}_{net}}{\dot{m}} = 317 \frac{\text{KJ}}{\text{Kg}} \quad \longleftarrow \text{Net}$$

<u>Problem 2</u> (5 points) Compute the coefficient of thermal expansion α and the isothermal compressibility β for an ideal gas.

<u>Solution:</u>

Idea gas equation PV=RT, so we have V(P,T)=RT/P
According to the definition of expansion coefficients

$$\alpha_P = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P = \frac{1}{v}\left(\frac{\partial (RT/P)}{\partial T}\right)_P = \frac{R}{Pv} = \frac{1}{T}$$

$$\beta_T = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_T = -\frac{1}{v}\left(\frac{\partial (RT/P)}{\partial P}\right)_T = -\frac{1}{v}\frac{-RT}{P^2} = \frac{1}{P}$$

Problem 3 (10 points) Use the methodology of your choice to compute an expression for $\left(\frac{\partial V}{\partial U}\right)_T$ in terms of T, P, α and β.

Hint: The differential forms of the energy functions are:

$$dU = TdS - PdV$$
$$dH = TdS + VdP$$
$$dF = -SdT - PdV$$
$$dG = -SdT + VdP$$

You will need to get V and U in terms of P and T. Start from (a) the expressions of dV in terms of dT and dP (you should know what that is), (b) derive an expression of dS in terms of dT and dP (use the appropriate Maxwell equation indicating from which of the four energy functions above is derived) and (c) get dU in terms of dT and dP. That should do it!

Solution:

Since T is one of the independent variables we can write that

$$\left(\frac{\partial V}{\partial U}\right)_T = \frac{\left(\frac{\partial V}{\partial P}\right)_T}{\left(\frac{\partial U}{\partial P}\right)_T} \qquad \textbf{(1)}$$

Solving for the resulting relations, we use equations for dU and dV

$$dV = V\alpha dT - V\beta dP \qquad \textbf{(2)}$$

The equation for dS is given as: dS= c_P/T dT – α v dP (the Maxwell equation used here is $\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P = -\alpha V$ **derived from dG = -S dT + V dP)**

From the above equation using Tds = du + P dv we derive the following:

$$dU = (C_P - PV\alpha)dT + V(P\beta - T\alpha)dP \qquad \textbf{(3)}$$

At constant T, the coefficients of dP are:

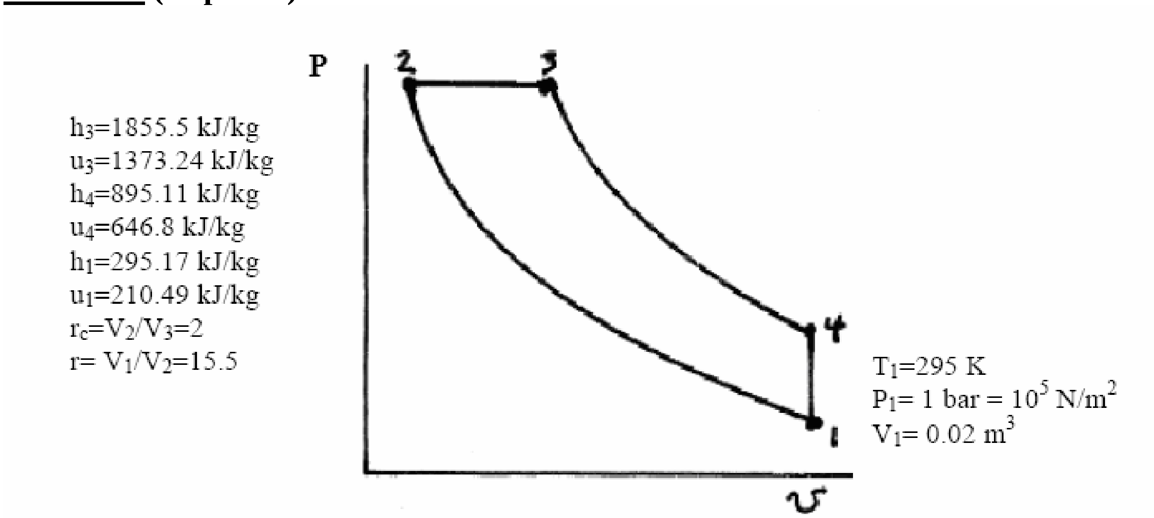
$$\left(\frac{\partial V}{\partial P}\right)_T = -V\beta \qquad \textbf{(4)}$$

$$\left(\frac{\partial U}{\partial P}\right)_T = V(P\beta - T\alpha) \qquad \textbf{(5)}$$

Substituting 4 and 5 into 1 and reducing we find that

$$\boxed{\left(\frac{\partial V}{\partial U}\right)_T = -\frac{\beta}{(P\beta - T\alpha)}}$$

<u>Problem 4</u> (25 points)



In the Diesel cycle above, the compression ratio is r = $\frac{V_1}{V_2}$ = 15.5, the cutoff ratio is r_c = $\frac{V_3}{V_2}$ = 2, and the initial (state 1) temperature, pressure and volume are T_1 = 295 K, p_1 = 1 bar and V_1=0.02 m^3. Additional specific enthalpy and specific heat data are provided above an din the Table given below (please note that not all this data is needed to answer the questions below).

(a) (5 points) Assuming that process 1-2 is isentropic, use the supplied table below to find the temperature at state 2.

(b) (10 points) Compute the heat added to the system (in KJ). For air assume that R = $\frac{8314}{28.97}\frac{\text{J}}{\text{kg}\cdot\text{K}}$.

(c) (5 points) What is the net work of the cycle?

(d) (5 points) Find the thermal efficiency of the cycle.

796 *Tables in SI Units*

TABLE A-22 Ideal Gas Properties of Air

T(K), h and u(kJ/kg), $s°$ (kJ/kg · K)									
				when $\Delta s = 0$[1]					
T	h	u	$s°$	p_r	v_r	T	h	u	$s°$
290	290.16	206.91	1.66802	1.2311	676.1	550	554.74	396.86	2.31809
295	295.17	210.49	1.68515	1.3068	647.9	560	565.17	404.42	2.33685
300	300.19	214.07	1.70203	1.3860	621.2	570	575.59	411.97	2.35531
305	305.22	217.67	1.71865	1.4686	596.0	580	586.04	419.55	2.37348
310	310.24	221.25	1.73498	1.5546	572.3	590	596.52	427.15	2.39140
800	821.95	592.30	2.71787	47.75	48.08	1400	1515.42	1113.52	3.36200
820	843.98	608.59	2.74504	52.59	44.84	1420	1539.44	1131.77	3.37901
840	866.08	624.95	2.77170	57.60	41.85	1440	1563.51	1150.13	3.39586
860	888.27	641.40	2.79783	63.09	39.12	1460	1587.63	1168.49	3.41247
880	910.56	657.95	2.82344	68.98	36.61	1480	1611.79	1186.95	3.42892

1. p_r and v_r data for use with Eqs. 6.43 and 6.44, respectively.

Solution:

a) **If the process is isentropic, then Eq. 6.44 applies and** $\frac{v_{r1}}{v_{r2}} = \frac{v_1}{v_2} = \frac{V_1}{V_2} = r$ **.**

From the supplied table at 295 K, v_{r1} = 647.9, so $v_{r2} = \frac{v_{r1}}{r} = \frac{647.9}{15.5} = 41.8$ **, so from the supplied table, T_2 = 840 K.**

b) **In the Diesel cycle, heat is added during the constant pressure process 2-3. From the first law,** $\Delta U = Q - W$ **, but since work is being done at constant pressure,** $W_{23} = \int_2^3 pdV = p(V_3 - V_2) = m \cdot p(v_3 - v_2)$

So $Q_{23} = m(u_3 - u_2) + m \cdot p(v_3 - v_2) = m[(u_3 + pv_3) - (u_2 + pv_2)]$**, but h = u + pv by definition, so**

$$Q_{23} = m(h_3 - h_2)$$

From the ideal gas equation of state,

$$m = \frac{p_1 V_1}{RT} = \frac{\left(1x10^5 \frac{N}{m^2}\right)\left(0.02m^3\right)}{\left(\frac{8314}{28.97} \frac{J}{kg \cdot K}\right)(295K)} = 0.0236kg$$

Therefore $Q_{in} = Q_{23} = 0.0236kg(1855.5 - 866.08)kJ/kg = 23.35kJ$

c) The net work of any cycle is equal to the net heat transfer, so $W_{cycle} = Q_{in} - Q_{out} = Q_{23} - Q_{41}$**. From a first law analysis with no work, since process 4-1 is at constant volume,** $Q_{41} = m(u_4 - u_1)$**.**
Therefore $W_{cycle} = 23.35kJ - 0.0236kg(646.48 - 210.49)kJ/kg = 13.06kJ$

d) The thermal efficiency of the cycle is $\eta = \frac{W_{cycle}}{Q_{in}} = \frac{13.06}{23.35} = 0.559 = 55.9\%$

Problem 5 **(15 points)** Read carefully the following statements and determine whether they are true or false and indicate your thermodynamic reasoning. For false statements and when possible, briefly amend the given statement to make it true.

a. A body in equilibrium and in thermal and mechanical contact with a reservoir at constant pressure and temperature will have the lowest possible value of Gibbs free energy for that body.
b. If two phases that are composed of the same kind of pure material are in equilibrium at constant pressure, then they must have the same value of the Gibbs free energy.
c. Melting of a fixed amount of a pure material at constant pressure is an endothermic process when the entropy of the liquid is greater than the entropy of the solid.
d. For a system composed of C components with chemical potentials μ_i for the i^{th} component and N_i is the number of molecules of the i^{th} species, $\sum_{i=1}^{c} \mu_i N_i$ will always have its smallest possible value.
e. If a system has no constraints other than being in equilibrium with a constant pressure reservoir and constant temperature reservoir, then that system is in equilibrium if there is at least one process that increases its Gibbs free energy.
f. The chemical potential of any species that can be exchanged between two phase will always be equal.
g. For the region of the phase diagram where two solid phases α and β of a binary alloy coexist in equilibrium, the number of independent degrees of freedom (not accounting for pressure dependence) is 2.
h. The Gibbs-Duhem equations reduce the number of degrees of freedom of a multicomponent system in equilibrium by C(f-1) where f is the number of phases and C the number of components.

Solutions:

(a) True. The change in Gibb's free energy for a body in equilibrium at constant T and P must be positive as a condition of equilibrium. The body could have a lower value of Gibb's free energy if the equilibrium at that T and P is not the absolute equilibrium.

(b) Almost True. They will have the same value of Gibb's free energy if the bodies have the same extent (i.e. the same number of moles). It would be correct to say that both the have the same molar Gibb's free energy.

(c) True. $\Delta\bar{G} = 0 = \Delta\bar{H} - T\Delta\bar{S}$ **. Therefore,** $\Delta\bar{H}$ **has the same sign as** $\Delta\bar{S}$

(d) Approaches Truth, $G = \sum_{i=1}^{C} \mu_i N_i$ **and** ***G*** **approaches its smallest value subject to the constraints of fixed T and P as the system approaches equilibrium.**

(e) The statement is not necessarily true. At equilibrium, all imaginable processes would have a positive change in the Gibb's free energy at equilibrium. The existence of a single process is not sufficient to specify equilibrium.

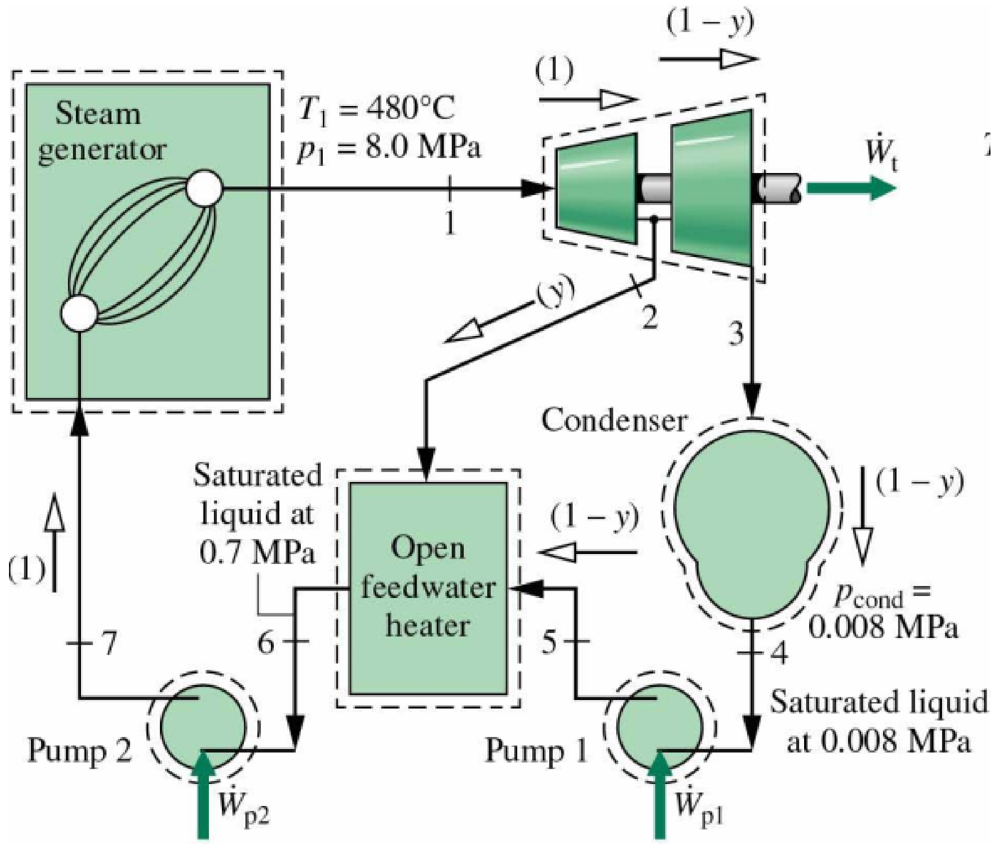
(f) False. The statement would be true if it were specified that the system was in equilibrium. Chemical potentials can vary spatially if the system is not in equilibrium.

(g) False: D=C+1-f = 2+1-2 = 1, only temperature can be changed – the composition of each of the two phases is fixed for a given temperature.

(h) False: There are only f Gibbs-Duhem equations, one for each phase.

Problem 6 (25 points) Consider a regenerate vapor power cycle with one open feed water heater. Steam enters the turbine at 8.0 MPa, 480 ^{o}C and expands to 0.7 MPa, where some of the steam is extracted and diverted to the open feed water heater operating at 0.7 MPa. The remaining steam expands through the second stage turbine to the condenser pressure of 0.008 MPa. Saturated liquid exits the open feed water heater at 0.7 MPa. The turbine stage and each pump operate isentropically. If the net power output of the cycle is 100 MW, evaluate the following:

(1) (10 points) The fraction y extracted from state 2
(2) (8 points) The thermal efficiency
(3) (7 points) The mass flow rate of steam entering the first stage turbine in kg/hr



Necessary data at different states:
State 1: h_1 = 3348.4 kJ/kg
State 2: h_2 = 2741.8 kJ/kg
State 3: h_3 = 2082.92 kJ/kg
State 4: h_4 = 173.88 kJ/kg, $v_4 = 1.0084\text{x}10^{-3}$ m^3/kg
States 5 & 6: Pressure constant at 0.7 MPa
State 6: h_6 = 697.22 kJ/kg, $v_6 = 1.1080\text{x}10^{-3}$ m^3/kg,
The steam generator operates at 8 MPa (states 7 and 1).

Solution

The specific enthalpy at state 5 is evaluated as

$$h_5 = h_4 + v_4 * (p_5 - p_4)$$

$$= 173.88 + (1.0084*10^{-3})\ m^3/kg * (0.7-0.008)\ MPa * \frac{10^6\ N/m^2}{1MPa} * \frac{1kJ}{10^3\ N.m} = 174.6\ kJ/kg$$

Similarly, the specific enthalpy at state 7 can be evaluated as

$$h_7 = h_6 + v_6 * (p_7 - p_6)$$

$$= 697.22 + 1.1080\text{x}10^{-3}\ m^3/kg * (8.0 - 0.7)\ MPa * \frac{10^6\ N/m^2}{1MPa} * \frac{1kJ}{10^3\ N.m} = 705.3\ kJ/kg$$

(a) fraction extracted, y = $\frac{h_6 - h_5}{h_2 - h_5} = \frac{697.22 - 174.6}{2741.8 - 174.6} = 0.2035$

(b) The total work output from the turbines is

$$\frac{\dot{W}_t}{\dot{m}} = (h_1 - h_2) + (1 - y) * (h_2 - h_3) = (3348.4 - 2741.8) + (1 - 0.2035) * (2741.8 - 2082.92)$$

= 1131.39 kJ/kg

The total pump work per unit mass passing through the first stage turbine is

$$\frac{\dot{W}_p}{\dot{m}} = (h_7 - h_6) + (1 - y) * (h_5 - h_4) = (705.3 - 697.22) + (1 - 0.2035) * (174.6 - 173.88)$$

= 8.65 kJ/kg

The heat added in the steam generator per unit mass passing through the first stage turbine is

$\frac{\dot{Q}_{in}}{\dot{m}} = (h_1 - h_7) = (3348.4 - 705.3)$ **=2643.1 kJ/kg**

Thus the thermal efficiency is $\eta = \frac{1131.39 - 8.65}{2643.1} = 0.4247$

(c) The net work output from the cycle per unit mass flowing through the first stage turbine is $\frac{\dot{W}_t}{\dot{m}} - \frac{\dot{W}_p}{\dot{m}}$ **=(1131.39-8.65)kJ/kg = 1122.74 kJ/kg.**

Thus, the mass flow rate is $\frac{100MW}{1122.74kJ/kg} * \frac{10^3\ kJ/s}{1MW}$ **= 89.06 kg/s = 3.206*10⁵ kg/hr**