Thursday October 12, 7:30 pm – 9:30 pm

Closed books and notes.

Wireless or any other form of internet/phone connection is strongly prohibited during the exam.

Answer all questions. Make sure your answers are legible. Circle your final answer.

The TAs and instructor will not respond to any questions during the exam. If you think that something is wrong with one of the problems below, please state your concern in your exam books.

Problem 1 **(15 points)** ***Multiple choice questions. Answer all questions – explanations are not needed and will not be graded, only provide your answer.***

(1) (3 points) A 2 kW electric resistance heater submerged in 5-kg water is turned on and kept on for 10 minutes. During the process, 300 kJ of heat is lost from the water. Assuming the specific heat of water to be 4.18 kJ/kg.^{o}C, the temperature rise of the water is

(a) 0.4 ^{o}C (b) 43.1 ^{o}C (c) 57.4 ^{o}C (d) 71.8 ^{o}C (e) 180.0 ^{o}C

Answer: Applying the first law of thermodynamics for this system

$Q - W = \Delta U$;
$Q = -300$ kJ,

$$W = -2\text{kW} \times 600\ \text{sec} \left| \frac{1\ \text{kJ}}{1\ \text{kW} \cdot \text{s}} \right| = -1200\ \text{kJ}$$

$\Delta U = mC\Delta T$ (m = mass of water = 5 kg, c = specific heat = 4.18 kJ/kg.^{o}C)

Therefore, $\Delta T = 43.1$ ^{o}C

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(2) (3 points) An ideal gas process for which $P v^k$ = constant, where $k=c_p/c_v$ is an (select the correct answer or answers)

(a) incompressible process (b) isobaric process (c) adiabatic process
(d) isentropic process (e) isothermal process

Answer: The correct answers are (c,d)

(3) (3 points) When a closed system with internal irreversibilities undergoes a process from state 1 to state 2, the entropy change, $S_2 - S_1$, and entropy production, σ, can be
a) negative and positive, respectively.

b) negative and zero, respectively.
c) positive and negative, respectively.
d) positive and zero, respectively.
e) all of the above.

Answer: (a) negative and positive respectively; σ is always positive for this process.

(4) (3 points) Air (considered an ideal gas) is compressed from room conditions to a specified pressure in a reversible manner by two compressors: one isothermal and the other adiabatic. If the entropy change of air is $\Delta s_{isothermal}$ during the reversible isothermal compression and $\Delta s_{adiabatic}$ during the reversible adiabatic compression, the correct statement regarding the entropy change of air per unit mass is

a) $\Delta s_{isothermal} = \Delta s_{adiabatic} = 0$
b) $\Delta s_{isothermal} = \Delta s_{adiabatic} > 0$
c) $\Delta s_{adiabatic} > 0$
d) $\Delta s_{isothermal} < 0$
e) $\Delta s_{isothermal} = 0$

Answer: (d) $\Delta s_{isothermal} < 0$;
For the reversible adiabatic compression process, $\Delta s_{adiabatic} = 0$. However, for the reversible isothermal compression process for an ideal gas, $\Delta s_{isothermal} = -R\ln(p_2/p_1) < 0$, since $p_2 > p_1$.

(5) (3 points) Water is at a temperature of 20 degrees C, and a pressure of 0.06 bar. What is the specific enthalpy?

a. 2567.4 kJ/kg
b. 86.30 kJ/kg
c. 83.96 kJ/kg
d. 21.54 kJ/kg

***Answer*:** Using the approximation for saturated liquids, $h(T,p) \sim h_f(T)$, so the answer is (c).

For each of problems 2-5, show all work. Solutions not supported by appropriate development will not be accepted. Report the number or type of the Table(s) used in your calculations as well as the input and output variables (data). Providing data or formulas with no justification will not be acceptable.

<u>Problem 2</u> (15 points)
Steam expands in an adiabatic turbine from 8 MPa and 480°C to a pressure of 0.7 bars at a rate of 1.8 kg/s. Compute the maximum possible power output of the turbine.

Solution: From the 1st Law, for a control volume enclosing the adiabatic turbine, we obtain:

$\dot{W} = \dot{m}(h_1 - h_2)$.

The maximum power output of the turbine is when the turbine is operating isentropically. Therefore: $h_2 = h_{2s}$.

h_1 = specific enthalpy at 8MPa, and 450°C. From Table A-4, h_1= 3348.4 kJ/kg. The specific entropy at state 1 is s_1 = 6.6586 kJ/kg-K.

Observe that at 0.7 bars, the saturated vapor has specific entropy 7.4797 KJ/Kg-K > 6.6586 kJ/kg-K. Therefore, the steam exits the turbine as a saturated mixture.

From Table A-3, calculate the quality for state 2s computed for the specific entropy 6.6586 KJ/Kgr-K at pressure p_2 = 0.7 bar. The quality x is given as

$$x= (6.6586 – 1.1919)/(7.4797-1.1919)=0.8694.$$

Using this quality, the specific enthalpy at exit state 2s is given as:

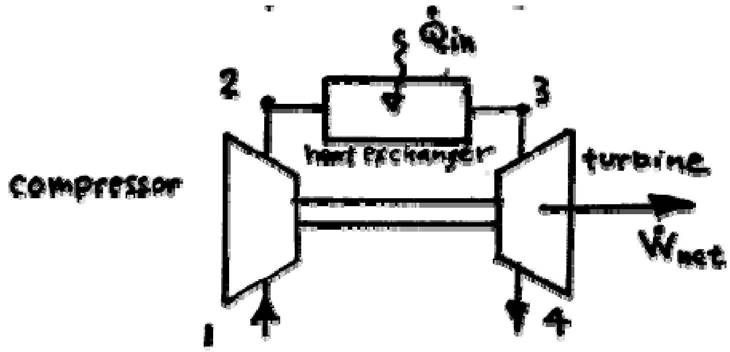
$$h_{2s} = 376.7 + 0.8694\ (2660-376.7) = 2361.83\ \text{kJ/kg}.$$

Therefore, maximum power = 1.8 * (3348.4-2361.83) = 1775 kW.

Problem 3 **(25 points)**

In a gas turbine operating at steady state, air enters the compressor with a mass flow rate of 6 kg/s at 0.95 bar and 295 K and exits at 5.7 bar. The air then passes through a heat exchanger before entering the turbine at 1200 K, 5.7 bar. Air exits the turbine at 0.95 bar. The compressor and turbine operate adiabatically and kinetic and potential energy effects can be ignored. Determine the net power developed by the plant, in kW, if

(a) (10 points) the compressor and turbine operate without internal irreversibilities.
(b) (15 points) the compressor and turbine isentropic efficiencies are 80% and 90%, respectively.



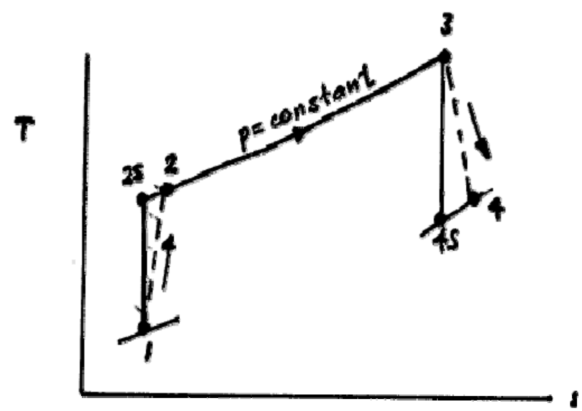
$P_1 = P_4 = 0.95\ bar \qquad P_2 = P_3 = 5.7\ bar$

$T_1 = 295\ K \qquad T_3 = 1200\ K$

$\dot{m} = 6\ \text{kg/s}$

Known: Operating data are provided for a gas turbine power plant operating at steady state.

Find: The net power developed when
(a) turbine and compressor are without internal irreversibility and
(b) The compressor and turbine isentropic efficiencies are 0.8 and 0.9, respectively.



Solution:

Schematic: The processes are shown on a T-s diagram, where the dashed lines indicate irreversible processes and states **2s** and **4s** represent the states for isentropic compression and expansion, respectively.

Assumptions:

1. The gas turbine power plant operates at steady state.
2. The compressor and turbine operate adiabatically.
3. KE and PE effects are negligible.
4. Air is modeled as an ideal gas.

Analysis:

(a) Turbine and compressor operate without any irreversibilities:
The mass and energy rate balances for a control volume about the compressor first and the turbine next, results in,

$$\frac{\dot{W}_{compressor}}{\dot{m}} = -(h_{2s} - h_1) \text{ and } \frac{\dot{W}_{turbine}}{\dot{m}} = (h_3 - h_{4s})$$

Further, $\frac{\dot{W}_{net}}{\dot{m}} = \frac{\dot{W}_{turbine}}{\dot{m}} + \frac{\dot{W}_{compressor}}{\dot{m}} = (h_3 - h_{4s}) - (h_{2s} - h_1)$

From Table A-22, h_1 = 295.17 kJ/kg; p_{r1} = 1.3068; h_3 = 1277.79 kJ/kg; p_{r3} = 238.0;
Then using Equation (6.43) in text, the reduced pressures, p_{r2}, p_{r4} can be calculated as

$p_{r2} = p_{r1}\frac{P_2}{P_1} = 1.3068 * \frac{5.7}{0.95} = 7.8408$; Interpolating in Table A-22 for p_{r2}, we get h_{2s} = 493.03 kJ/kg.

Also, $p_{r4} = p_{r3}\frac{P_4}{P_3} = 238.0 * \frac{0.95}{5.7} = 39.67$; Interpolating in Table A-22 for p_{r2}, we get h_{4s} = 780.32 kJ/kg.

Therefore, $\frac{\dot{W}_{net}}{\dot{m}}$ = (1277.79 – 780.32) – (493.03 – 295.17) = 497.47 – 197.86
= 299.61 kJ/kg

=> $\dot{W}_{net}$ = 6 kg/s * 299.61 kJ/kg = 1797.66 kW

(b) Turbine and compressor operating at 0.9 and 0.8 isentropic efficiencies respectively:
$\eta_t = 0.9;\ \ \eta_c = 0.8$

In this case, from part (a); we see that

$$\frac{\dot{W}_{net}}{\dot{m}} = \frac{\dot{W}_{turbine}}{\dot{m}} + \frac{\dot{W}_{compressor}}{\dot{m}} = (h_3 - h_4) - (h_2 - h_1) \qquad \rightarrow \text{I}$$

Now, using the isentropic efficiencies of the compressor and turbine, we get {check Equations (6.48) and (6.50) in text}

$$(h_3 - h_4) = \eta_t\,(h_3 - h_{4s}) \text{ and } (h_2 - h_1) = \frac{(h_{2s} - h_1)}{\eta_c} \qquad \rightarrow \text{II}$$

Therefore, from Equation I and II, we get

$$\frac{\dot{W}_{net}}{\dot{m}} = \eta_t(h_3 - h_{4s}) - \frac{(h_{2s} - h_1)}{\eta_c}$$

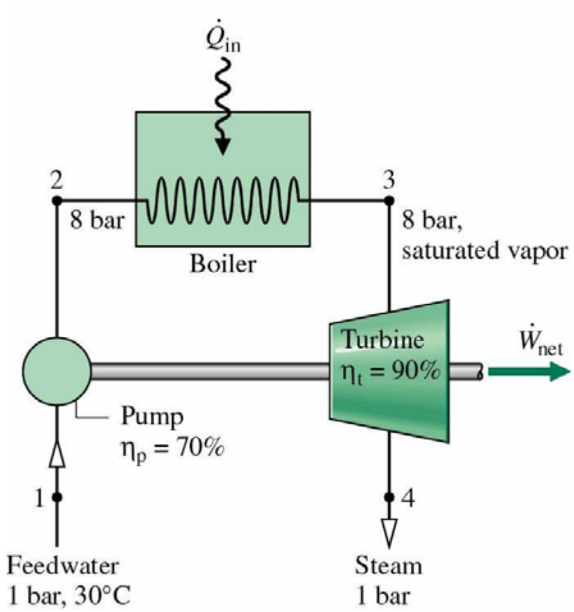
$$= 0.9 * 497.47 - \frac{197.86}{0.8} = 447.72 - 247.33 = 200.4\frac{kJ}{kg}$$

=> $\dot{W}_{net}$ = 6 kg/s * 200.4 kJ/kg = 1202.4 kW

Problem 4 (30 points)

The figure shows three devices operating at steady state: a pump, a boiler, and a turbine. The turbine provides the power required to drive the pump and also supplies power to other devices. For adiabatic operation of the pump and turbine, and ignoring kinetic and potential energy effects, determine, in kJ per kg of steam flowing:

a) (10 points) the work required by the pump
b) (10 points) the heat transfer to the boiler
c) (10 points) the net work developed by the turbine

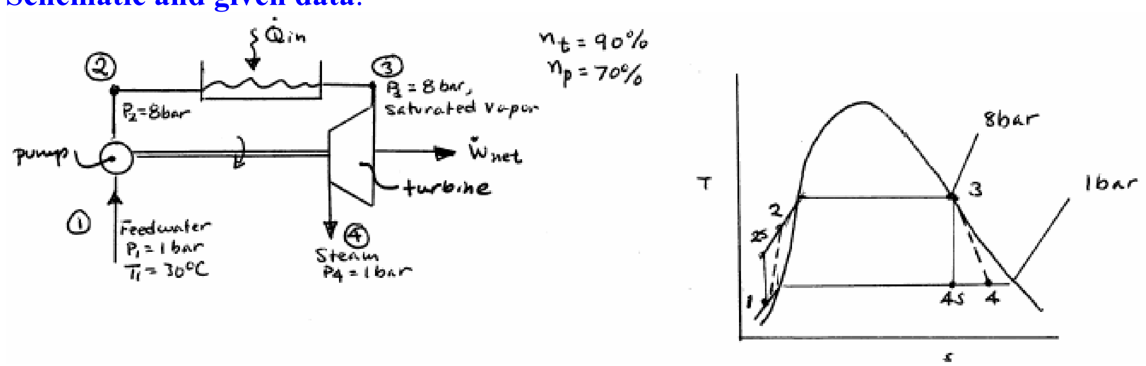


Solution:

Known: Steady state operating data are provided for a pump, a boiler and a turbine in series.

Find: Determine in KJ per Kg of steam flowing (a) the pump work, (b) the net work developed by the turbine and (c) the heat transfer to the boiler.

Schematic and given data:



Find: (1) Control volumes enclosing the pump, the boiler and the turbine are at steady-state. For the pump and turbine we neglect all heat transfers to or from the surroundings $\dot{Q}_{cv} = 0$. (2) All kinetic and potential energy effects are negligible.

Analysis:

For the pump, Eq. 6.53b from the text can be invoked to evaluate the work in the internally reversible process. With data from Table A-2, $h_1 \approx = h_f(T_1) = 125.79$ KJ/Kgr, $v_1 = v_f(T_1) = 1.0043 \times 10^{-3}$ m^3/Kgr. Then

$$\left(\frac{\dot{W}_P}{\dot{m}}\right)_{\substack{int\\rev}} \approx -v_1 \Delta p = \left(\frac{1.0043}{10^3} \frac{m^3}{kg}\right)(8-1)\times 10^5 \frac{N}{m^2} \left|\frac{1\,kJ}{10^3\,N\cdot m}\right| = -0.7 \frac{kJ}{kg}$$

Then using the isentropic pump efficiency, we can compute the following:

$$\eta_p = \frac{(\dot{W}_P/\dot{m})_{s=c}}{(-\dot{W}_p/\dot{m})} \Rightarrow \frac{\dot{W}_P}{\dot{m}} = \frac{-0.7\,kJ/kg}{0.7} = -1 \frac{kJ}{kg}$$ ← PUMP

Mass and energy rate balances reduce to give:

$$0 = \dot{Q}_{cv}^{\,0} - \dot{W}_P + \dot{m}(h_1 - h_2)$$

Thus

$$h_2 = (-\dot{W}_P/\dot{m}) + h_1 = 126.79\ kJ/kg$$

For the boiler, mass and energy rate balances reduce to give:

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_2 = 2769.1 - 126.79 = 2642.3\ kJ/kg$$ ← BOILER

where h_3 is from Table A-3.

For the turbine, mass and energy rate balances reduce to give:

$$\frac{\dot{W}_t}{\dot{m}} = h_3 - h_4$$

or on introducing the isentropic turbine efficiency:

$$\frac{\dot{W}_t}{\dot{m}} = \eta_t (h_3 - h_{4s})$$

From Table A-3, $s_3 = s_{4s} = 6.6628$ KJ/Kgr K. The quality at 4_s is:

$$x_{4s} = \frac{6.6628 - 1.3026}{7.3594 - 1.3026} = 0.885$$

Then, $h_{4s} = 417.46 + 0.885\ (2258) = 2415.8$ KJ/Kgr. Accordingly,

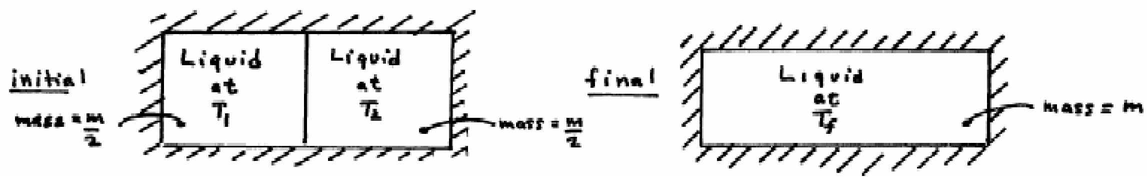
$$\frac{\dot{W}_t}{\dot{m}} = 0{,}9\,(2769.1 - 2415.8) = 318 \text{ KJ/Kg}$$

Of this amount, 1 KJ/Kgr is required by the pump leaving

$$\frac{\dot{W}_{net}}{\dot{m}} = 317 \frac{\text{KJ}}{\text{Kg}} \leftarrow \text{Net}$$

<u>Problem 5</u> (15 points)

An isolated system of known total mass m if formed by mixing equal masses of the same liquid that are initially at the known temperatures T_1 and T_2. The liquid is assumed to be incompressible with known constant specific heat c.



(a) (5 points) Derive the final temperature T_f of the liquid in terms of T_1 and T_2 (just stating the answer with no derivation will not result in any credit).

(b) (10 points) Compute the amount of the entropy produced in terms of m, c, T_1 and T_2 (just stating the answer with no derivation will not result in any credit).

Solution:

Known: Isolated system of total mass m, formed by mixing equal masses of the same liquid initially at temperatures T_1 and T_2.

Find: (a) Final temperature, (b) Entropy produced.
Schematic:

Assumptions:

1. The system is isolated and liquid is assumed to be an ideal fluid.
2. The liquid is incompressible with constant specific heat, c.

Analysis:

(a) Consider the energy balance of this isolated system, $\Delta U = Q - W = 0$**, because the system is isolated. Therefore,**

$$\Delta U = 0$$

$$\Rightarrow mu(T_f) - \frac{m}{2} * \left(u(T_1) + u(T_2)\right) = 0$$

$$\Rightarrow \frac{m}{2} * \left(u(T_f) - u(T_1)\right) + \frac{m}{2} * \left(u(T_f) - u(T_2)\right) = 0$$

By assumption (2), Δu = c*ΔT and therefore,

$$\frac{mc}{2} * \left(T_f - T_1\right) + \frac{mc}{2} * \left(T_f - T_2\right) = 0$$

$$\Rightarrow T_f = \frac{T_1 + T_2}{2}$$

(b) Entropy balance for the isolated system gives,

$$\Delta S = \int_1^2 \frac{\delta Q}{T} + \sigma$$

Since system is isolated, there is no heat transfer and

$$\sigma = \Delta S = m * s_f - \frac{m}{2} * (s_1 + s_2) = \frac{m}{2} * \left[(s_f - s_1) + (s_f - s_2)\right]$$

$$= \frac{mc}{2} * \left[\ln\left(\frac{T_f}{T_1}\right) + \ln\left(\frac{T_f}{T_2}\right)\right] = mc \ln\left[\frac{T_f}{\sqrt{T_1 T_2}}\right]$$

$$= mc \ln\left[\frac{T_1 + T_2}{2\sqrt{T_1 T_2}}\right]$$