HOMEWORK 9

Handed out: Monday, 22 October 2007
Due: Monday, 29 October 2007 by 5 pm

Problem 1: Consider air (ideal gas) at 300 K and 0.86 m^3/kg. The state of air changes to 302 K and 0.87 m^3/kg as a result of some disturbance. Estimate the change in the pressure of air using the relation

$$dP = (\partial P/\partial T)_\upsilon\, dT + (\partial P/\partial \upsilon)_T\, d\upsilon \,.$$

Problem 2: Use du = Tds – Pdv and one of Maxwell's relations to find an expression for $(\partial u/\partial P)_T$ that only has properties P, v, T and α and β involved. What is the value of that partial derivative if you have an ideal gas?

Problem 3: Start from dh = Tds + vdP and use one of Maxwell's equation to get $(\partial h/\partial v)_T$ in terms of properties P, v and T. Then use $C_v = (\partial u/\partial T)_v = T\,(\partial s/\partial T)_v$ to also find an expression for $(\partial h/\partial T)_v$.

Problem 4: From Eqs. $C_P = (\partial h/\partial T)_P = T\,(\partial s/\partial T)_P$ and $C_v = (\partial u/\partial T)_v = T\,(\partial s/\partial T)_v$ and the knowledge that Cp > Cv what can you conclude about the slopes of constant v and constant P curves in a T-s diagram? Notice that we are looking at functions T(s) (with P or v given).

Problem 5: This problem has several parts. Be sure you read carefully and follow the suggested steps.

Consider the various forms of the combined 1st and 2nd laws of thermodynamics:

dU=TdS-PdV, dH=TdS+VdP, dG=-SdT+VdP, dF=-SdT-PdV

From these equations you can write down and use the corresponding Maxwell equations as needed.

a) Using the equations above and the definition of C_v, show that

$$dS = \frac{C_V}{T} dT + \left(\frac{\partial P}{\partial T}\right)_V dV$$

b) Using the equations above and the definition of C_P and α, show that

$$dS = \frac{C_P}{T} dT - \alpha V dP$$

c) Start with the ``cyclic relation'' derived in class and the definitions of α and β to show that $\left(\frac{\partial P}{\partial T}\right)_V = \frac{\alpha}{\beta}$

d) Equate the results from Part 1 and 2 and use part 3 to show the following:

$$dV = \frac{C_p - C_v}{T}\frac{\beta}{\alpha}dT - \beta V dP$$

e) In class we derive in expression of dV in terms of dT and dP using the definitions of α and β (Hint: to arrive at this equation, start from $dV = \left(\frac{\partial V}{\partial T}\right)_P dT + \left(\frac{\partial V}{\partial P}\right)_T dP$). Equate the simplification of this expression with the result in part 4 above to show that:

$$C_p - C_v = T\frac{\alpha^2 v}{\beta}$$

f) Consider an ideal gas (PV=RT). Derive simplified expressions for α and β (in terms of P,T or V) and then simplify the equation in Part 5 to a familiar form for ideal gases.

Problem 6: Use the methodology of your choice to compute an expression for $\left(\frac{\partial V}{\partial U}\right)_T$ in terms of T, P, α and β.

Problem 7: Which of the following relation(s) is correct?

(a) $dU = TdS - PdV$ **(b)** $dS = \frac{C_p}{T}dT - V\alpha dP$

(c) $dH = C_p dT + V(1 - T\alpha)dP$ **(d)** $dF = -\left(S + \frac{PV}{T}\right)dT - VdP$

Problem 8: Consider the following equation $\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V = -1$. Which of the following statements is true and why?

(a) the equation is valid only for ideal gases (PV=RT),
(b) the equation is valid only for polytropic processes PV^n=const,
(c) the equation is valid for ideal gases (PV=RT) in isentropic processes (PV^n=const), \
(d) for all P-V-T state relations in solids, gases and liquids
(e) valid for all P-V-T state equations of gases and liquids but not solids

Problem 9:

(a) Using the combined 1st and 2nd laws, dU = T dS – P dV, the definition of C_V and the Maxwell relation, $\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$, prove the following thermodynamic relation (where familiar notation is used):

$$dU = C_v dT + \left[T\left(\frac{\partial P}{\partial T}\right)_V - P \right] dV$$ → (Equation 1)

(b) Applying the concept of exact differential to Equation 1

show that: $\left(\frac{\partial C_v}{\partial v}\right)_T = T\left(\frac{\partial^2 P}{\partial T^2}\right)_V$ → (Equation 2)

(c) We have used in many occasions in this course that the specific energy U of an ideal gas is only a function of temperature T. Prove that this is indeed the case starting from Equations (1) and (2) above and using PV=RT for an ideal gas.