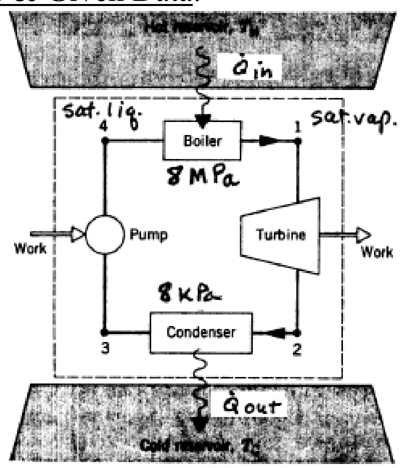
SOLUTIONS TO HOMEWORK 7 & 8

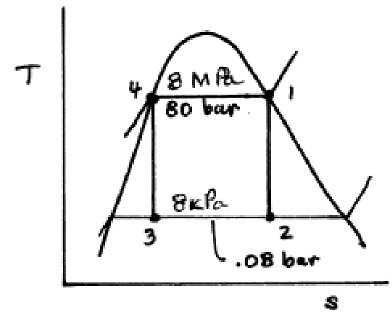
Problem 1

Known: Water is the working fluid in a Carnot vapor power cycle. The states at the boiler and turbine inlet are specified and the condenser pressure is known.

Find: Determine (a) the thermal efficiency, (b) the back work ratio, (c) the heat transfer to the working fluid per unit mass passing through the boiler, and (d) the heat transfer from the working fluid per unit mass passing through the condenser.

Schematic & Given Data:





Assumptions: (1) Each component is analyzed as a control volume at steady state. (2) All processes of the working fluid are internally reversible. (3) The turbine and the pump operate adiabatically. (4) Kinetic and potential energy effects are negligible.

Analysis: First, fix each of the principal states.

State 1: P_1 = 80 bar, Sat. Vapor → h_1 = 2758.0 kJ/kg, s_1 = 5.7432 kJ/kgK

State 2: P_2 = 0.08 bar, $s_2 = s_1$ → $x_2 = (s_2 - s_{f2})/(s_{g2} - s_{f2}) = 0.6745$, h_2 = 1794.8 kJ/kg

State 3: P_3 = 0.08 bar, $s_3 = s_4$ = 3.2068 kJ/kgK → $x_3 = (s_3 - s_{f3})/(s_{g3} - s_{f3}) = 0.3423$, h_3 = 996.46 kJ/kg

State 4: P_4 = 80 bar, Sat. Liquid → h_4 = 1316.6 kJ/kg

(a) The thermal efficiency of the Carnot cycle is given by Eq. 5.8. With data from Table A-3; T_H = T(sat@80bar) = 568.25 K, and T_C = T(sat@0.08bar) = 314.66K. Thus

$$\eta_{\max} = 1 - \frac{T_C}{T_H} = 1 - \frac{314.66K}{568.25K} = 0.4463 = \underline{44.63\%}$$

(b) The back work ratio is

$$bwr = \frac{(\dot{W}_p/\dot{m})}{(\dot{W}_t/\dot{m})} = \frac{h_4 - h_3}{h_1 - h_2} = \frac{1316.6 - 996.46}{2758.0 - 1794.8} = \frac{320.14}{963.2} = 0.332 = \underline{33.2\%}$$

(c) From mass and energy rate balances for the boiler,

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_1 - h_4 = 2758.0\frac{kJ}{kg} - 1316.6\frac{kJ}{kg} = \underline{1441.4\frac{kJ}{kg}}$$

(d) From mass and energy rate balances for the condenser,

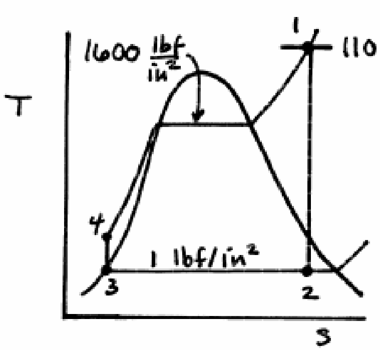
$$\frac{\dot{Q}_{out}}{\dot{m}} = h_2 - h_3 = 1794.8\frac{kJ}{kg} - 996.46\frac{kJ}{kg} = \underline{798.34\frac{kJ}{kg}}$$

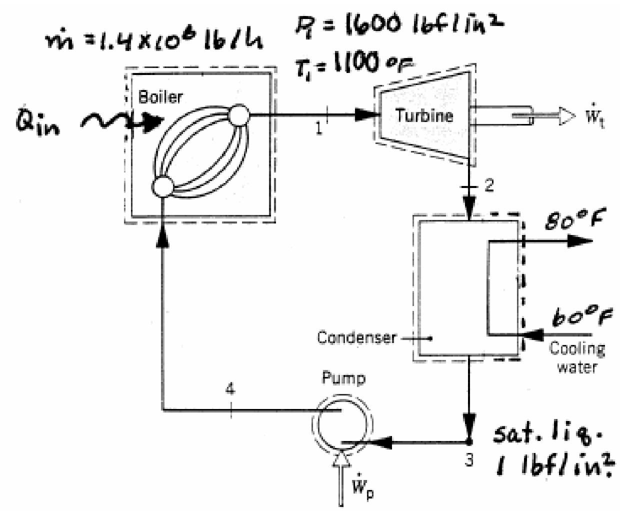
Problem 2

Known: Water is the working fluid in an ideal Rankine cycle. The turbine inlet state and the condenser pressure are known. Also, the mass flow rate of steam entering the turbine is given.

Find: Determine (a) the net power developed, (b) the thermal efficiency, and (c) the mass flow rate of cooling water.

Schematic & Given Data:





Assumptions: See Example 8.1, Assumptions 1-4.

Analysis: First, fix each of the principal states.

State 1: P_1 = 1600 lbf/in^2, T_1 = 1100 °F → h_1 = 1547.7 Btu/lb, s_1 = 1.6315 Btu/lb°R

State 2: P_2 = 1 lbf/in^2, $s_2 = s_1$ → $x_2 = (s_2 - s_{f2})/(s_{g2} - s_{f2})$ = 0.8122, h_2 = 911.2 Btu/lb

State 3: P_3 = 1 lbf/in^2, Sat. Liquid, h_3 = 69.74 Btu/lb

State 4: $h_4 = h_3 + v_3(P_4 - P_3)$ = 69.74 Btu/lb + (0.01614 ft^3/lb)[144 in^2/1 ft^2][1 Btu/788 ft·lbf](1600 – 1)lbf/in^2 = 69.74 + 4.78 = 74.52 Btu/lb

(a) The net power developed by the cycle is

$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_p = \dot{m}[(h_1 - h_2) - (h_4 - h_3)]$

$= (1.4 \times 10^6\, lb/h)[(1547.7 - 911.2) - (4.78)] Btu/lb = \underline{8.84 \times 10^8\, Btu/h}$

(b) To find the cycle thermal efficiency, first find the heat transfer rate to the working fluid passing through the system generator.

$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = (1.4 \times 10^6\, lb/h)(1547.7 - 74.52) Btu/lb$

$= \underline{2.06 \times 10^9\, Btu/h}$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_{cycke}}{\dot{Q}_{in}} = \frac{8.84 \times 10^8}{2.06 \times 10^9} = 0.429 = \underline{42.9\%}$$

(c) For the control volume enclosing the condenser

$0 = \dot{Q}_{CV} - \dot{W}_{CV} + \dot{m}(h_2 - h_3) + \dot{m}_{CW}(h_{CW,in} - h_{CW,out})$

$(\dot{Q}_{CV} = \dot{W}_{CV} = 0)$

Where m_{cw} is the mass flow rate of the cooling water. Thus,

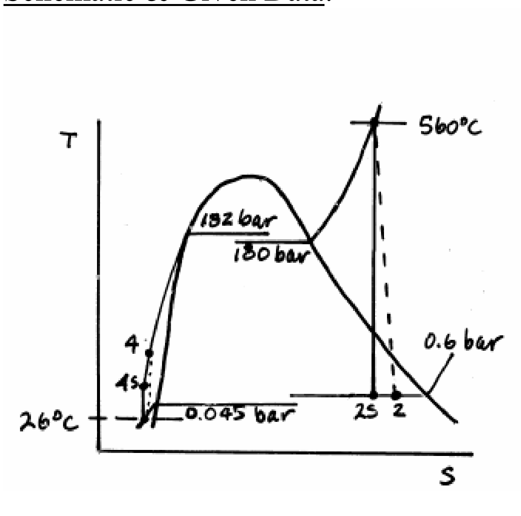
$$\dot{m}_{cw} = \frac{(1.4 \times 10^6\, lb/h)(911.2 - 69.74) Btu/lb}{(48.09 - 28.08) Btu/lb} = \underline{5.89 \times 10^7\, lb/h}$$

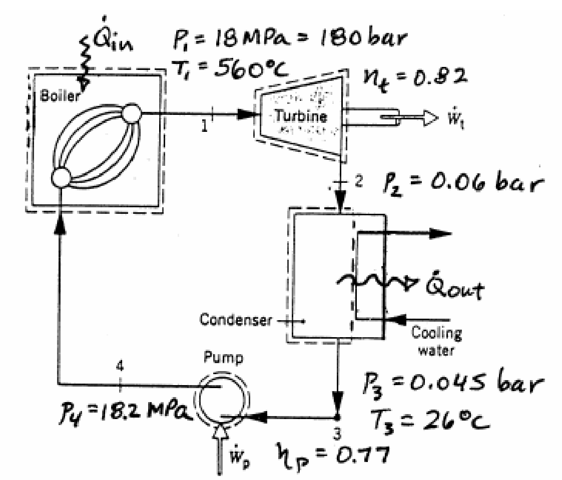
Problem 3

Known: Water is the working fluid in a simple vapor power plant. Data are known at various locations

Find: Determine (a) the net work per unit mass of steam flow, (b) the heat transfer per unit mass of steam passing through the boiler, (c) the thermal efficiency, and (d) the heat transfer per unit mass of steam passing through the condenser.

Schematic & Given Data:





Assumptions: (1) Each control volume shown is at steady state. (2) The turbine and the pump operate adiabatically. (3) Kinetic and potential energy effects can be neglected.

Analysis: First, fix each of the principal states.

State 1: P_1 = 180 bar, T_1 = 560 °C → h_1 = 3444.4 kJ/kg, s_1 = 6.4392 kJ/kgK

State 2: First, P_2 = 0.06 bar, $s_{2s} = s_1$ → x_{2s} = 0.75783, h_{2s} = 1982.4 kJ/kg

Using the turbine efficiency, $n_t = (h_1 - h_2)/(h_1 - h_{2s})$, thus

$h_2 = h_1 - n_t(h_1 - h_{2s})$ = 2245.6 kJ/kg

State 3: From Table A-2, P_3 > P(Sat.@26°C). Thus, state 3 is a sub-cooled liquid state. Since the pressure is low, h3 = h_f(26°C)=109.07 kJ/kg, and $v_3 = v_f$(26°C) = $1.0032\text{x}10^{-3}$ m^3/kg.

State 4: First, $h_{4s} = h_3 + v_3(P_4 - P_3)$ = 127.32 kJ/kg.

Using the pump efficiency, $n_p = (h_{4s} - h_3)/(h_4 - h_3)$

$h_4 = h_3 + (h_{4s} - h_3)/ n_p$ = 132.78 kJ/kg

(a) The net work per unit mass of steam flow is

$$\frac{\dot{W}_{cycle}}{\dot{m}} = \frac{\dot{W}_t}{\dot{m}} - \frac{\dot{W}_p}{\dot{m}} = (h_1 - h_2) - (h_4 - h_3)$$

$$= (3444.4 - 2245.6) - (132.78 - 109.07) = \underline{1175.1 kJ/kg}$$

(b) For control volume enclosing the steam passing through the boiler

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_1 - h_4$$

$$= (3444.4 - 132.78) kJ/kg = \underline{3311.6 kJ/kg}$$

(c) The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{1175.1}{3311.6} = 0.355 = \underline{35.5\%}$$

(d) The heat transfer to cooling water passing through the condenser per kg of steam condensed is

$$\frac{\dot{Q}_{out}}{\dot{m}} = h_2 - h_3 = (2245.6 - 109.07) kJ/kg$$

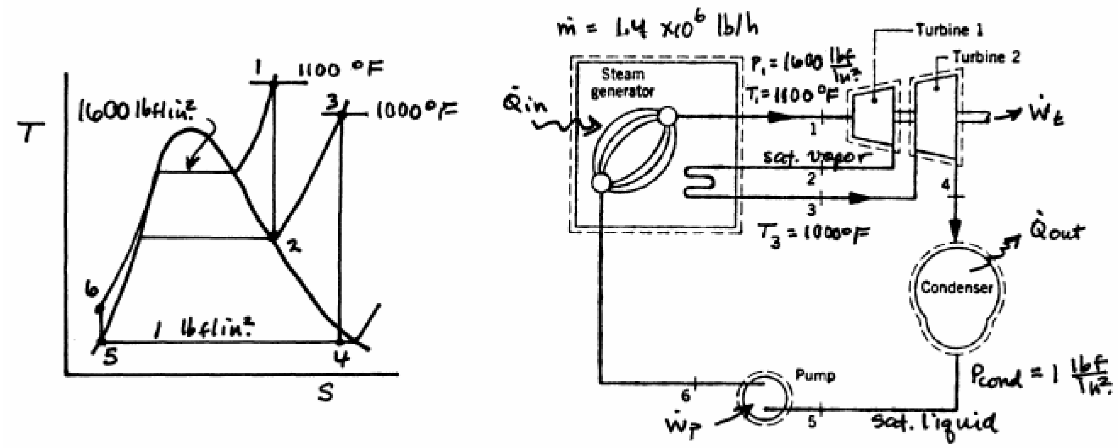
$$= 2136.5 \underline{kJ/kg}$$

Problem 4

Known: The ideal Rankine cycle of Problem 2 is modified to include reheat. The mass flow rate is the same as in Problem 2.

Find: Determine for the modified cycle (a) the net power developed, (b) the rate of heat transfer to the working fluid in the reheat process, and (c) the thermal efficiency.

Schematic & Given Data:



Assumptions: Same as Example 8.3

Analysis: First, fix each of the principal states.
State 1: $P_1 = 1600$ lbf/in^2, $T_1 = 1100$ °F → $h_1 = 1547.7$ Btu/lb, $s_1 = 1.6315$ Btu/lb°R
State 2: $s_2 = s_1$, Sat. Vapor → Interpolating in Table A-3E; $P_2 = 70.5$ lbf/in^2, $h_2 = 1181.1$ Btu/lb
State 3: $P_3 = 70.5$ lbf/in^2, $T_3 = 1000$ °F → $h_3 = 1532.9$ Btu/lb, $s_3 = 1.9605$ Btu/lb°R
State 4: $P_4 = 1$ lbf/in^2, $s_4 = s_3$ → $x_4 = 0.9905$, $h_4 = 1095.9$ Btu/lb
State 5: $P_5 = 1$ lbf/in^2, Sat. Liquid, $h_5 = 69.74$ Btu/lb
State 6: $h_6 = h_5 + v_6(P_6 - P_5) = 69.74$ Btu/lb + (0.01614 ft^3/lb)[144 in^2/1 ft^2][1 Btu/788 ft·lbf](1600 – 1)lbf/in^2 = 69.74 + 4.78 = 74.52 Btu/lb

(a) The net power developed by the cycle is

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_p = \dot{m}\left[(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)\right]$$
$$= \left(1.4 \times 10^6\, lb/h\right)\left[(1547.7 - 1181.1) + (1532.9 - 1095.9) - (74.52 - 69.74)\right] Btu/lb = \underline{1.118 \times 10^9\, Btu/h}$$

(b) For the reheat process

$$\dot{Q}_{reheat} = \dot{m}(h_3 - h_2) = \left(1.4 \times 10^6\, lb/h\right)(1532.9 - 1181.1) Btu/lb$$
$$= \underline{4.93 \times 10^8\, Btu/h}$$

(c) The thermal efficiency is

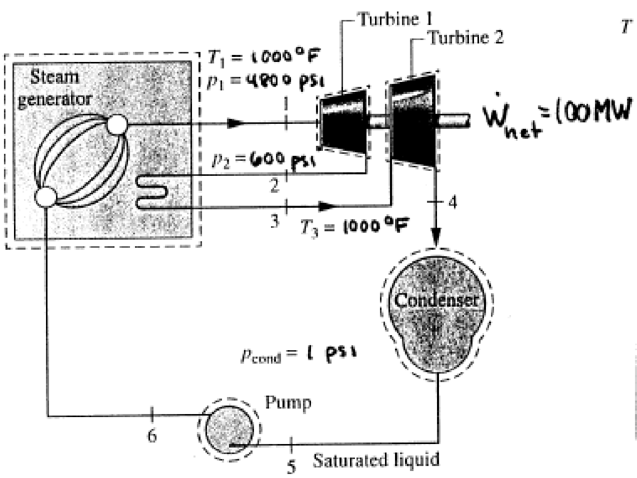
$$\eta = \frac{\dot{W}_{cycke}}{\dot{Q}_{in}} = \frac{\dot{W}_{cycke}}{\dot{m}\left[(h_1 - h_6) + (h_3 - h_2)\right]} = \frac{1.118 \times 10^9}{1.4 \times 10^6\left[(1547.7 - 74.52) + (1532.9 - 1181.1)\right]}$$
$$= 0.438 = \underline{43.8\%}$$

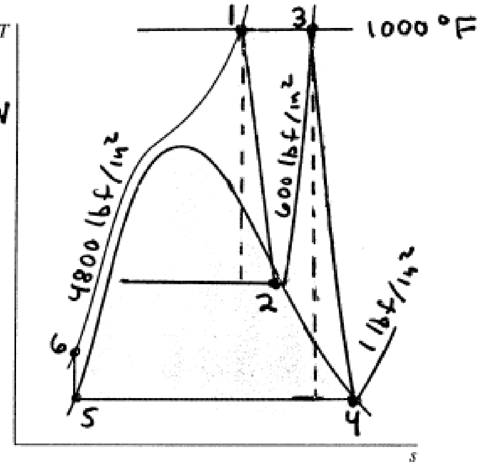
Problem 5

Known: Water is the working fluid in an ideal Rankine cycle modified to include two turbine stages with reheat between the stages. The turbine and pump efficiencies are 85%.

Find: Determine (a) the rate of heat transfer to the working fluid passing through the steam generator, (b) the rate of heat transfer from the working fluid passing through the condenser, and (c) the cycle efficiency.

Schematic & Given Data:





Assumptions: Same as in Example 8.3

Analysis: First, fix each of the principal states.

State 1: P_1 = 4800 lbf/in^2, T_1 = 1000 °F → h_1 = 1317.4 Btu/lb, s_1 = 1.4078 Btu/lb°R

State 2: P_2 = 600 lbf/in^2, $s_{2s} = s_1$ →, x_{2s} = 0.95, h_{2s} = 1167.49 Btu/lb,
$n_t = (h_1 - h_2)/(h_1 - h_{2s})$, h_2 = 1189.98 Btu/lb

State 3: P_3 = 600 lbf/in^2, T_3 = 1000 °F → h_3 = 1517.8 Btu/lb, s_3 = 1.7155 Btu/lb°R

State 4: P_4 = 1 lbf/in^2, $s_{4s} = s_3$ → x_{4s} = 0.8577, h_{4s} = 958.37 Btu/lb,
$n_t = (h_3 – h_4)/(h_3 – h_{4s})$, h_4 = 1042.28 Btu/lb

State 5: P_5 = 1 lbf/in^2, Sat. Liquid, h_5 = 69.74 Btu/lb, v_5 = 0.01614 ft^3/lb

State 6: P_6 = 4800 lbf/in^2, $h_6 = h_5 + v_6(P_6 – P_5)/n_p$ = 69.74 Btu/lb + (0.01614 ft^3/lb)[144 in^2/1 ft^2][1 Btu/788 ft·lbf](4800 – 1)lbf/in^2 /0.85 = 69.74 + 14.34 = 84.08 Btu/lb

Next, determine the flow rate of the working fluid.

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_p = \dot{m}[(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)]$$

$$\dot{m} = \frac{\dot{W}_{net}}{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)}$$

$$= \frac{(100MW)\left(\frac{1000kJ/s}{1MW}\right)\left(\frac{1Btu}{1.0551kJ}\right)}{[(1317.4 - 1189.98) + (1517.8 - 1042.28) - (84.08 - 69.74)]Btu/lb} = 161.02lb/s$$

(a) The rate of heat transfer to the working fluid in the steam generator is

$$\dot{Q}_{in} = \dot{m}(h_1 - h_6 + h_3 - h_2) = (161.2lb/s)(1317.4 - 84.08 + 1517.8 - 1189.98)Btu/lb$$

$$= 2.514 \times 10^5\ Btu/s = \underline{265.23MW}$$

(b) The rate of heat transfer from the working fluid in the condenser is

$$\dot{Q}_{out} = \dot{m}(h_4 - h_5) = (161.2lb/s)(1042.28 - 69.74)Btu/lb\left[\frac{1.0551}{1000}\right]$$

$$= \underline{165.23MW}$$

(c) The thermal efficiency is

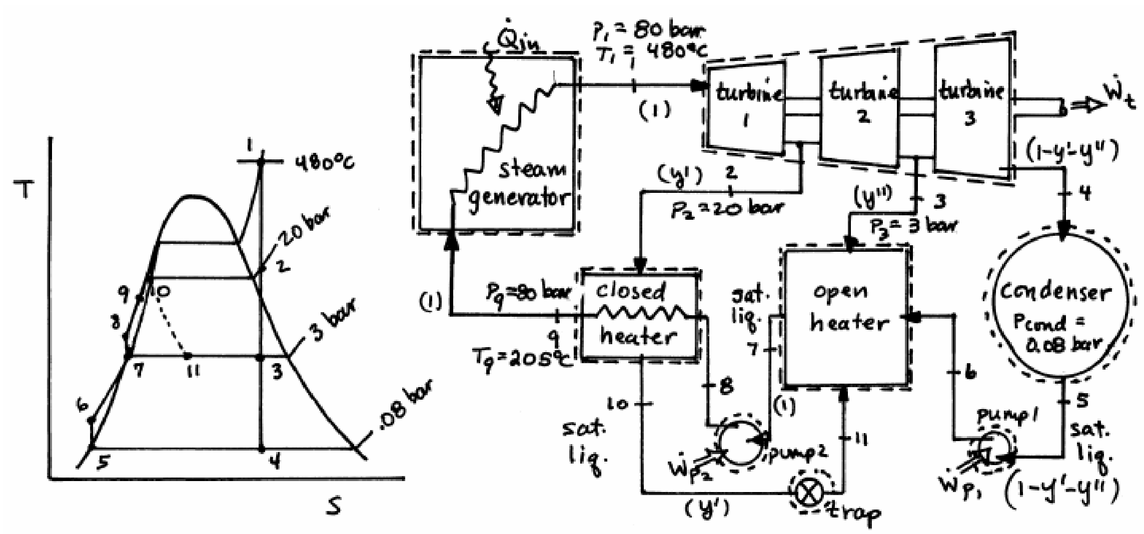
$$\eta = \frac{\dot{W}_{cycke}}{\dot{Q}_{in}} = \frac{100MW}{265.23MW} = 0.377 = \underline{37.7\%}$$

Problem 6

Known: Water is the working fluid in a regenerative vapor power cycle with one closed and one open feedwater heater. Data are known at various locations and the net power output is given.

Find: Determine (a) the thermal efficiency and (b) the mass flow rate of steam entering the first turbine stage.

Schematic & Given Data:



Assumptions: Same as Example 8.6, except no reheater.

Analysis: First, fix each of the principal states.

State 1: $P_1 = 80$ bar, $T_1 = 480$ °C → $h_1 = 3348.4$ kJ/kg, $s_1 = 6.6586$ kJ/kgK
State 2: $P_2 = 20$ bar, $s_2 = s_1$ → $h_2 = 2963.5$ kJ/kg

State 3: $P_3 = 3$ bar, $s_3 = s_1$ → $x_3 = 0.9373$, $h_3 = 2589.6$ kJ/kg
State 4: $P_4 = 0.08$ bar, $s_4 = s_1$ → $x_4 = 0.7944$, $h_4 = 2082.9$ kJ/kg
State 5: $P_5 = 0.08$ bar, Sat. Liq. → $h_5 = 173.88$ kJ/kg
State 6: $h_6 = h_5 + v_5(P_6 - P_5) = 173.88 + 0.294 = 174.17$ kJ/kg
State 7: $P_7 = 3$ bar, Sat. Liq. → $h_7 = 561.47$ kJ/kg
State 8: $h_8 = h_7 + v_7(P_8 - P_7) = 561.47 + 8.26 = 569.73$ kJ/kg
State 9: $P_9 = 80$ bar, $T_9 = 205$ °C → Table A-5, $h_9 = 878.34$ kJ/kg
State 10: $P_{10} = 20$ bar, Sat. Liq. → $h_{10} = 908.79$ kJ/kg
State 11: Throttling Process → $h_{11} = h_{10} = 908.79$ kJ/kg

(a) Begin by determining the flow rate ratios y' and y''. Energy and mass rate balances for the closed feedwater heater reduce to

$$0 = y'(h_2 - h_{10}) + (h_8 - h_9)$$

$$y' = \frac{h_9 - h_8}{h_2 - h_{10}} = \frac{878.34 - 569.73}{2963.5 - 908.79} = 0.1502$$

And, for the open feedwater heater

$$0 = y''h_3 + (1 - y' - y'')h_6 + y'h_{11} - h_7$$

$$y'' = \frac{y'(h_6 - h_{11}) + (h_7 - h_6)}{(h_3 - h_6)} = \frac{(0.1502)(174.17 - 908.79) + (561.47 - 174.17)}{(2589.6 - 174.17)} = 0.1147$$

For the turbine stages

$$\frac{\dot{W}_t}{\dot{m}} = (h_1 - h_2) + (1-y')(h_2 - h_3) + (1-y'-y'')(h_3 - h_4)$$
$$= (3348.4 - 2963.5) + (.8498)(2963.5 - 2589.6) + (.7351)(2589.6 - 2082.9)$$
$$= 1075.1 kJ/kg$$

For the pumps

$$\frac{\dot{W}_p}{\dot{m}} = (h_8 - h_7) + (1-y'-y'')(h_6 - h_5)$$
$$= (569.73 - 561.47) + (.7351)(174.17 - 173.88) = 8.47 kJ/kg$$

For the working fluid passing through the steam generator

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_1 - h_9 = 3348.4 - 878.34 = 2470.1 kJ/kg$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_t/\dot{m} - \dot{W}_p/\dot{m}}{\dot{Q}_{in}/\dot{m}} = 0.4318 = \underline{43.18\%}$$

(b) To find the mass flow rate,

$$\dot{W}_{cycle} = \dot{m}\left[(\dot{W}_t/\dot{m}) - (\dot{W}_p/\dot{m})\right]$$

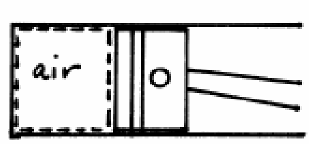
$$\dot{m} = \frac{100 \times 10^3 kW}{(1075.1 - 8.47) kJ/kg}\left|\frac{3600 s}{1 h}\right|\left|\frac{1 kJ/s}{1 kW}\right| = \underline{3.375 \times 10^5 kg/h}$$

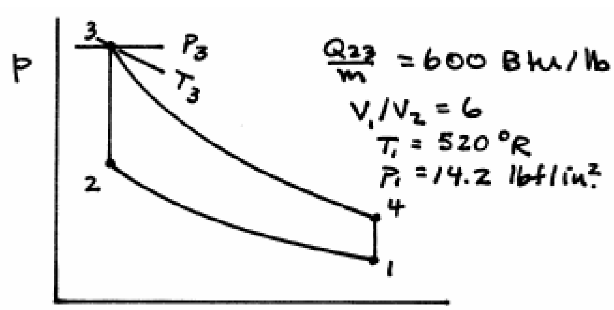
Problem 7

Known: An air-standard Otto cycle has a known compression ratio and a specified state at the beginning of compression. The heat addition per unit mass of air is given.

Find: Determine (a) the maximum temperature, (b) the maximum pressure, and (c) the thermal efficiency.

Schematic & Given Data:





Assumptions: Same as Example 9.1

Analysis: (a) To determine T3, begin by fixing state 2. For the isentropic compression

$$v_{r2} = v_{r1}\frac{V_2}{V_1} = \frac{158.58}{6} = 26.43$$

Where data for air is obtained from Table A-22E. Thus, for v_{r2} = 26.43, T_2 = 1051.4 °R, u_2 = 181.74 Btu/lb. The energy balance for process 2-3 is

$$m(u_3 - u_2) = Q_{23}$$

$$u_3 = \frac{Q_{23}}{m} + u_2 = 600\frac{Btu}{lb} + 181.74\frac{Btu}{lb} = 781.74\frac{Btu}{lb}$$

Thus, from table A-22E, T_3 = 3861 °R

(b) To find P_3, first determine P_2. For the isentropic compression
$P_2 = P_1$ (Pr2/Pr1) = (14.2)(14.78/1.2147) = 172.78 lbf/in^2.
Now, since $V_3 = V_2$; $P_3 = (T_3/T_2)P_2$ = 634.5 lbf/in^2

(c) To determine the thermal efficiency, first find the net work. That is,

$$\frac{W_{cycle}}{m} = \frac{Q_{23}}{m} - \frac{Q_{41}}{m} = \frac{Q_{23}}{m} - (u_4 - u_1)$$

To fix state 4, consider the expansion process

$$v_{r4} = v_{r3}\frac{V_4}{V_3} = v_{r3}\frac{V_1}{V_2} = (.5095)(6) = 3.057$$

Thus, T_4 = 2228 °R and u_4 = 414.35, and the net work is

$$\frac{W_{cycle}}{m} = 600 - (414.35 - 88.62) = 274.27 Btu/lb$$

Finally, the thermal efficiency is

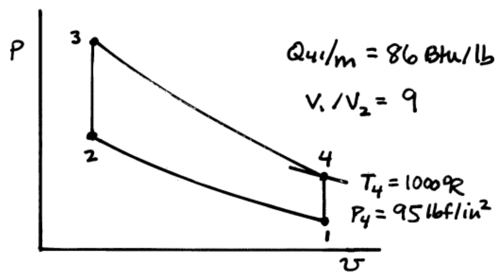
$$\eta = \frac{W_{cycle}/m}{Q_{23}/m} = \frac{274.27}{600} = 0.457 = 45.7\%$$

Problem 8

Given: A cold air-standard Otto cycle has a known compression ratio and a specified state at the end of the expansion process. The heat rejection per unit mass of air is also given. k = 1.4.

Find: Determine (a) the net work per unit mass of air (b) the thermal efficiency, and (c) the mean effective pressure.

Schematic:



Analysis:

Using the given heat rejection, Q_{41}/m = 86 Btu/lb:

$$\frac{Q_{41}}{m} = u_4 - u_1 \text{ and } \Delta u = c_v \Delta T \Rightarrow \frac{Q_{41}}{m} = c_v(T_4 - T_1)$$

From Table A-20E, c_v = 0.172 Btu/lb.°R at k = 1.4. Therefore:

$$T_1 = T_4 - \frac{Q_{41}}{m \cdot c_v} = 1000 - \frac{86}{0.172} = 500°R \qquad \text{(state 1 is defined)}$$

For the isentropic compression (1 → 2): $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{(k-1)}$

Therefore, $T_2 = T_1\left(\frac{V_1}{V_2}\right)^{(k-1)} = 500(9)^{(1.4-1)} = 1204.1°R$ (state 2 is defined)

Also, for the isentropic expansion (2 → 3): $\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{(k-1)}$

Therefore, $T_3 = T_2\left(\frac{V_1}{V_2}\right)^{(k-1)} = 1204.1(9)^{(1.4-1)} = 2408.2°R$ (state 3 is defined)

(a) The net work per unit mass of air is:

$$\frac{W_{cycle}}{m} = \frac{Q_{23}}{m} - \frac{Q_{41}}{m} = c_v\left(T_3 - T_2\right) - \frac{Q_{41}}{m} = (0.172)(2408.2 - 1204.1) - 86 = 121.11 Btu/lb$$

➔**W_{cycle}/m = 121.11 Btu/lb**

(b) The thermal efficiency is:

$$\eta = \frac{W_{cycle}/m}{Q_{23}/m} = \frac{W_{cycle}/m}{c_v\left(T_3 - T_2\right)} = \frac{121.11}{207.11} = 0.585$$

➔ **η = 58.5 %**

(c) The mean effective pressure is:

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}/m}{v_1 - v_2} = \frac{W_{cycle}/m}{v_1\left(1 - v_2/v_1\right)}$$

The specific volume ($v_1 = v_4$) is given by:

$$v_4 = \frac{RT_4}{p_4} = \frac{\left(\frac{1545}{28.97}\frac{ft.lbf}{lb.°R}\right)(1000°R)}{\left(95 lbf/in^2\right)}\left|\frac{1 ft^2}{144 in^2}\right| = 3.898 ft^3/lb$$

Therefore:

$$mep = \frac{(121.11 Btu/lb)}{\left(3.898 ft^3/lb\right)(1 - 1/9)}\left|\frac{1 ft^3}{144 in^2}\right|\left|\frac{778 ft.lbf}{1 Btu}\right| = 188.8 lbf/in^2$$

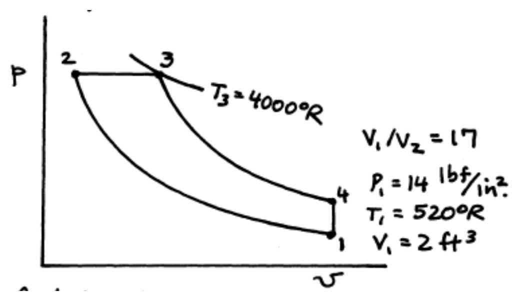
➔ **mep = 188.8 lbf/in^2**

Problem 9

Given: An air-standard Diesel cycle has a known compression ratio and a specified state at the beginning of compression. The maximum cycle temperature is given.

Find: Determine (a) the net work, (b) the thermal efficiency, (c) the mean effective pressure, and (d) the cutoff ratio.

Schematic:



Analysis:

Use data from Table A-22E to fix each principal state of the cycle.

State 1: $T_1 = 520°R \Rightarrow u_1 = 88.62$ Btu/lb, $v_{r1} = 158.58$ (from table)

State 2: For the isentropic compression:

$$\frac{v_{r2}}{v_{r1}} = \frac{V_2}{V_1} \Rightarrow v_{r2} = v_{r1}\left(\frac{V_2}{V_1}\right) = 158.58\left(\frac{1}{17}\right) = 9.3282$$

Interpolating in the tables, $T_2 = 1534.5°R$ and $h_2 = 378.32$ Btu/lb

State 3: $T_3 = 4000°R \Rightarrow h_3 = 1088.3$ Btu/lb and $v_{r3} = 0.4518$

State 4: For the isentropic expansion:

$$\frac{P_2V_2}{P_3V_3} = \frac{RT_2}{RT_3} \Rightarrow \frac{V_2}{V_3} = \frac{T_2}{T_3}$$

$$\frac{V_4}{V_3} = \left(\frac{V_4}{V_2}\right)\left(\frac{V_2}{V_3}\right) = \left(\frac{V_1}{V_2}\right)\left(\frac{V_2}{V_3}\right) = \left(\frac{V_1}{V_2}\right)\left(\frac{T_2}{T_3}\right) = (17)\left(\frac{1534.5}{4000}\right) = 6.544$$

$$\frac{v_{r4}}{v_{r3}} = \frac{V_4}{V_3} \Rightarrow v_{r4} = v_{r3}\left(\frac{V_4}{V_3}\right) = 0.4518(6.544) = 2.9466$$

Interpolating in the tables, $T_4 = 2253.7°R$ and $u_4 = 421.25$ Btu/lb

(a) For the cycle, $W_{cycle} = Q_{cycle}$. Therefore,

$W_{cycle} = Q_{23} - Q_{41} = m[(h_3 - h_2) - (u_4 - u_1)]$

Evaluating m: $$m = \frac{P_1V_1}{RT_1} = \frac{(14.0 lbf/in^2)(2 ft^3)}{\left(\frac{1545}{28.97}\frac{ft.lbf}{lb.°R}\right)(520°R)}\left|\frac{144 in^2}{1 ft^2}\right| = 0.1454 lb$$

$W_{cycle} = (.1454 \text{ lb})*[(1088.3 - 378.32) - (421.25 - 88.62)]$ Btu/lb

➔ **W_{cycle} = 54.87 Btu**

(b) The thermal efficiency is

$$\eta = \frac{W_{cycle}}{Q_{23}} = \frac{W_{cycle}}{m(h_3 - h_2)} = \frac{54.87}{(.1454)(1088.3 - 378.32)} = 0.531$$

➔ **η = 53.1 %**

(c) The mean effective pressure is

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}}{V_1(1 - V_2/V_1)} = \frac{(54.87 Btu)}{(2\,ft^3)(1 - 1/17)}\left|\frac{1\,ft^2}{144 in^2}\right|\left|\frac{778\,ft.lbf}{1 Btu}\right| = 157.5 lbf/in.^2$$

➔ **mep = 157.5 lbf/in.^{2}**

(d) The cutoff ratio is determined as follows:

$$\frac{P_3 V_3}{P_2 V_2} = \frac{RT_3}{RT_2} \Rightarrow \frac{V_3}{V_2} = \frac{T_3}{T_2}$$

$$r_c = \frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{4000}{1534.5}$$

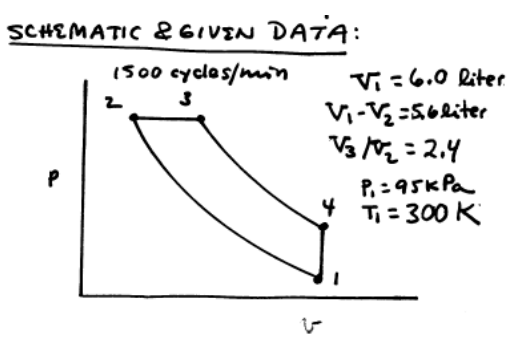
➔ **r_c = 2.61**

Problem 10

Given: Operating data are provided for an internal combustion engine modeled as an air-standard Diesel cycle.

Find: Determine the net work per cycle, the power developed, and the thermal efficiency.

Schematic:



Analysis:

Find the compression ratio:

$V_2 = V_1 - 5.6 = 6.0 - 5.6 = 0.4$ liter → $V_1/V_2 = 6.0/0.4 = 15$

Use data from Table A-22 to fix each principal state of the cycle.

State 1: $T_1 = 300K \Rightarrow u_1 = 214.07$ kJ/kg, $v_{r1} = 621.2$ (from table)

State 2: For the isentropic compression:

$$\frac{v_{r2}}{v_{r1}} = \frac{V_2}{V_1} \Rightarrow v_{r2} = v_{r1}\left(\frac{V_2}{V_1}\right) = 621.2\left(\frac{1}{15}\right) = 41.41$$

Interpolating in the tables, $T_2 = 843K$ and $h_2 = 869.63$ kJ/kg

State 3: $\frac{P_3V_3}{P_2V_2} = \frac{RT_3}{RT_2} \Rightarrow \frac{T_3}{T_2} = \frac{V_3}{V_2} \Rightarrow T_3 = (2.4)(843) = 2023K$

Interpolating in the tables, $h_3 = 2280.85$ and $v_{r3} = 2.6743$

State 4: For the isentropic expansion:

$$\frac{v_{r4}}{v_{r3}} = \frac{V_4}{V_3} = \left(\frac{V_4}{V_2}\right)\left(\frac{V_2}{V_3}\right) = \left(\frac{V_1}{V_2}\right)\left(\frac{V_2}{V_3}\right) = (15)\left(\frac{1}{2.4}\right) = 6.25$$

$$v_{r4} = v_{r3}\left(\frac{V_4}{V_3}\right) = 2.6743(6.25) = 16.714$$

Interpolating in the tables, $u_4 = 884.96$ kJ/kg

Evaluating m: $m = \frac{P_1V_1}{RT_1} = \frac{(95{,}000N/m^2)(6liters)}{\left(\frac{8314}{28.97}\frac{Nm}{kgK}\right)(300K)}\left|\frac{10^{-3}m^3}{1liter}\right| = 6.62\times10^{-3}kg$

For the cycle, $W_{cycle} = Q_{cycle}$. Therefore, $W_{cycle} = Q_{23} - Q_{41} = m[(h_3 - h_2) - (u_4 - u_1)]$

$W_{cycle} = (6.62\text{x}10^{-3})*[(2280.85 - 869.63) - (884.96 - 214.07)]$

→ **W_{cycle} = 4.901 kJ per cycle**

The power developed is:

$$\dot{W}_{cycle} = \left(1500\frac{cycles}{\min}\right)\left(4.901\frac{kJ}{cycle}\right)\left|\frac{1\min}{60s}\right|\left|\frac{1kW}{1kJ/s}\right| = 121.53kW$$

➔ **W_{cycle}/s = 121.53 kW**

The thermal efficiency is:

$$\eta = \frac{W_{cycle}}{Q_{23}} = \frac{4.901}{(6.62 \times 10^{-3})(2289.85 - 869.63)} = 0.525$$

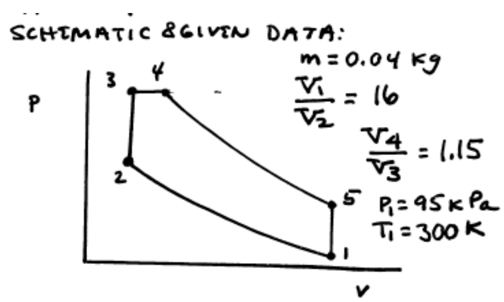
➔ **η = 52.5 %**

Problem 11

Given: Operating data are provided for an air-standard dual cycle. $P_3 = 2.2P_2$

Find: Determine (a) the heat addition at constant volume and at constant pressure, (b) the net work, (c) the heat rejected, (d) the thermal efficiency.

Schematic:



Analysis:
Use data from Table A-22 to fix each principal state of the cycle.

State 1: T_1 = 300K => u_1 = 214.07 kJ/kg, v_{r1} = 621.2 (from table)

State 2: For the isentropic compression:

$$\frac{v_{r2}}{v_{r1}} = \frac{V_2}{V_1} \Rightarrow v_{r2} = v_{r1}\left(\frac{V_2}{V_1}\right) = 621.2\left(\frac{1}{16}\right) = 38.825$$

Interpolating in the tables, T_2 = 862.4K and u_2 = 643.35 kJ/kg

State 3: $\frac{P_3V_3}{P_2V_2} = \frac{RT_3}{RT_2} \Rightarrow \frac{T_3}{T_2} = \frac{P_3}{P_2} \Rightarrow T_3 = (2.2)(862.4) = 1897.3K$

Interpolating in the tables, u_3 = 1580 kJ/kg and h_3 = 2124 kJ/kg

State 4: $\frac{P_4V_4}{P_3V_3} = \frac{RT_4}{RT_3} \Rightarrow \frac{T_4}{T_3} = \frac{V_4}{V_3} \Rightarrow T_4 = (1.15)(1897.3) = 1897.3K$

Interpolating in the tables, h_4 = 2480.6 kJ/kg and v_{r4} = 2.0707

State 5: For the isentropic expansion:

$$\left(\frac{V_5}{V_4}\right) = \left(\frac{V_5}{V_3}\right)\left(\frac{V_3}{V_4}\right) = \left(\frac{V_1}{V_2}\right)\left(\frac{V_3}{V_4}\right) = (16)\left(\frac{1}{1.15}\right) = 13.913$$

$$v_{r5} = v_{r4}\left(\frac{V_5}{V_4}\right) = 2.0707(13.913) = 28.810$$

Interpolating in the tables, u_5 = 721.2 kJ/kg

(a) Applying energy balances:

$Q_{23} = m(u_3 - u_2) = 0.04*(1580 - 643.35)$
$Q_{34} = m(h_4 - h_3) = 0.04*(2480.6 - 2124)$

➔ **Q_{23} = 37.47 kJ/kg, Q_{34} = 14.26 kJ/kg**

(b) For any cycle, $W_{cycle} = Q_{cycle}$

$W_{cycle} = (Q_{23} + Q_{34}) - Q_{51} = (37.47 + 14.76) - 0.04(721.2 - 214.07)$

➔ **W_{cycle} = 31.44 kJ**

(c) The heat rejection is $Q_{51} = m(u_5 - u_1)$

➔ **Q_{51} = 20.29 kJ**

(d) The thermal efficiency

$$\eta = \frac{W_{cycle}}{Q_{23} + Q_{34}} = \frac{31.44}{37.47 + 14.26} = 0.608$$

➔ **η = 60.8 %**