SOLUTIONS TO HOMEWORK 5

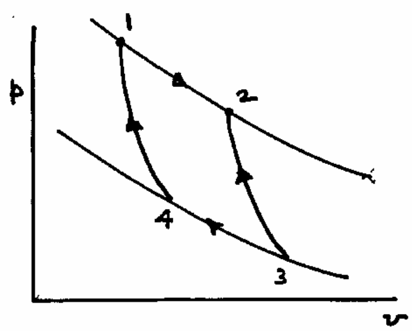
Problem 1 (Carnot Cycle): One kilogram of air as an ideal gas executes a Carnot power cycle having a thermal efficiency of 60%. The heat transfer to the air during the isothermal expansion is 40KJ. At the end of the isothermal expansion, the pressure is 5.6 bar and the volume is $0.3m^3$ .Determine

(a) The maximum and minimum temperatures for the cycle, in K.
(b) The pressure and volume at the beginning of the isothermal expansion in bar and m^3, respectively.
(c) The work and heat transfer for each of the four processes in KJ.
(d) Sketch the cycle on p-v coordinates.

Known: 1 kg of air undergoes a Carnot cycle for which, $\eta = 0.6$.
Find: Determine the minimum and maximum temperatures, the pressure and volume at the beginning of the isothermal expansion, the work and heat transfer for each process, and sketch the cycle on pv coordinates.

Schematic and given data:

$\eta = 60\%$
$Q_{12} = 40kJ$
$P_2 = 5.6 bar$
$V_2 = 0.3m^3$



Summary:

process	Q	W
1-2	40	40
2-3	0	256.7
3-4	-16	-16
4-1	0	-256.7
cycle	24	24

Assumption: The system shown in the schematic consists of air modeled as an ideal gas.

Analysis: (a) using the ideal gas model equation of state $T_2 = \frac{P_2 V_2}{mR} \Rightarrow T_2 = 585.4K$

Then since

$$\eta = 1 - \frac{T_C}{T_H} \Rightarrow T_C = T_H(1-\eta) = T_2(1-\eta) = 234.2K$$

$$T_3 = T_4 = T_C$$

(b) For process 1-2, $Q_{12} = 40KJ$. An energy balance leads $m(u_2 - u_1) = Q_{12} - W_{12}$
But since internal energy of and ideal gas depends on temperature and $T_1 = T_2$ and $W_{12} = Q_{12}$. Further $W_{12} = \int_1^2 pdV = \int_1^2 \frac{mRT_H}{V} dV = mRT_H \ln \frac{V_2}{V_1}$. Solving and inserting values:

$$\ln\frac{V_2}{V_1}=\frac{W_{12}}{T_H}=0.2381\Rightarrow V_1=0.24m^3$$

Since $T_1=T_2$ $p_1V_1=mRT, p_2V_2=mRT\Rightarrow p_2V_2=p_1V_1\Rightarrow p_1=\frac{p_2V_2}{V_1}=7bar$

(c) For process 2-3: $Q_{23}=0$. An energy balance reduces to $W_{23}=m(u_2-u_3)$

With data from Table A-22, $W_{23}=1kg(423.7-167.0)=256.7kJ$.

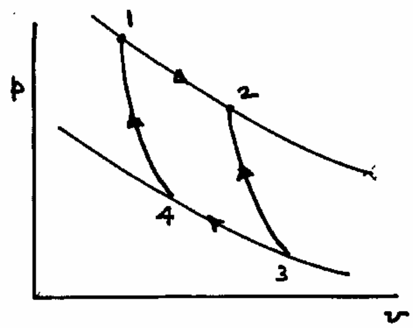
For process 3-4 $W_{34}=Q_{34}$. Also Eq. 5.6 is applicable

$$\frac{|Q_{34}|}{T_C}=\frac{|Q_{12}|}{T_H}\Rightarrow|Q_{34}|=0.4(40KJ)=16KJ\Rightarrow Q_{34}=-16KJ, W_{34}=-16KJ$$

For process 4-1, $Q_{41}=0$. An energy balance reduces to

$W_{41}=m(u_4-u_1)=m(u_3-u_2)=-256.7KJ$

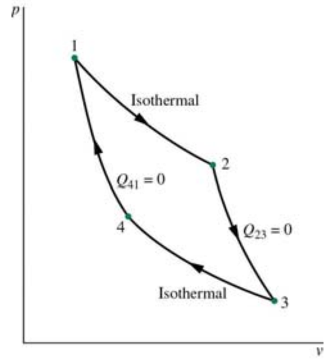
(d)



Summary:

process	Q	W
1-2	40	40
2-3	0	256.7
3-4	-16	-16
4-1	0	-256.7
cycle	24	24

Problem 2 (Carnot Cycle): : The pressure-volume diagram of a Carnot power cycle executed by an ideal gas with constant specific heat ratio of k is shown here. Demonstrate that



(a) $V_4V_2 = V_1V_3$

(b) $$\frac{T_2}{T_3} = \left(\frac{p_2}{p_3}\right)^{\frac{k-1}{k}}$$

(c) $$\frac{T_2}{T_3} = \left(\frac{V_3}{V_2}\right)^{k-1}$$

Known: A Carnot cycle is executed by an ideal gas with constant specific heat ratio k.

Find: Show that:

$V_4V_2 = V_1V_3$

$$\frac{T_2}{T_3} = \left(\frac{p_2}{p_3}\right)^{\frac{k-1}{k}}$$

$$\frac{T_2}{T_3} = \left(\frac{V_3}{V_2}\right)^{k-1}$$

Assumptions:

1) The system shown in the figure consists of an ideal gas.
2) The specific heat ratio k is constant (required for part (b) only).
3) The system undergoes a Carnot cycle.

Analysis: (a) The thermal efficiency of the cycle can be written as:

$$\eta_{cycle} = \frac{W_{cycle}}{Q_{in}}$$

where $W_{cycle} = Q_H - Q_C$. For the cycle shown above, $Q_H = Q_{12}$ and $Q_C = Q_{34}$. Since the internal energy of an ideal gas depends only on temperature, and energy balance for process 1-2 reduces to $U_2\text{-}U_1 = Q_{12} - W_{12}$ where $U_2 = U_1$. Thus, $Q_{12} = W_{12}$. Therefore,

$$\eta_{cycle} = 1 - \frac{Q_{34}}{Q_{12}} = 1 - \frac{W_{34}}{W_{12}}$$

Furthermore,

$$W_{12} = \int_1^2 pdV = \int_1^2 \frac{mRT_H}{V} dV = mRT_H \ln\left(\frac{V_2}{V_1}\right)$$

Similarly for W_{34}.

$$W_{34} = \int_3^4 pdV = \int_3^4 \frac{mRT_C}{V} dV = mRT_C \ln\left(\frac{V_3}{V_4}\right)$$

Thus, the thermal efficiency is

$$\eta_{cycle} = 1 - \frac{mRT_C \ln\left(V_3 / V_4 \right)}{mRT_H \ln\left(V_2 / V_1 \right)}$$

However, for a Carnot cycle the thermal efficiency is also defined as $\eta_{cycle} = 1 - \dfrac{T_C}{T_H}$

Thus, with algebra we find that

$$\frac{\ln\left(V_3 / V_4 \right)}{\ln\left(V_2 / V_1 \right)} = 1 \Rightarrow \underline{\underline{V_4 V_2 = V_3 V_1}}$$

(b) and (c) Process 2-3 is adiabatic; therefore a energy balance in differential form reads:

$$dU = \underbrace{\delta Q}_{0} - \delta W$$

where $\delta W = pdV$ and with assumption (1) $dU = mc_v dT$. Collecting these results and using $PV = mRT$ and $c_v = R / k-1$ we find that:

$$\frac{1}{k-1} d \ln T = -d \ln V$$

Integrating, and assuming k is a constant (assumption (2)) gives:

$$\ln\left(\frac{T_3}{T_2} \right) = -\ln\left(\frac{V_3}{V_2} \right)^{k-1} \Rightarrow \underline{\underline{\frac{T_2}{T_3} = \left(\frac{V_3}{V_2} \right)^{k-1}}}$$

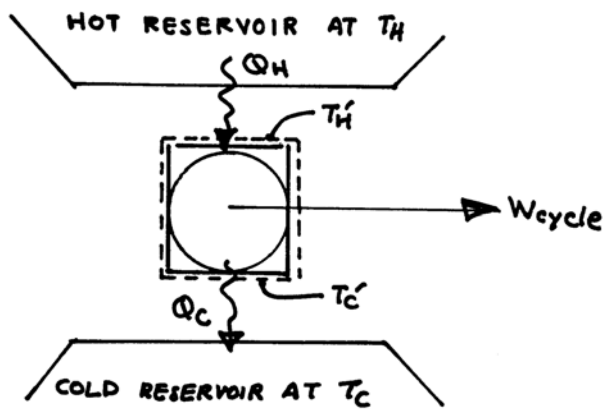
Finally, using $V = mRT / p$

$$\frac{T_2}{T_3} = \left(\frac{p_2}{p_3} \frac{T_2}{T_3} \right)^{k-1} \Rightarrow \underline{\underline{\frac{T_2}{T_3} = \left(\frac{p_2}{p_3} \right)^{\frac{k-1}{k}}}}$$

Problem 3:

Known: A system undergoes a cycle while receiving Q_H at T'_H and discharging Q_C at T'_C. Q_H and Q_C are with hot and cold reservoirs at T_H and T_C, respectively.

Find: (a) Determine an expression for W_{cycle} in terms of Q_H, T'_H, T'_C and σ.
(b) State relationships of T'_H to T_H and T'_C to T_C.
(c) Obtain an expression for W_{cycle} when there are (i) no internal irreversibilities (ii) no irreversibilities.



Schematic and Given Data:

Analysis: (a) An energy balance gives $W_{cycle} = Q_H - Q_C$. ….(1)
An entropy balance gives

$$\Delta S_{cycle} = \frac{Q_H}{T'_H} - \frac{Q_C}{T'_C} + \sigma_{cycle} \quad \text{….(2)}$$

where σ_{cycle} is the amount of entropy produced within the system. $\Delta S = 0$, since the system undergoes a cycle. Solving Eq.(2) for QC and substituting in Eq. (1) for W_{cycle},

$$W_{cycle} = Q_H\left(1 - \frac{T'_C}{T'_H}\right) - T'_C\,\sigma_{cycle} \quad \text{….(3)}$$

(b) For heat transfer to occur from the hot reservoir to the system, $T_H \geq T'_H$.
For heat transfer to occur from the system to the cold reservoir, $T'_C \geq T_C$.

(c) If there were no irreversibilities within the system during the cycle, the term σ_{cycle} vanishes in Eq.(3) leaving,

$$W_{cycle} = Q_H\left(1 - \frac{T'_C}{T'_H}\right) \quad \text{….(4)}$$

External irreversibilities are associated with heat transfer between the reservoirs and the system. If these are also absent, $T_H = T'_H$ and $T'_C = T_C$. Eq.(4) then becomes,

$$W_{cycle} = Q_H\left(1 - \frac{T_C}{T_H}\right)$$

which is the maximum theoretical work that can be obtained.

Problem 4 (Compute entropy increase): A cylindrical rod of length L insulated on its lateral surface is initially in contact at one end with a wall at temperature T_H and at the other end with a wall at a lower temperature T_C. The temperature within the rod initially varies linearly with position z according to $T(z) = T_H - (\frac{T_H - T_C}{L})z$. The rod is then insulated on its ends and eventually comes to a final equilibrium state where the temperature is T_f. Evaluate T_f in terms of T_H and T_C and show that the amount of entropy produced is

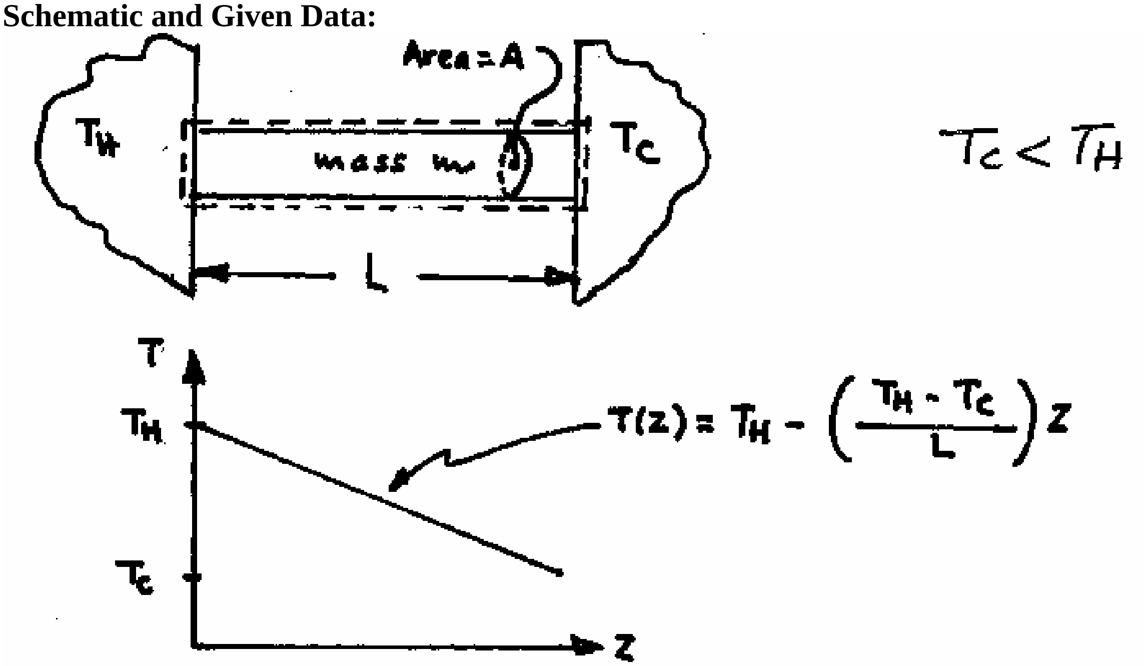
$$\sigma = mc\left(1 + \ln T_f + \frac{T_C}{T_H - T_C}\ln T_C - \frac{T_H}{T_H - T_C}\ln T_H\right)$$

where *c* is the specific heat of the rod.

Known: The temperature within the rod initially varies linearly with position z according to $T(z) = T_H - (\frac{T_H - T_C}{L})z$. The rod is then insulated on its ends and eventually comes to a final equilibrium state where the temperature is T_f.

Find: Evaluate T_f in terms of T_H and T_C and the amount of entropy produced.

Schematic and Given Data:



Analysis: The final temperature can be determined using an energy balance which reduces to give $\Delta U = \not{Q} - \not{W} = 0$. Each element of rod dz changes temperature from T(z) to the final temperature T_f, and thus contributes to the change in internal energy

$$\Delta U = \int_0^L du = \int_0^L c(T_f - T(z))dm = \int_0^L c(T_f - T(z))\rho A dz$$
$$= \int_0^L c(T_f - T_H + (\frac{T_H - T_C}{L})z)\rho A dz$$
$$= \rho A c\left[(T_f - T_H)z + \frac{T_H - T_C}{L}\frac{z^2}{2}\right]_0^L$$
$$= \rho A c L\left[(T_f - T_H) + \frac{T_H - T_C}{2}\right]_0^L$$

Since $\Delta U = \not{Q} - \not{W} = 0$, we have $T_f = \frac{T_H + T_C}{2}$.

To find the entropy production, an entropy balance reduces to give

$$\Delta S = \underbrace{\int_1^2 \frac{\delta Q}{T}}_{0} + \sigma = \sigma$$

$$Eq6.24 \Rightarrow dS = dm \times c \ln\frac{T_f}{T(z)} = \rho A c \ln\frac{T_f}{T(z)} dz$$
$$\Rightarrow \sigma = \Delta S = \int_0^L dS = \int_0^L \rho A c \ln\frac{T_f}{T(z)} dz$$
$$= \rho A c \int_0^L \left[\ln T_f - \ln T(z)\right] dz = \rho A c\left[L \ln T_f - \int_0^L \ln T(z) dz\right]$$

Since $T(z) = T_H - (\frac{T_H - T_C}{L})z$**,** $dT = -(\frac{T_H - T_C}{L})dz \Rightarrow dz = -\frac{L}{T_H - T_C}dT$

So

$$\int_0^L \ln T(z) dT = \int_{T_H}^{T_C} \ln T(z)\frac{-L}{T_H - T_C}dT = \frac{L}{T_H - T_C}\int_{T_C}^{T_H} \ln T(z)dT = \frac{L}{T_H - T_C}\left[T\ln T - T\right]_{T_C}^{T_H}$$
$$= L\left[\frac{T_H \ln T_H}{T_H - T_C} - \frac{T_C \ln T_C}{T_H - T_C} - 1\right]$$

Substitute into

$$\sigma = \rho A c\left[L \ln T_f - \int_0^L \ln T(z) dz\right]$$

We have

$$\sigma = mc\left(1 + \ln T_f + \frac{T_C}{T_H - T_C}\ln T_C - \frac{T_H}{T_H - T_C}\ln T_H \right)$$

Problem 5 (*): At steady state, an insulated mixing chamber receives two liquid streams of the same substance at temperatures T_1 and T_2 and mass flow rates $\dot{m}_1$ and $\dot{m}_2$, respectively. A single stream exits at T_3 and $\dot{m}_3$. Using an incompressible substance model with constant specific heat c, obtain an expression for

(a) T_3 in terms of T_1, T_2, and the ratio of mass flow rates $\dot{m}_1/\dot{m}_3$.

(b) the rate of entropy production per unit of mass exiting the chamber in terms of c, T_1/T_2, and $\dot{m}_1/\dot{m}_3$.

Known: An insulated mixing chamber of steady state receives liquid streams of the same substance at $\dot{m}_1$, T_1 and, $\dot{m}_2$, T_2. A single stream exits at $\dot{m}_3$, T_3.

Find: a) Evaluate T_3
b) Evaluate $\sigma/\dot{m}_3$

Assumptions:

1. The control volume is at steady state.
2. No heat or work transfer for the control volume.
3. The liquid streams are modeled as incompressible with constant specific heat c and negligible effects of pressure.

Analysis: a) At steady state, mass balance reads $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$. Energy balance equation results in

$0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$. Combining with mass balance gives,

$$0 = \dot{m}_1 (h_1 - h_2) + \dot{m}_3 (h_2 - h_3)$$

=> $0 = \dot{m}_1 c(T_1 - T_2) + \dot{m}_3 c(T_2 - T_3)$ => $T_3 = T_2 + (\dot{m}_1/\dot{m}_3)(T_1 - T_2)$

b) Entropy balance reduces at steady state (Equation 6.39 with assumptions of no heat transfer) reduces to:

$0 = \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}$; Using the mass balance and rearranging, we obtain:

=> $0 = \dot{m}_1 (s_1 - s_2) + \dot{m}_3 (s_2 - s_3) + \dot{\sigma}$

=> (using equation 6.24 from the text) $0 = \dot{m}_1 c \ln(T_1/T_2) + \dot{m}_3 c \ln(T_2/T_3) + \dot{\sigma}_3$

Solving for $\frac{\dot{\sigma}}{\dot{m}_3}$, we obtain the following:

$$\frac{\dot{\sigma}}{\dot{m}_3} = c\left[\frac{\dot{m}_1}{\dot{m}_3}\ln\left(\frac{T_2}{T_1}\right) + \ln\left(\frac{T_3}{T_2}\right)\right]$$

Replacing $T_3 = T_2 + \frac{\dot{m}_1}{\dot{m}_3}(T_1 - T_2)$, we obtain the desired result.

$$\frac{\dot{\sigma}}{\dot{m}_3} = c\left[\frac{\dot{m}_1}{\dot{m}_3}\ln\left(\frac{T_2}{T_1}\right) + \ln\left(1 + \frac{\dot{m}_1}{\dot{m}_3}\left(\frac{T_1}{T_2}\right) - \frac{\dot{m}_1}{\dot{m}_3}\right)\right]$$

Problem 6(Use tables): Employing the ideal gas model determine the change in specific entropy between the indicated states, in kJ/(kg K). Solve two ways: Use the appropriate ideal gas table, and a constant specific heat value from Table A-20.

(a) air, $p_1 = 100kPa, T_1 = 20^oC, p_2 = 100kPa, T_2 = 100^oC$

(b) air, $p_1 = 1bar, T_1 = 27^oC, p_2 = 3bar, T_2 = 377^oC$

(c) carbon dioxide, $p_1 = 150kPa, T_1 = 30^oC, p_2 = 300kPa, T_2 = 300^oC$

(d) carbon monoxide, $T_1 = 300K, v_1 = 1.1m^3/kg, T_2 = 500K, v_2 = 0.75m^3/kg$

(e) nitrogen, $p_1 = 2MPa, T_1 = 800K, p_2 = 1MPa, T_2 = 300K$

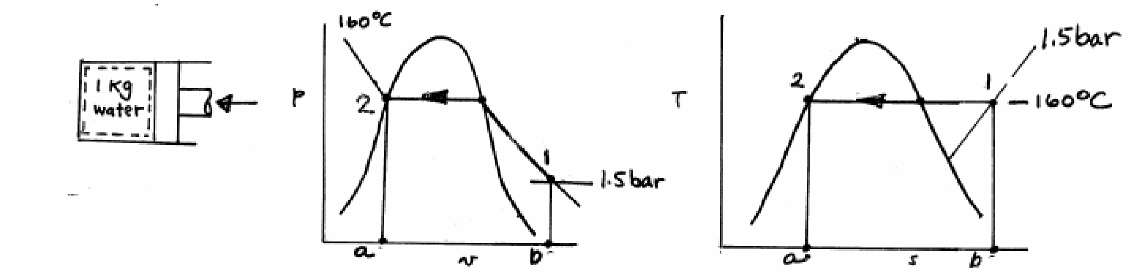
case	Ideal Gas Table $\Delta s = s^0(T_2) - s^0(T_1) - R\ln\frac{p_2}{p_1}$	Constant Specific Heat $\Delta s = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$
(a)	With s^0 data from Table A22 $\Delta s = 1.92119 - 1.678298 - 0 = 0.24289 kJ/kgK$	With c_p at 333K from Table A20 $\Delta s = 1.007\ln\frac{373}{293} - 0 = 0.2431 kJ/kgK$
(b)	With s^0 data from Table A22 $\Delta s = 2.49364 - 1.70203 - \frac{8.314}{28.97}\ln\frac{2}{1} = 0.47632 kJ/kgK$	With c_p at 475K from Table A20 $\Delta s = 1.0245\ln\frac{650}{300} - \frac{8.314}{28.97}\ln\frac{3}{1} = 0.47684 kJ/kgK$
(c)	With s^0 data from Table A23 and M from Table A1 $\Delta s = \frac{241.033 - 214.284 - 8.314\ln\frac{300}{150}}{44.01} = 0.4769 kJ/kg$	With c_p at 438K from Table A20 $\Delta s = 0.9686\ln\frac{573}{303} - \frac{8.314}{44.01}\ln\frac{300}{150} = 0.4862 kJ/kg$
(d)	With pv=RT $\frac{p_2}{p_1} = \frac{T_2}{T_1}\frac{V_1}{V_2} = \frac{500}{300}\frac{1.1}{0.75} = 2.444$ With s^0 data from Table A23 and M from Table A1 $\Delta s = \frac{212.719 - 197.723 - 8.314\ln 2.444}{28.01} = 0.2701 kJ/kg$	With c_p at 400K from Table A20 $\Delta s = 1.047\ln\frac{500}{300} - \frac{8.314}{28.01}\ln 2.444 = 0.2696 kJ/kg$
(e)	With s^0 data from Table A23 and M from Table A1 $\Delta s = \frac{191.682 - 220.907 - 8.314\ln\frac{1}{2}}{28.02} = -0.8373 kJ/kgK$	With c_p at 550K from Table A20 $\Delta s = 1.065\ln\frac{300}{800} - \frac{8.314}{28.02}\ln\frac{1}{2} = -0.8389 kJ/kgK$

Problem 7 (Reversible process): One kilogram of water initially at 160 ^oC, 1.5 bar undergoes an isothermal, internally reversible compression process to the saturated liquid state. Determine the work and heat transfer, in each in kJ. Sketch the process on p-v and T-s coordinates. Associate the work and heat transfer with areas on these diagrams.

Known: One kg of water undergoes an isothermal process between two specified states.

Find: Determine the heat transfer and the work. Sketch the process on p-v and T-s coordinates.

Schematic & Given Data:



Assumptions:
1. As shown in the accompanying figure, the system consists of 1 kg of water.
2. The compression takes place isothermally and without internal irreversibilities.
3. There is no change in kinetic and potential energy between the end states.

Analysis: Using assumption 2, Eq.(6.25) becomes $Q = \int_1^2 TdS = mT(s_2 - s_1)$. Then with data from Tables A-2 and A-4, $Q = 1kg(433K)(1.9427 - 7.4665)\ kJ/kg.K = -2391.8\ kJ$

The magnitude of the heat transfer is represented by area 1-2-a-b-1 on the T-s diagram.
The energy balance reduces to give W = Q – m(u_2 – u_1).
With data from Tables A-2 and A-4, W = -2391.8 – 1 kg(674.86 – 2595.2) kJ/kg = -471.5 kJ.

Alternaltely, $W = \int_1^2 PdV$. The magnitude of the work is represented by the area 1-2-a-b-1 on the p-v diagram.

Problem 8 (Entropy change): For each of the following systems, specify whether the entropy change during the indicated process is positive, negative, zero, or indeterminate.

(a) One kilogram of water vapor undergoing an adiabatic compression process
(b) Two pounds mass of nitrogen heated in an internally reversible process
(c) One kilogram of Refrigerant 134a undergoing an adiabatic process during which it is stirred by a paddle wheel.
(d) One-pound mass of carbon dioxide cooled isothermally.
(e) Two pounds mass of oxygen modeled as an ideal gas undergoing a constant pressure process to a higher temperature
(f) Two kilograms of argon modeled as an ideal gas undergoing an isothermal process to a lower pressure.

(a)

$\Delta S = \underbrace{\int_1^2 \frac{\delta Q}{T}}_{0} + \sigma = \sigma \geq 0$. It can be + or 0 depending on the nature of the process.

Indeterminate!

(b) Nitrogen heated internally reversibly.

$\Delta S = \int_1^2 \frac{\delta Q}{T} + \not{\sigma} = \int_1^2 \frac{\delta Q}{T} > 0$. Increase!

(c) R134a stirred adiabatically

$$\Delta S = \underbrace{\int_1^2 \frac{\delta Q}{T}}_{0} + \sigma = \sigma > 0$$

Stir would increase entropy.

(d) CO_2 cooled isothermally

$\Delta S = \underbrace{\int_1^2 \frac{\delta Q}{T}}_{<0} + \underbrace{\sigma}_{>=0}$. Indeterminate!

(e) Ideal gas undergoing a constant pressure process T2>T1
Using Eq. 6.21(a)

$\Delta s = s^0(T_2) - s^0(T_1) - R\ln\frac{p_2}{p_1} = s^0(T_2) - s^0(T_1) > 0$ Increase!

(f) Constant temperature p2<p1

$\Delta s = s^0(T_2) - s^0(T_1) - R\ln\frac{p_2}{p_1} = -R\ln\frac{p_2}{p_1} > 0$ Increase!