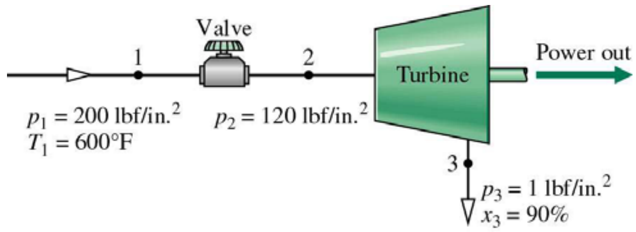
## SOLUTION TO HOMEWORK 4

**Problem 1**

Given: Data are provided for a valve and turbine in series, each operating at steady state.
Find: For the turbine, determine the temperature at the inlet and the power developed per unit mass of steam flowing ($T_2$ and $\dot{W}/\dot{m}$)

Schematic:




Assumptions:
1) Control volumes at steady state enclose the valve and the turbine, respectively.
2) The expansion across the valve is a throttling process.
3) Heat transfer with the surroundings and kinetic/potential energy effects are negligible.

Analysis:

We can find $T_2$ from the tables if we know what $h_2$ is. For the valve, the expansion acts as a throttling process so that $h_1 = h_2$. Using table A-4E (there is steam at point 1), we can determine that $h_1$ = 13221.1 Btu/lb with the given $p_1$ and $T_1$. Therefore $h_2$ = 1322.1 Btu/lb, and we can find $T_2$ by interpolating at $p_2$ = 120 lbf/in.$^2$

*Part (a)* ➔ **$T_2$ = 589°F**

Mass balance for the turbine: $dm/dt = 0 = \dot{m}_2 - \dot{m}_3 \Rightarrow \dot{m}_2 = \dot{m}_3 = \dot{m}$

Energy balance: $dE/dt = 0 = \dot{Q} - \dot{W} + \dot{m}\left[(h_2 - h_3) + (V_2^2 - V_3^2)/2 + g(z_2 - z_3)\right]$

Using assumption (3): $\dot{W} = \dot{m}[(h_2 - h_3)] \Rightarrow \dot{W}/\dot{m} = (h_2 - h_3)$

Using table A-3E:

$h_3 = h_f - x_3 h_{fg}$

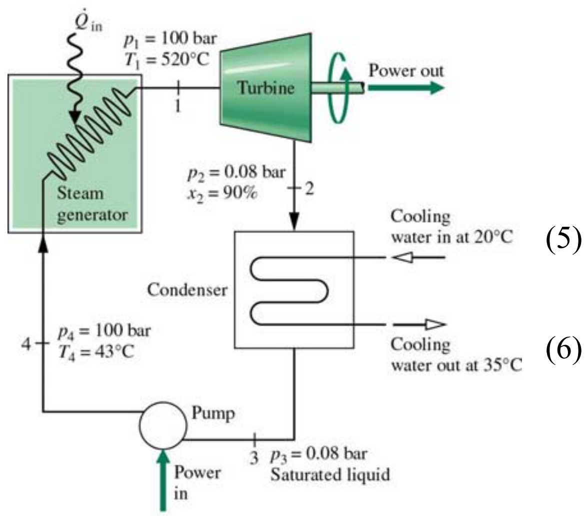
$h_3 = 69.74 + 0.9(1036) = 1002.1$ Btu/lb

Therefore, $\dot{W}/\dot{m} = (h_2 - h_3) = (1322.1 - 1002.1)$

*Part (b)* ➔ $\dot{W}/\dot{m} = 320$ **Btu/lb**

**Problem 2**

Given: Data are provided for a simple vapor power plant at steady state.
Find: Determine the thermal efficiency and the mass flow rate of the cooling water through the condenser ($\eta$ and $\dot{m}_{cw}$)
Schematic:




Assumptions:
1) Control volumes enclosing each of the four components are at steady state.
2) For each control volume, kinetic/potential energy effects are negligible as are all stray heat transfers

Analysis:

For any power cycle, the thermal efficiency is the ratio of the net work developed to the heat added.

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{W}_{turbine} - |\dot{W}_{pump}|}{\dot{Q}_{in}}$$

Applying energy rate balances to the turbine, pump, and steam generator, respectively, yields:

$$\dot{W}_{turbine} = \dot{m}\left(h_1 - h_2\right)$$

$$\left|\dot{W}_{pump}\right| = \left|\dot{m}\left(h_3 - h_4\right)\right|$$

$$\dot{Q}_{in} = \dot{m}\left(h_1 - h_4\right)$$

From Table A-4 (steam), $h_1$ = 3425.1 kJ/kg at the given $p_1$ and $T_1$
From Table A-3, $h_2 = h_f + x_3 h_{fg}$ = 173.88 + 0.9(2403.1) = 2336.7 kJ/kg at the given $p_2$
From Table A-3 (saturated liquid), $h_3$ = 173.88 kJ/kg at the given $p_3$
From Table A-5 (condensed liquid), $h_4$ = 188.9 kJ/kg at the given $p_4$ and $T_4$

$$\eta = \frac{\dot{m}(h_1 - h_2) - \left|\dot{m}(h_3 - h_4)\right|}{\dot{m}(h_1 - h_4)} = \frac{(3425.1 - 2336.7) - \left|(173.9 - 188.9)\right|}{(3425.1 - 188.9)}$$

*Part (a)* ➔**η = 0.332 (33.2%)**

Applying mass rate balances to the condenser only, we get:

$$\dot{m}_2 = \dot{m}_3 = 109 \text{ kg/s and } \dot{m}_5 = \dot{m}_6 = \dot{m}_{cw}$$

Using assumption (2) and applying energy rate balance to the condenser:

$$0 = \dot{m}_2(h_2 - h_3) + \dot{m}_{cw}(h_5 - h_6)$$

$$\dot{m}_{cw} = \frac{\dot{m}_2(h_2 - h_3)}{(h_6 - h_5)}$$

Since water is incompressible, we can approximate enthalpy as $h = h_f(T)$.

Therefore, using Table A-2:

$h_5 = h_5(20°C) = 83.96$ kJ/kg
$h_6 = h_6(35°C) = 146.08$ kJ/kg

$$\dot{m}_{cw} = \frac{109(2336.7 - 173.9)}{(146.08 - 83.96)}$$

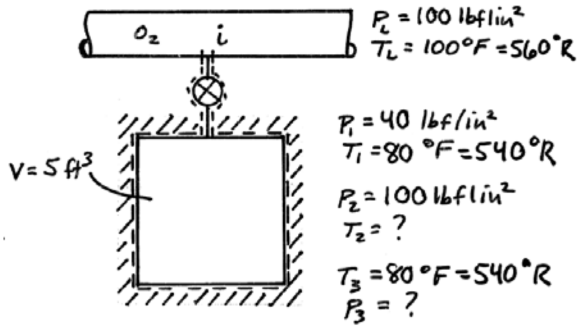
*Part (b)* ➔ $\dot{m}_{cw} = 3759$ **kg/s**

**Problem 3**

Given: An insulated tank containing oxygen is connected to a supply line carrying oxygen. A valve between the tank and the line is opened and has flows into the tank until the pressure reaches that of the line. The valve is closed and eventually the tank contents cool back to their initial temperature.

Find: Determine (a) the tank temperature when the valve closes and (b) the final pressure in the tank ($T_2$ and $p_3$)

Schematic:




Assumptions:

1) For the control volume shown, $\dot{W}_{cv} = 0$.
2) There is no heat transfer during the filling process.
3) Kinetic and potential effects are negligible.
4) The state of the $O_2$ in the supply line remains constant.
5) The $O_2$ behaves as an ideal gas.

Analysis:

For the filling process, the mass rate balance takes the form:

$$dm_{cv}/dt = \dot{m}_i$$

Using assumptions (1), (2), and (3), the energy rate balance reduces to:

$$\frac{dU_{cv}}{dt} = \dot{m}_i\, h_i$$

The specific enthalpy in the supply line is constant. Combining the mass and energy rate balances, and integrating, we get:

$$m_2u_2 - m_1u_1 = h_i(m_2 - m_1)$$

Using the ideal gas equation of state:

$$m_1 = \frac{p_1V}{RT_1}, m_2 = \frac{p_2V}{RT_2}$$

Combining and rearranging terms yields:

$$u_2 = h_i + \left(\frac{T_2 p_1}{T_1 p_2}\right)(u_1 - h_i)$$

Using the given data for $p_L$ and $T_L$, we can determine $h_i$ and $u_1$ from Table A-23E.

$$h_i = \left(\frac{3886.6 Btu/lbmol}{32 lb/lbmol}\right) = 121.46 Btu/lb$$

$$u_1 = \left(\frac{2673.8 Btu/lbmol}{32 lb/lbmol}\right) = 83.56 Btu/lb$$

Inserting known values into the above equation yields:

$$u_2 = 121.46 - (0.02807)T_2$$

So the final temperature must satisfy the above expression. On the other hand, it is also needed to satisfy the data from Table A-23E since it is an idea gas model.

Since $u_2$ depends on $T_2$, the final temperature can be found by iteration using the data from Table A-23E and the above expression. The result is:

**➔$T_2$ = 661°R = 201°F**

An alternative way to solve this temperature is to first plot the line using the above expression. Then we can also plot the curve using the data from Table A-23E. The intersection point of the computed line and the curve from the data of the Table is the needed solution.

To determine the final pressure, we must first find the mass in the tank after the valve closes:

$$m_2 = \frac{p_2 V}{RT_2} = \frac{(100 lbf/in^2)(5 ft^3)}{\left(\frac{1545}{32}\frac{ft \cdot lbf}{lb.°R}\right)(661°R)}\left|\frac{144 in^2}{1 ft^2}\right| = 2.256 lb$$

Since $m_2 = m_3$, the final pressure is just:

$$p_3 = \frac{m_2 R T_3}{V} = \frac{(2.256)\left(\frac{1545}{32}\right)(540)}{(5)|144|}$$

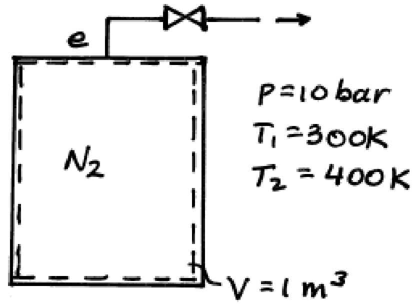
**➔$p_3$ = 81.7 lbf/in.$^2$**

**<u>Problem 4</u>**

<u>Given</u>: Heat transfer occurs to nitrogen gas contained in a rigid tank. Gas escapes through a pressure relief valve, maintaining constant pressure in the tank. The initial and final temperatures are specified.

Find: Determine the mass of nitrogen that escapes and the amount of energy transfer by heat ($m_e$ and $Q_{cv}$)

Schematic:




Assumptions:

1) For the control volume shown, $\dot{W}_{cv} = 0$.
2) The state in the control can be assumed to be uniform at any time during the process.
3) Kinetic and potential effects are negligible.
4) The nitrogen behaves as an ideal gas with constant specific heats evaluated at 350K.

Analysis:

The mass rate balance takes the form $dm_{cv}/dt = -\dot{m}_e$

Therefore, the mass that escapes is:

$$\int_1^2 \dot{m}_e \, dt = -\int_{m_1}^{m_2} dm_{cv} = m_1 - m_2$$

$$\Rightarrow m_e = m_1 - m_2$$

With the ideal gas equation of state, $m_1 = \frac{pV}{RT_1}, m_2 = \frac{pV}{RT_2}$

Therefore, $m_e = \frac{pV}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right] = \frac{(10bar)(1m^3)}{\left(\frac{8.314}{28.01}\frac{kJ}{kg \cdot K}\right)}\left[\frac{1}{300K} - \frac{1}{400K}\right]\left|\frac{10^5 N/m^2}{1bar}\right|\left|\frac{1kJ}{10^3 Nm}\right|$

➔ $\mathbf{m_e = 2.807\ kg}$

Using assumptions (1) and (3), the energy rate balance reduces to:

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{m}_e h_e$$

Using the following relationships: $U_{cv}$ = mu, $h_e$ = u + RT, and $dm_{cv}/dt = -\dot{m}_e$ , the above rate balance equation becomes:

$$m\frac{du}{dt} + u\frac{dm}{dt} = \dot{Q}_{cv} + (u + RT)\frac{dm}{dt}$$

This simplifies to: $\dot{Q}_{cv} = m\frac{du}{dt} - RT\frac{dm}{dt}$

For an ideal gas: $du = c_v dT$ and $m = pV/RT$.

$$\frac{dm}{dT} = -\frac{pV}{RT^2}, dm = -\left(\frac{pV}{R}\right)\left(\frac{dT}{T^2}\right)$$

Therefore, we get:

$$\dot{Q}_{cv} = \left(\frac{pV}{RT}\right)c_v\frac{dT}{dt} + \left(\frac{pV}{RT}\right)R\frac{dT}{dt} = \left(\frac{pV}{R}\right)(c_v + R)\left(\frac{1}{T}\right)\frac{dT}{dt}$$

Since $c_v + R = c_p$:

$$\dot{Q}_{cv}\, dt = \left(\frac{pVc_p}{R}\right)\frac{dT}{T}$$

From Table A-20, we determine that $c_p$ = 1.041 kJ/kgK at 350K.

$$\int_1^2 \dot{Q}_{cv} dt = \left(\frac{pVc_p}{R}\right)\int_{T_1}^{T_2}\frac{dT}{T}$$

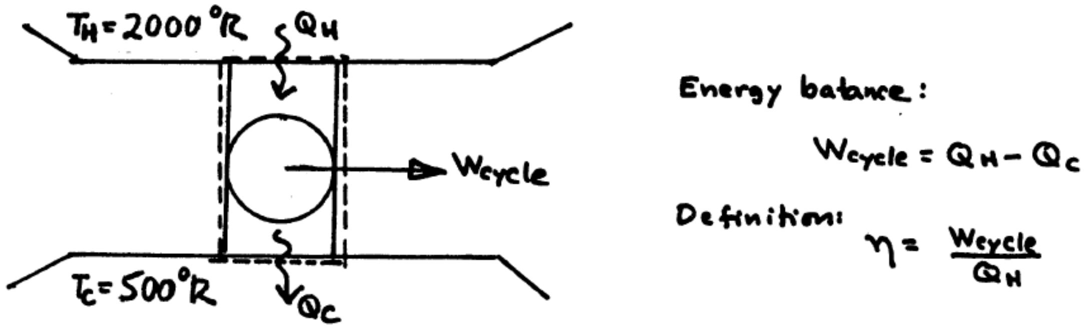
$$Q_{cv} = \left(\frac{pVc_p}{R}\right)(\ln(T_2) - \ln(T_1)) = \left(\frac{pVc_p}{R}\right)\ln\left(\frac{T_2}{T_1}\right)$$

$$Q_{cv} = \frac{(10bar)(1m^3)(1.041kJ/kg\cdot K)}{\left(\frac{8.314}{28.01}\frac{kJ}{kg\cdot K}\right)}\ln\left(\frac{400}{300}\right)\left|\frac{10^5 N/m^2}{1bar}\right|\left|\frac{1kJ}{10^3 Nm}\right|$$

➔ **$Q_{cv}$ = 1008.94 kJ**

**Problem 5**

Given: Operating data are provided for a system undergoing a power cycle while receiving and discharging energy by heat transfer with two thermal reservoirs at specified temperatures.
Find: For each of four sets of data, determine if the cycle is reversible, irreversible, or impossible.
Schematic:




Assumptions:
The system shown in the figure undergoes a power cycle.
Analysis:
a) $Q_H = 800$ Btu, $W_{cycle} = 480$ Btu
The maximum thermal efficiency for any power cycle under the stated conditions is given by:

$$\eta_{max} = 1 - \frac{T_C}{T_H} = 1 - \frac{500}{2000} = 0.75 \ \ (75\%)$$

For the given data:

$$\eta_{max} = \frac{W_{cycle}}{Q_H} = \frac{480}{800} = 0.60 \ \ (60\%)$$

**➔Since $\eta < \eta_{max}$, this cycle is irreversible.**

b) $Q_H = 800$ Btu, $Q_C = 200$ Btu
In this case, it is convenient to use the thermal efficiency in the following form:

$$\eta_{max} = 1 - \frac{Q_C}{Q_H} = 1 - \frac{200}{800} = 0.75 \ \ (75\%)$$

**➔Since $\eta = \eta_{max}$, this cycle is reversible.**

c) $W_{cycle} = 800$ Btu, $Q_C = 200$ Btu
Expressing the thermal efficiency in terms of given quantities:

$$\eta_{max} = \frac{W_{cycle}}{W_{cycle} + Q_C} = \frac{800}{800 + 200} = 0.80 \ \ (80\%)$$

**➔Since $\eta > \eta_{max}$, this cycle is impossible.**

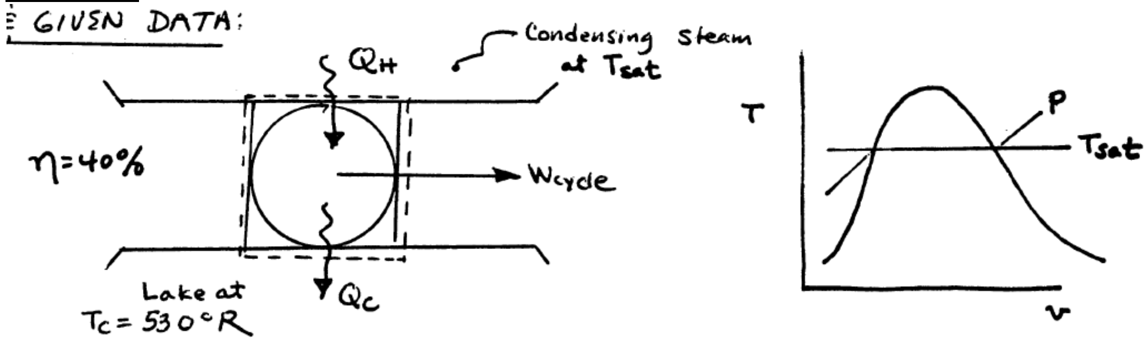
d) $\eta = 50\%$
**➔Since $\eta < \eta_{max}$, this cycle is irreversible.**

**Problem 6**

Given: A system undergoes a power cycle while receiving energy by heat transfer from condensing steam and discharging energy by heat transfer to a lake at 70°F. For the cycle, η = 40%.

Find: Determine the lowest possible condensing steam temperature and the corresponding pressure.

Schematic:




Assumptions:

1) The system shown in the figure undergoes a power cycle
2) The condensing steam and the lake play the roles of the hot and cold reservoirs, respectively.

Analysis:

Since $p_{sat}$ decreases as $T_{sat}$ decreases, the lowest possible condensing steam pressure corresponds to the lowest possible condensing steam temperature. However, the thermal efficiency must be less than, or equal to, the thermal efficiency of a reversible power cycle operating between reservoirs at the specified temperatures:

$$0.40 \le 1 - \frac{T_C}{T_H} = 1 - \frac{530}{T_{sat}}$$

$$0.60 \ge \frac{530}{T_{sat}} \Rightarrow T_{sat} \ge 883°R$$

➔ **The lowest possible saturation temperature is $T_{sat}$ = 883°R (423°F)**

Using $T_{sat}$ and Table A-2E, we can interpolate to determine the corresponding saturation pressure.

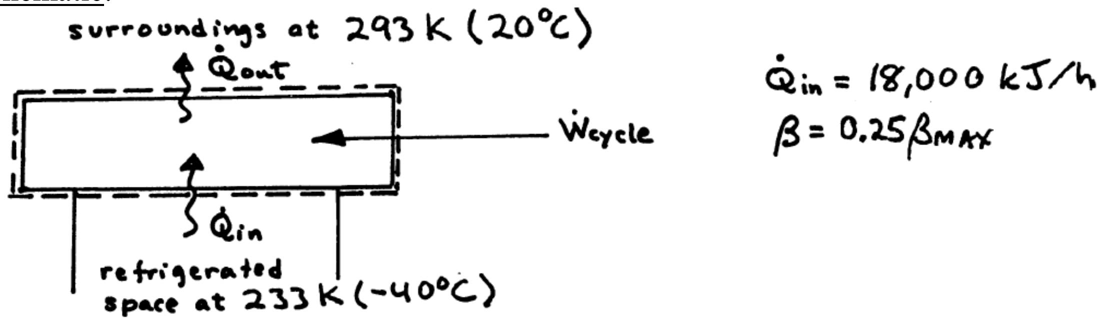
➔ **Saturation pressure, $p_{sat}$ = 319 lbf/in.$^2$**

**Problem 7**

Given: A refrigerator maintains a refrigerated space at a specified temperature. Data for operation at steady state are provided.

Find: Determine the power input required

Schematic:




Assumptions:
1) The system shown in the figure undergoes a refrigeration cycle.
2) The data are for operation at steady state.
3) The refrigerated space and the surroundings play the roles of the cold and hot reservoirs, respectively.

Analysis:

The coefficient of performance of a reversible refrigeration cycle operating between reservoirs at $T_H$ and $T_C$ is given by:

$$\beta_{\max} = \frac{T_C}{T_H - T_C} = \frac{233}{293-233} = 3.88$$

$$\beta = 0.25\beta_{\max} = 0.25 * 3.88 = 0.97$$

$$\beta = \frac{\dot{Q}_C}{\dot{W}_{cycle}} \Rightarrow \dot{W}_{cycle} = \frac{\dot{Q}_C}{\beta} = \frac{18{,}000\frac{kJ}{h}}{0.97}\left|\frac{1h}{3600s}\right|\left|\frac{1kW}{1kJ/s}\right|$$

➔ $\dot{W}_{cycle} = 5.15kW$