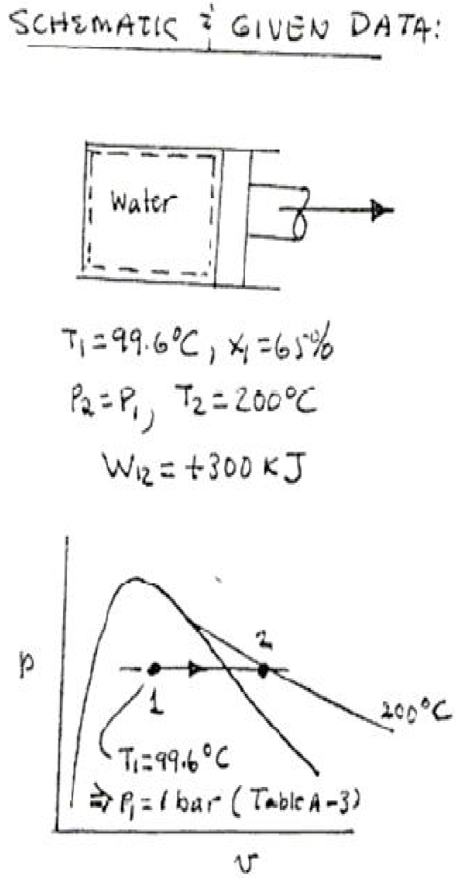
SOLUTIONS TO HOMEWORK 3

Problem 1

Known: Data are provided for a process of water contained in a piston cylinder assembly
Find: Determine the mass of water and Q for the process



Assumption:

1. the water in the piston-cylinder assembly is the closed system
2. the process occurs at constant pressure
3. volume change is the only work mode.
4. changes in kinetic and potential energy are negligible.

Analysis: a) since volume change is the only work mode $W_{12} = \int_1^2 PdV$ on $W_{12} = mP(v_1 - v_2)$

Thus, $m = \dfrac{W_{12}}{P(v_1 - v_2)}$ = $\dfrac{300kJ}{(105\dfrac{N}{m_2})(2.172 - 1.1015)\dfrac{m^3}{kg}} \times \dfrac{10^3 Nm}{1kJ}$ = 2.80kg

where $v_1 = v_f + x_1(v_g - v_f) = (\dfrac{1.0432}{10^3}) + 0.65(1.694 - \dfrac{1.0432}{10^3}) = 1.1015\dfrac{m^3}{kg}$ (table A-3)

$v2 = 2.172\dfrac{m^3}{kg}$ (table A-4)

b) An energy balance reduces to

$$\Delta U + \Delta KE + \Delta PE = Q_{12} - W_{12}$$

or

$$Q_{12} = W_{12} + m(u_2 - u_1)$$

where $u_1 = u_f + x(u_g - u_f) = 417.36 + 0.65(2506.1 - 417.36) = 1775.04\dfrac{kJ}{kg}$ (table A-3)

$u_2 = 2658.1\dfrac{kJ}{kg}$

$$\therefore Q12 = 300kJ + 2.8kg[2658.1 - 1775.04]\frac{kJ}{kg} = 2772.57kJ$$

Problem 2

a) Using the ideal gas equation of state

$$m = \frac{PV}{RT} = \frac{(10MPa)(0.05m^3)}{(\frac{8.314}{28.01}\frac{kJ}{kg \cdot K})(252K)} \times \frac{10^6 N/m^2}{1MPa} \times \frac{1kJ}{10^3 N \cdot m} = 6.685kg$$

b) Using data from table A-1

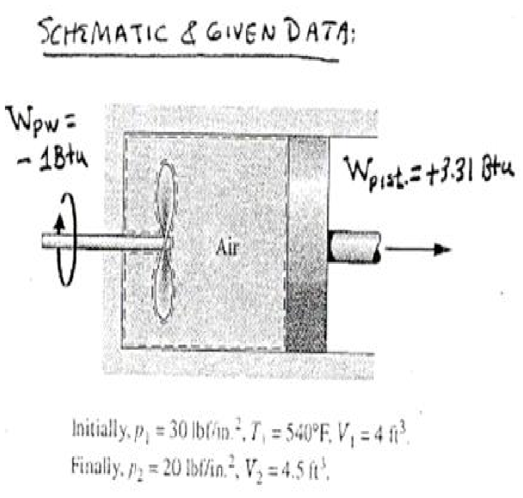
$$P_R = \frac{P}{P_c} = \frac{100bar}{33.9bar} = 2.95 \quad \text{and} \quad T_R = \frac{T}{T_C} = \frac{252K}{126K} = 2.00 \quad \text{Fig A-2 ,} \; v_R' = 0.65$$

$$v_R' = \frac{v}{RT_c / P_c} \Rightarrow v = \frac{v_R' RT_c}{P_c} \Rightarrow v = \frac{(0.65)(\frac{8314J}{28.01kg \cdot K})(126K)}{33.9bar(\frac{10^5 N/m^2}{1bar})} = 7.171 \times 10^{-3} m^3/kg$$

And $m = \frac{V}{v} = \frac{0.05m^3}{7.171 \times 10^{-3} m^3/kg} = 6.973kg$

Assuming that the chart value is correct, the ideal gas model under-predicts the mass by about 4.1%

Problem 3



Known: Data are provided for air contained in a piston-cylinder assembly fitted with a paddle wheel

Find: For the process of the air, find temperature at the final state and Q

Assumptions

1. The air within the piston-cylinder assembly is the closed system
2. the changes in kinetic and potential energy for the process play no role
3. Air is modeled as an ideal gas

Analysis: a) Using the ideal gas equation of state, $P_1V_1 = mRT_1$, $P_2V_2 = mRT_2$

$$\frac{T_1}{T_2} = \frac{P_2V_2}{P_1V_1} = \frac{(20lbf/in^2)}{(30lbf/in^2)} \times \frac{4.5ft^3}{4ft^3} = 0.75 \Rightarrow T_2 = 0.75(1000°R) = 750°R$$

b)Reducing an energy balance,

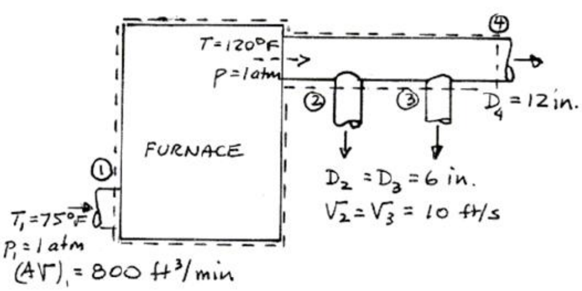
$$\Delta U + \Delta KE + \Delta PE = Q_{12} - W_{12} \Rightarrow Q_{12} = W_{12} + m(u_2 - u_1)$$

Where $W_{12} = W_{pw} + W_{pist} = (-1Btu) + (+3.31Btu) = 2.31Btu$. The specific internal energy values are obtained from table A-22E: $\begin{matrix} u_1 = 172.43Btu/lb \\ u_2 = 128.25Btu/lb \end{matrix}$. The mass, m, is obtained from the ideal gas equation of state

$$m = \frac{P_1 V_1}{(\bar{R}/M)T_1} = \frac{30 \times 144 \dfrac{lbf}{ft^2} \times 4\,ft^3}{(\dfrac{1545\,ft \cdot lbf}{28.97lb \cdot {}^\circ R})(1000^\circ R)} = 0.324lb$$

Finally, $Q_{12} = 2.31Btu + (0.324lb)(128.25 - 172.43)\dfrac{Btu}{lb} = -12Btu$

Problem 4



Known: Air enters a furnace operating at steady sate and is to a duct system consisting of three ducts. Data are known at the inlet and each of the discharge ducts

Find: Determine (a) the mass flow rate entering the furnace, (2)the volumetric flow rate in each of the 6-in. exit ducts, (c) the velocity in the 12-in. exit duct

Assumption: 1. the control shown on the accompanying sketch is at steady state.

2. The air behaves as an ideal gas

3. The temperature and pressure in each duct are the same as the temperature and pressure of the air delivered to the duct system

Analysis: (a) Using Eq 4.4b, with the ideal gas equation

$$m_1 = \frac{(AV)_1}{v_1} = \frac{P_1 (AV)_1}{RT_1}$$

$$= \frac{(1atm)(800\,ft^3/\text{min})}{(\dfrac{1545\,ft \cdot lbf}{28.97lb^\circ R})(535^\circ R)} \times \frac{14.696lbf\ /in^2}{1atm} \times \frac{144in^2}{1\,ft^2} \times \frac{1\text{min}}{60s} = 0.989lb/s$$

(b)Since $D_2 = D_3$ and $V_2 = V_3$

$$(AV)_2 = (AV)_3 = (\frac{\pi D^2}{4})V = (\frac{\pi(\dfrac{6}{12})^2\ ft^2}{4})(10\,ft/s)(60s/1\text{min}) = 117.8\,ft^3/\text{min}$$

(c) Applying the mass balance to get

$$\frac{dm_{cv}}{dt} = \dot{m}_1 - \dot{m}_2 - \dot{m}_3 - \dot{m}_4 = 0 \Rightarrow \dot{m}_4 = \dot{m}_1 - \dot{m}_2 - \dot{m}_3 \quad (*)$$

Where $\dot{m}2 = \dot{m}3 = \frac{(AV)}{v} = \frac{P(AV)}{RT} = \frac{(14.696)(117.8)}{(\frac{1545}{28.97})(580)} \times (\frac{144}{60}) = 0.1343 lb/s$

From (*)

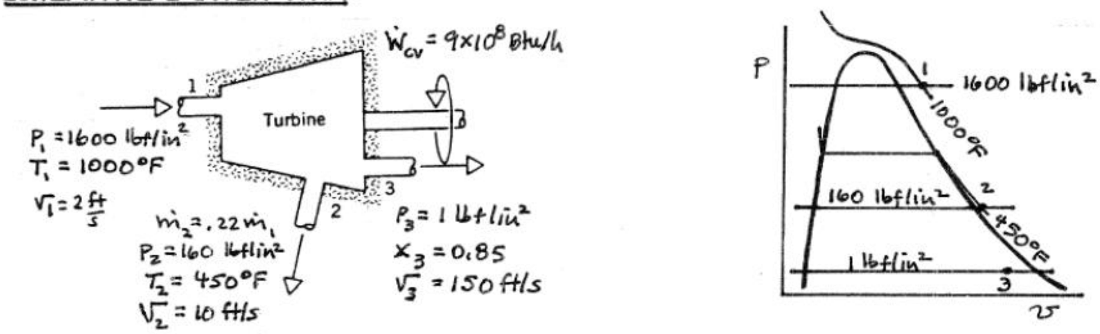
$\dot{m}_4 = 0.989 - 2(1.343) = 0.7204 lb/s$

Finally, from $\dot{m}_4 = \frac{A_4 V_4}{v_4}$

$$V_4 = \frac{v_4 \dot{m}_4}{A_4} = \frac{(RT_4)(\dot{m}_4)}{(P_4)(\frac{\pi D_4^2}{4})} = \frac{(\frac{1545}{28.97})(580)(0.7704)}{(14.696)(\frac{\pi (12)^2}{4})} = 13.41 ft/s$$

Problem 5

SCHEMATIC & GIVEN DATA:



Known: steam passes through an extraction turbine operating at steady state with known inlet and exit conditions. The power output is specified

Find: Determine (a) the inlet mass flow rate (b)the diameter of the extraction duct

Assumption:

1. A control volume enclosing the turbine is at steady state
2. For the control volume, heat transfer and potential energy effects are negligible

Analysis: To find $\dot{m}_1$, apply a mass rate balance: $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$

Then, since $\frac{\dot{m}_2}{\dot{m}_1} = 0.22$, we have $\frac{\dot{m}_3}{\dot{m}_1} = 0.78$. Next apply an energy rate balance

$0 = -\dot{W}_{cv} + \dot{m}_1 (h_1 + \frac{V_1^2}{2}) - \dot{m}_2 (h_2 + \frac{V_2^2}{2}) - \dot{m}_3 (h_3 + \frac{V_3^2}{2})$ where the potential energy terms are omitted by assumption 2.

Solving $\dot{m}_1 = \dfrac{\dot{W}_{cv}}{(h_1 + \frac{V_1^2}{2}) - 0.22(h_2 + \frac{V_2^2}{2}) - 0.78(h_3 + \frac{V_3^2}{2})}$

From table A-4E, h_1=1487Btu/lb, h_2=1246.1Btu/lb. With table A-3E data, $h_3 = h_{f3} + x_3(h_{g3} - h_{f3}) = 69.74 + 0.85(1036) = 950.3 Btu/lb$

Thus,

$$\dot{m}_1 = \frac{(9\times10^8 Btu/h)}{(1487\frac{Btu}{lb} + \frac{(2\,ft/s)^2}{2} \times \frac{1lbf}{32.2lb\cdot ft/s^2(778)}) - 0.22(1246.1 + \frac{(10)^2}{2(32.2)(778)}) - 0.78(950.3 + \frac{(150)^2}{2(32.2)(778)})}$$

=1.91 x 10^6 lb/h

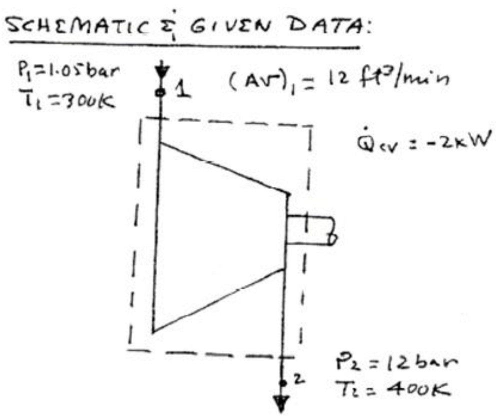
**Unit conversion is necessary for the velocity components in the denominator since it is in ft^2/s^2. To convert that into Btu/lb, which is the unit for enthalpy, first of all, multiply 1Btu/778 ft lbf and to get rid of lbf, multiply 1 lbf / (32.2 lb ft/ s^2). Then the unit will come out as Btu/lb. (1Btu = 778 ft lbf and 1 lbf = 32.2 lb ft/ s2 from the conversion factors chart in the book)

Every component in the denominator is in Btu/lb. In the end, (Btu/h) / (Btu/lb) ➔ lb/h

Since $A_2 = \dfrac{\pi d^2}{4}$

$$d_2 = \sqrt{\frac{4A_2}{\pi}} = \sqrt{\frac{(4)(37.68\,ft^2)}{\pi}} = 6.93\,ft$$

Problem 6



Assumptions:

1. The control volume shown in the sketch is at steady state
2. The air is modeled as an ideal gas
3. Kinetic and potential energy effects are neglected

Analysis: reducing Eq. 4. 20a

$\dot{W}_{cv} = \dot{Q}_{cv} + \dot{m}(h_1 - h_2)$

Where

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{(AV)_1 P_1}{RT_1} = \frac{(12\,ft^3/\min)(1.05\times105 N/m^2)}{(\frac{8314 N\cdot m}{28.97 kg\cdot K})(300K)} \times (\frac{1\min}{60s}) = 0.2436 kg/s$$

With enthalpy data from table A-22, Eq(1) gives

$$\dot{W}_{cv} = -2kW + (0.2439 kg/s)(300.19 - 400.98)\frac{kJ}{kg}(\frac{1kW}{1kJ/s}) = -26.6kW$$

Problem 7

Known: Ammonia and air pass in separate streams through a heat exchanger at steady state, for which data are provided

Find: Determine the mass flow rate of the air

Assumptions

1. A control volume enclosing the heat exchanger is at steady state
2. For the control volume, $\dot{W}cv = 0$, heat transfer can be ignored, and kinetic/potential energy effects are negligible.
3. Air is modeled as an ideal gas

Analysis: Since the streams flow separately, the conservation of mass principle indicates as steady state: $\begin{matrix} \dot{m}_1 = \dot{m}_2 \equiv \dot{m}_R, \\ \dot{m}_3 = \dot{m}_4 \equiv \dot{m}_A \end{matrix}$, energy balance reads

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_R[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(Z_1 - Z_2)] + \dot{m}_A[h_3 - h_4 + \frac{V_3^2 - V_4^2}{2} + g(Z_3 - Z_4)]$$

➔ $$\dot{m}_A = \dot{m}_R[\frac{h_1 - h_2}{h_4 - h_3}]$$

From table A-15, h_1=1542.89kJ/kg. From table A-14, h_2=352.97kJ/kg. From table A-22, h_3=290.16kJ/kg, h_4=315.27kJ/kg. Then

$$\dot{m}_A = (450)\frac{kg}{h}[\frac{1542.89 - 352.97}{315.27 - 290.16}](\frac{1h}{60\min}) = 355.4kg/\min$$