# SOLUTIONS TO HOMEWORK 1

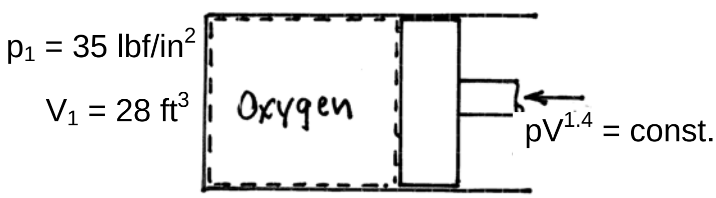
**Problem 1:**

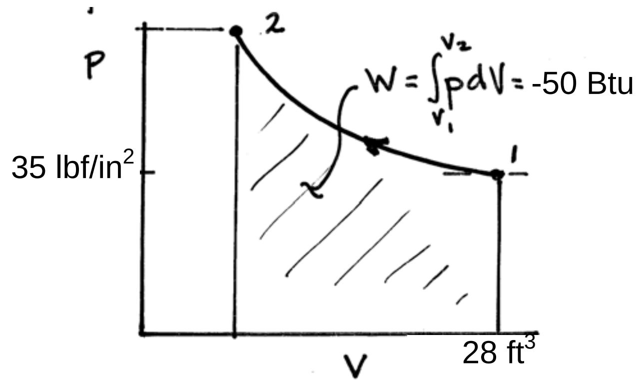
**Given**: Oxygen in a piston-cylinder process undergoes a process where $pV^{1.4} = \text{constant}$. The total work done is known.

**Find**: The final volume and pressure

**Assumptions**: (1) $O_2$ is a closed system (2) process is polytropic.

**Analysis**:




We start with $W = \int_{V_1}^{V_2} p\,dV$, where W is the work done in a piston-cylinder assembly.

To determine $V_2$, substitute the p-V relation into this equation and integrate

$W = \int_{V_1}^{V_2} \frac{\text{constant}}{v^{1.4}} dV = \text{constant}\frac{v_2^{-.4} - v_1^{-.4}}{-.4}$. But we know, constant = $pV^{1.4}$ thus the

$\text{constant} = P_1 V_1^{1.4}$

Using this expression and solving for $V_2$, we get

$$V_2^{-.4} = \frac{-0.4\,W}{P_1 V_1^{1.4}} + V_1^{-0.4} = \frac{(-.4)(-50\text{ Btu})}{(35\text{ lbf/in}^2)(28\text{ ft}^3)^{1.4}}\left|\frac{778\text{ ft.lbf}}{1\text{ Btu}}\right|\left|\frac{1\text{ ft}^2}{144\text{ in}^2}\right| + (28\text{ ft}^3)^{-.4} = 0.2928$$

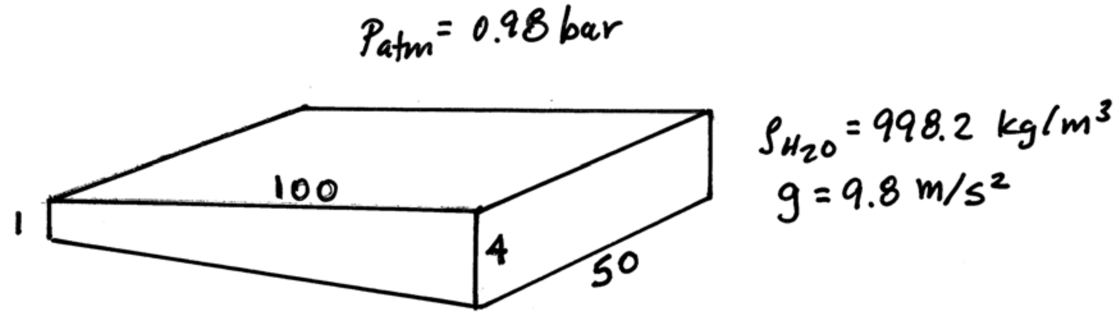
$V_2 = 21.56\text{ ft}^3$

Now, we use the p-V relation to get $P_2$

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{1.4} = \left(35\frac{\text{lbf}}{\text{in}^2}\right)\left(\frac{28}{21.56}\right)^{1.4} = 50.47\text{ lbf/in}^2$$

The shaded portion in the p-V diagram shows the work done.

**Problem 2**:




**Given**: Dimensions of the swimming pool = 100 m x 50 m and depth varying from 1 m to 4m, density of water = 998.2 kg/m$^3$, atmospheric pressure = 0.98 bar.

**Find**: total force at the bottom of the pool and pressure on the floor at the center of the pool

**Analysis**: Total pressure at the bottom of the pool = weight of water inside the pool + downward force exerted by atmosphere on the surface of the water.

$$\mathbf{F_{total} = F_{atm} + F_{grav}}$$

To find the weight of water, find the total mass of water inside the pool as follows:

$$m = \rho V = 998.2\frac{\text{kg}}{\text{m}^3}\left[(1)(100)(50)+\frac{1}{2}(3)(100)(50)\right]\text{m}^3$$

$$= (998.2)(12500) = 1.25\text{x}10^7\text{kg}$$

Gravitational force exerted by water is given by,

$$\mathbf{F_{grav}} = \text{mg} = 1.25\text{x}10^7\text{kg x } 9.81\ \frac{\text{m}}{\text{s}^2}\left|\frac{1\text{N}}{1\text{ kgms}^{-2}}\right|\left|\frac{1\text{kN}}{10^3\text{N}}\right| = 1.226\text{x}10^5\text{ kN}$$

$$\text{F}_{\textbf{atm}} = \text{P}_{\text{atm}}\ \text{A}_{\text{surface}} = (0.98\text{ bar})\ (100\text{x}50)\text{m}^2\left|\frac{10^5\ \text{N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kN}}{10^3\text{N}}\right| = 4.9\text{x}10^5\text{ kN}$$

Therefore, the total force acting on the bottom of the pool surface is

$\mathbf{F_{total}}$ = 1.226 x $10^5$ kN + 4.9 x $10^5$ kN = 6.126 x $10^5$ kN

Depth at the center of the pool is $h_{center}$ = 2.5 m. Therefore, the pressure at the center of the pool is given by, $\mathbf{P_{center}} = P_{atm} + \rho g h_{center}$

$$= (0.98\text{ bar})\left|\frac{100\text{ kPa}}{1\text{ bar}}\right| + \left(998.2\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(2.5\text{ m})\left|\frac{1\text{ N}}{1\text{ kg}\cdot\text{ms}^{-2}}\right|\left|\frac{1\text{ kpa}}{10^3\text{ Nm}^{-2}}\right| = 122.5\text{ kPa}$$

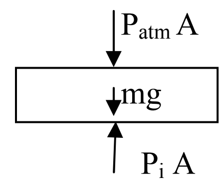
**Problem 3**

**Given**: Frictionless piston cylinder arrangement
Cross-section of the piston = $0.5m^2$
Air trapped inside is at 1000K and 400kPa.
2 processes: Process 1 – till piston hits the stops
Process 2 – after the piston hits the stops

**Assumptions**:

1. Process happens very slowly
2. Friction between the piston and Cylinder wall is ignored
3. Let the mass of the weight + piston be m.

**Analysis**:




Part A) Consider the free body diagram of the piston.
Perform a force balance:

$$P_i A = P_{atm} A + mg$$

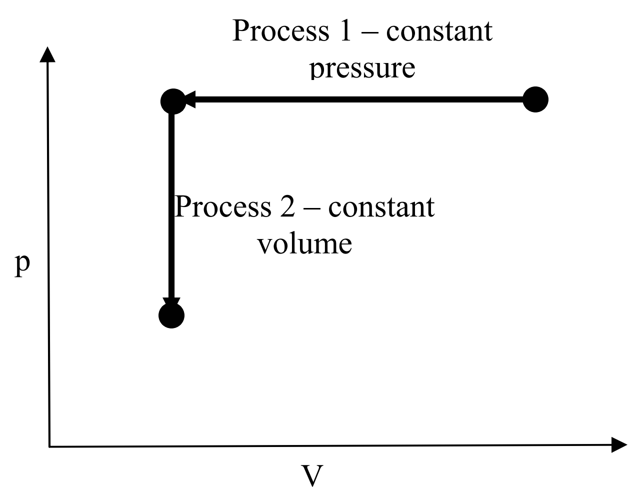
Therefore, in process 1,

$$P_i = \text{constant} = P_{atm} + mg / A$$

Thus the pressure remains constant during this process (isobaric process). In essence we assume that the process happens quasi-statically and that the piston is at all times in mechanical equilibrium.

**Remark:** If you are wondering how the pressure remains constant as the gas is cooled and compressed at the same time, think what will be the case for an ideal gas PV=RT with constant P. As you reduce the temperature (cooling the gas), V is reduced (i.e. the gas is compressed). Similar behavior is expected from non-ideal gases.

Note that once the piston hits the stops, it does not move upon further cooling. Therefore, the volume is fixed in Process 2.
Part B) Sketch the two processes on p-V diagram?



Part C) What is the work done in both the processes?

In process 1: Using Eq 2.17; $W = \int_{v1}^{v2} p\,dv = p*(V_2 - V_1) = p*(h_2 - h_1)*\text{Area}$

$= 400 \text{ kPa} * (10^3 \text{ N/m}^2 / 1\text{kPa}) * (-1\text{m}) * 0.5 \text{ m}^2 * (1\text{kJ}/10^3 \text{ N.m})$

$\mathbf{= -200 \text{ kJ}}$

In process 2: $W = \int_{v1}^{v2} p\,dv$; Since the volume is the same, the net work is **0kJ**.

**Problem 4:**

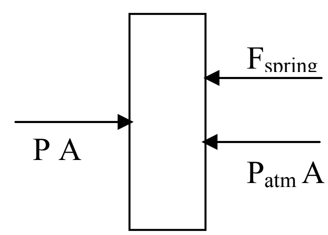
**Given**: Warm air cools slowly in a piston cylinder assembly from a known initial volume to a known final volume. During the process, a spring exerts a force on the piston that varies linearly from a known initial value to a final value of zero. The initial volume $V_1 = 0.003 \text{ m}^3$, final volume $V_2 = 0.002 \text{ m}^3$.

**Find**: The initial and final pressures of the air and the work.

**Assumptions**:

1. The air is a closed system.
2. Process occurs slowly no acceleration.
3. No friction between piston and cylinder wall.
4. Spring force varies linearly with volume (zero when $V = V_2$).

**Analysis**: The initial and final pressures of the air are determined from a free body diagram of the piston. Consider the force balance along the horizontal direction.




Initially: $P_1 = P_{atm} + F_{spring}/A$

$= 100 \text{ kPa} + 900\text{N}/0.018\text{m}^2 * (1\text{kPa}/10^3\text{N/m}^2)$

$= 150 \text{ kPa}$

Finally: The force exerted by the spring is zero.

Therefore, $P_2 = P_{atm} = 100 \text{ kPa}$

Work is determined from $W = \int_{V1}^{V2} p\,dV$

But the pressure at any time in the cylinder is $p = P_{atm} + F_{spring}/A$

And $F_{spring}$ (from assumption 4) $= k*(V-0.002)$ (k being a constant)

$F_{spring}(\text{initial}) = k(0.003 - 0.002) = 900 \text{ N}$

Therefore, $k = 9\text{x}10^5 \text{ N/m}^3$

Finally, $W = \int_{V1}^{V2} \{ P_{atm} + (900/0.001) *(V-0.002)/( 0.018 * 10^3) \} dV$

$= \int_{V1}^{V2} \{ 100 + 50000V - 100 \} dV$

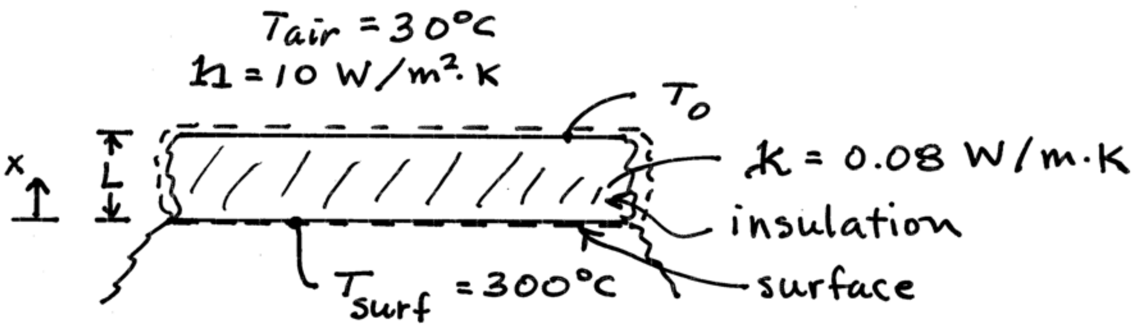
$= 250000 \text{ kPa} * (V_1^2 - V_2^2)$

$$= -0.125\ \text{kPa*m}^3 \left|\frac{1000\ \text{N/m}^2}{1\ \text{kPa}}\right|\left|\frac{1\ \text{kJ}}{1000\ \text{N}\cdot\text{m}}\right|$$

$$= -0.125\ \text{kJ}$$

The negative sign indicates that work is done on the air (system) by the piston (surroundings).

**Problem 5**:




**Given :** A hot surface and a insulation. Energy transfer occurs from the outer surface of the insulation to the surrounding air.

**Find :** To determine the minimum thickness of insulation to maintain the outer surface temperature less than or equal to 60 $^oC$.

**Assumptions**:

1. The system is at steady state.
2. Newton's law of cooling applies for heat transfer from the insulation to the air.
3. The temperature distribution throughout the insulation is linear.
4. The thermal conductivity of the insulation is uniform.

**Analysis**:

In the insulation, energy is transferred by conduction. We have,

$\dot{Q}_x = -kA\frac{dT}{dx} = kA\frac{(T_{surf} - T_0)}{L}$, where `k' is thermal conductivity and A area of insulation material perpendicular to the paper.

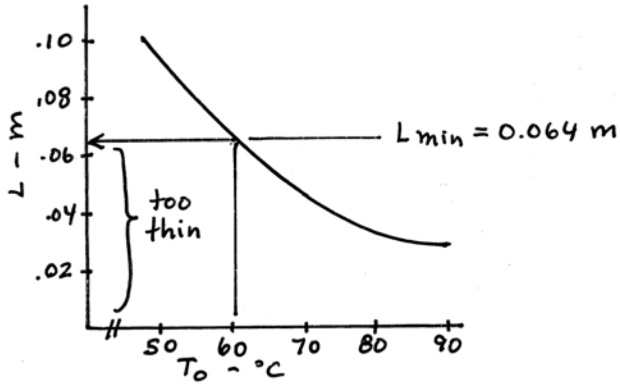
On the outer surface, energy transfer occurs by convection from insulation to air.

$\dot{Q}_0 = hA(T_0 - T_{air})$, where `h' is the convection heat transfer coefficient.

At steady state, $\dot{Q}_x = \dot{Q}_0 \Rightarrow \frac{kA(T_{surf} - T_0)}{L} = hA(T_0 - T_{air})$

Solving for L, we obtain $L = \frac{k(T_{surf} - T_0)}{h(T_0 - T_{air})}$

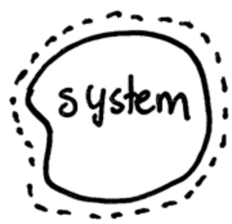
Using the given data, we obtain the minimum value of L to be 0.064 m to satisfy the required constraint on $T_0$. A plot of L versus $T_0$ is given below :




**Problem 6**:

**Given** : A closed system undergoes a process with a known heat transfer rate, and the power varies as a given function of time.

**Find** : Determine (i) the rate of change of system energy at t = 0.7 h and (ii) the change of system energy after 2.5 h.




**Assumption** : The system is closed

**Analysis** : (i) The time rate of change of energy is at any time t is given by

$$\frac{dE}{dt} = \dot{Q} - \dot{W}. \quad \text{At t = 0.7 h,}$$

$$\left.\frac{dE}{dt}\right|_{t = 0.6\text{ h}} = \left[\dot{Q} - (-9t)\right]_{t = 0.7\text{ h}} = (-10\text{ kW}) - (-9\text{x}0.7)\text{ kW} = -3.7\text{ kW}$$

(ii) The change in system energy is obtained by integrating the expression for dE/dt over the time period of 2.5h. That is

$$\Delta E = \int_{t=0}^{t=2.5h} \left(\dot{Q} - \dot{W}\right) dt = \dot{Q}\Delta t - \left[\int_{t=0}^{t=1h} (-9t)\, dt + \int_{t=1h}^{t=2.5h} -9\, dt\right]$$

$$= (-10)(2.5) - \left[\frac{-9}{2}t^2\right]_0^1 - \left[(-9)\, t\right]_1^{2.5} = -25 - [(-4.5)] - [(-9)(2.5-1)]$$

$$= -7\text{ kWh}$$

Therefore, $\Delta E = (-7\text{ kWh}) \left|\frac{1\text{ kJ/s}}{1\text{ kW}}\right|\left|\frac{3600\text{ s}}{1\text{ h}}\right| = -25,200\text{ kJ}$

(a) At t = 0.6 h, the energy of the system is decreasing at the rate of 3.7 kW because the rate of heat transfer out exceeds the rate of heat transfer in by work.
(b) The negative sign denotes a net decrease of energy over time period.