Thursday November 15, 7:30 pm – 9:30 pm

Closed books and notes.

The notation is all problems is the one used in class and further explanations are not needed. Answer all questions. Make sure your answers are legible.

The TAs and instructor will not respond to any questions during the exam. If you think that something is wrong with one of the problems below, please state your concern in your exam books.

Problem 1 (20 points) *Answer each question carefully providing all necessary details.*

1. (3 points) For the following combined form of the first and second laws of thermodynamics, dG=-SdT+VdP, write down the corresponding Maxwell equation as well as the two coefficient relations.

Solution: (a)

$$dG = -SdT + VdP \quad \Rightarrow \quad -\left(\frac{\partial S}{\partial P}\right)_T = \left(\frac{\partial V}{\partial T}\right)_P, S = -\left(\frac{\partial G}{\partial T}\right)_P, V = \left(\frac{\partial G}{\partial P}\right)_T$$

2. (5 point) Use the methodology of your choice to compute an expression for $\left(\frac{\partial V}{\partial U}\right)_T$ in terms of T, P, α and β.

Answer: Since T is one of the independent variables we can write that

$$\left(\frac{\partial V}{\partial U}\right)_T = \frac{\left(\frac{\partial V}{\partial P}\right)_T}{\left(\frac{\partial U}{\partial P}\right)_T} \qquad (1)$$

Solving for the resulting relations, we use equations for dU and dV

$$dV = V\alpha dT - V\beta dP \qquad (2)$$

The equation for dS is given as: dS= c_P/T dT – α v dP (the Maxwell equation used here is $\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P = -\alpha V$ derived from dG = -S dT + V dP in part 1 above)

From the above equation using TdS = dU+ P dV, we derive the following:

$$dU = (C_P - PV\alpha)dT + V(P\beta - T\alpha)dP \qquad (3)$$

At constant T, the coefficients of dP are (from eqs 2 and 3, respectively):

$$\left(\frac{\partial V}{\partial P}\right)_T = -V\beta \qquad (4)$$

$$\left(\frac{\partial U}{\partial P}\right)_T = V\left(P\beta - T\alpha\right) \qquad (5)$$

Substituting 4 and 5 into 1 and reducing we find that

$$\boxed{\left(\frac{\partial V}{\partial U}\right)_T = -\frac{\beta}{\left(P\beta - T\alpha\right)}}$$

3. (5 points) From Eqs. $C_P = (\partial h/\partial T)_P = T\,(\partial s/\partial T)_P$ and $C_v = (\partial u/\partial T)_v = T\,(\partial s/\partial T)_v$ and the knowledge that Cp > Cv what can you conclude about the slopes of constant v and constant P curves in a T-s diagram? Notice that we are looking at functions T(s) with P or v given.

Solution

The slopes of constant v or constant P curves in a T-s diagram are T lines as a function of s with either v or P held constant. Therefore, the slopes of these lines are $(\partial T/\partial s)_v$ and $(\partial T/\partial s)_P$.

From the definitions of c_v and c_p:

$c_v = (\partial u/\partial T)_v = T\,(\partial s/\partial T)_v$ (for the derivation on the right we used: du = Tds – Pdv)
$c_p = (\partial h/\partial T)_P = T\,(\partial s/\partial T)_P$ (for the derivation on the right we used: dh = Tds + vdP)

we get using the inversion relation (e.g. $(\partial T/\partial s)_v = 1 / (\partial s/\partial T)_v$, etc.).

$(\partial T/\partial s)_v = T/c_v$ and
$(\partial T/\partial s)_P = T/c_p$

Since $c_p > c_v$ (recall the general relation $C_p - C_v = T\frac{\alpha^2 v}{\beta} > 0$), we can see that

$(\partial T/\partial s)_v > (\partial T/\partial s)_p$

➔ Therefore, we can conclude that the T(s) curves of constant volume are steeper than the T(s) curves of constant pressure.

4. (3 points) The Clapeyron equation as discussed in lecture is used to:

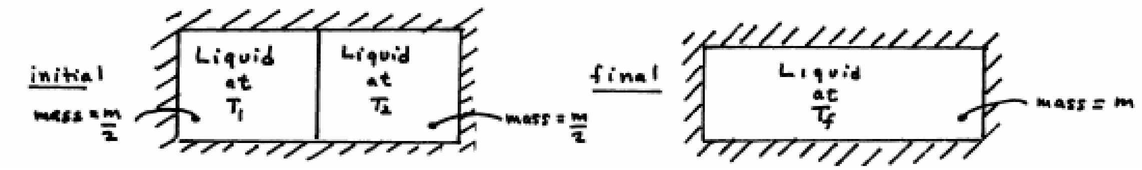
   a) compute the entropy change in polytropic processes
   b) determine the enthalpy change associated with phase transformations
   c) derive the Maxwell equations

d) compute the relation between the specific heats C_v and C_p
e) compute the thermal expansion coefficient

Solution: (b)

<u>Problem 2</u> (10 points)

An isolated system of mass m if formed by mixing two equal masses (m/2) of the same liquid initially at the temperatures T_1 and T_2 (see Figure below). Eventually the system attains an equilibrium state (temperature $T_f=(T_1+T_2)/2$). **Each mass is incompressible with constant specific heat c.** Compute the amount of entropy produced in terms of m, c, T_1 and T_2.



Hint: To compute entropy changes for each mass, consider the combined 1st ($\delta Q = dU + p\,dV$) and 2nd ($\delta Q = TdS$) laws of thermodynamics as well as the definition of specific heat.

<u>Solution:</u>

An entropy balance gives $\Delta S = \cancelto{0}{\int_1^2 \left(\frac{\delta Q}{T}\right)_b} + \sigma$

Or, $\sigma = ms_f - \left[\frac{m}{2}s_1 + \frac{m}{2}s_2\right] = \frac{m}{2}\left[\left(s_f - s_1\right) + \left(s_f - s_2\right)\right]$

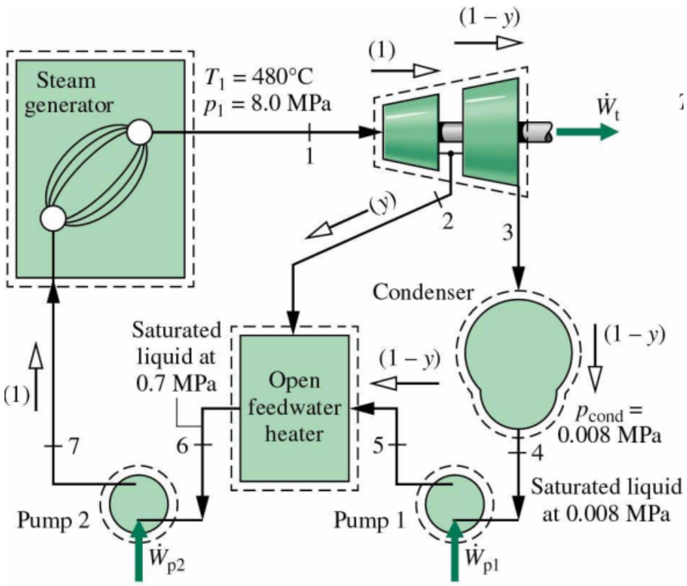
Using Eq. (6.28) in text, $\sigma = \frac{m}{2}c\left[\ln\frac{T_f}{T_1} + \ln\frac{T_f}{T_2}\right] = \frac{mc}{2}\ln\left(\frac{T_f^2}{T_1T_2}\right)$

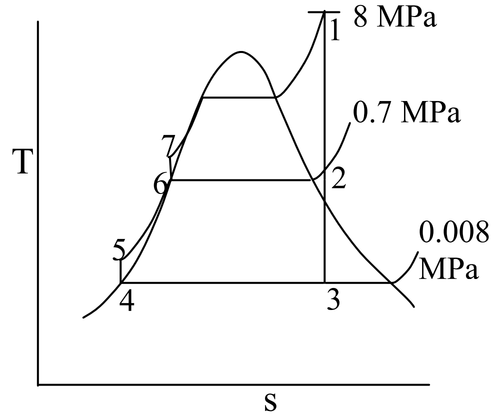
$$\sigma = mc\ln\left[\frac{T_f}{\left(T_1T_2\right)^{1/2}}\right] = mc\ln\left[\frac{T_1+T_2}{2\left(T_1T_2\right)^{1/2}}\right]$$

<u>Problem 3</u> (25 points)

Consider a regenerative vapor power cycle with one open feed water heater. Steam enters the turbine at 8.0 MPa, 480 °C and expands to 0.7 MPa, where some of the steam is extracted and diverted to the open feed water heater operating at 0.7 MPa. The remaining steam expands through the second stage turbine to the condenser pressure of 0.008 MPa. Saturated liquid exits the open feed water heater at 0.7 MPa. The turbine stage and each pump operate isentropically. The net power output of the cycle is 100 MW.

(1) (4 points) Provide the T-S diagram clearly indicating all states.
(2) (7 points) The fraction y extracted from state 2
(3) (7 points) The thermal efficiency
(4) (7 points) The mass flow rate of steam entering the first stage turbine in kg/hr





Necessary data at different states (please note that some data are artificial):
State 1: $h_1 = 3348.4$ kJ/kg
State 2: $h_2 = 2741.8$ kJ/kg
State 3: $h_3 = 2082.92$ kJ/kg
State 4: $h_4 = 173.88$ kJ/kg, $v_4 = 1.0084\text{x}10^{-3}$ m^3/kg
States 5 & 6: Pressure constant at 0.7 MPa
State 6: $h_6 = 697.22$ kJ/kg, $v_6 = 1.1080\text{x}10^{-3}$ m^3/kg

Solution:
The specific enthalpy at state 5 is evaluated as
$h_5 = h_4 + v_4*(p_5 – p_4)$

$$= 173.88 + (1.0084*10^{-3})\ \text{m}^3/\text{kg} * (0.7-0.008)\ \text{MPa} * \frac{10^6\ N/m^2}{1MPa} * \frac{1kJ}{10^3\ N.m} = 174.6\ \text{kJ/kg}$$

Similarly, the specific enthalpy at state 7 can be evaluate

$h_7 = h_6 + v_6*(p_7 – p_6)$

$$= 697.22 + 1.1080\text{x}10^{-3}\ \text{m}^3/\text{kg} * (8.0 – 0.7)\ \text{MPa} * \frac{10^6\ N/}{1MI}\ \frac{1kJ}{10^3\ N.m} = 705.3\ \text{kJ/kg}$$

3

(b) fraction extracted, y = $\frac{h_6 - h_5}{h_2 - h_5} = \frac{697.22 - 174.6}{2741.8 - 174.6} = 0.2035$

(c) The total work output from the turbines is

$$\frac{\dot{W}_t}{\dot{m}} = (h_1 - h_2) + (1-y)*(h_2 - h_3) = (3348.4 - 2741.8) + (1 - 0.2035)*(2741.8 - 2082.92)$$

= 1131.39 kJ/kg

The total pump work per unit mass passing through the first stage turbine is

$$\frac{\dot{W}_p}{\dot{m}} = (h_7 - h_6) + (1-y)*(h_5 - h_4) = (705.3 - 697.22) + (1 - 0.2035)*(174.6 - 173.88)$$

= 8.65 kJ/kg

The heat added in the steam generator per unit mass passing through the first stage turbine is

$$\frac{\dot{Q}_{in}}{\dot{m}} = (h_1 - h_7) = (3348.4 - 705.3) = 2643.1 \text{ kJ/kg}$$

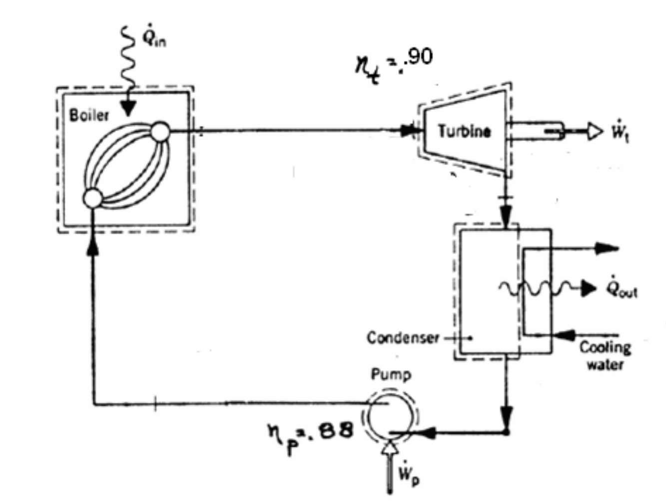
Thus the thermal efficiency is $\eta = \frac{1131.39 - 8.65}{2643.1} = 0.4247$

(d) The net work output from the cycle per unit mass flowing through the first stage turbine is $\frac{\dot{W}_t}{\dot{m}} - \frac{\dot{W}_p}{\dot{m}}$ **=(1131.39-8.65)kJ/kg = 1122.74 kJ/kg.**

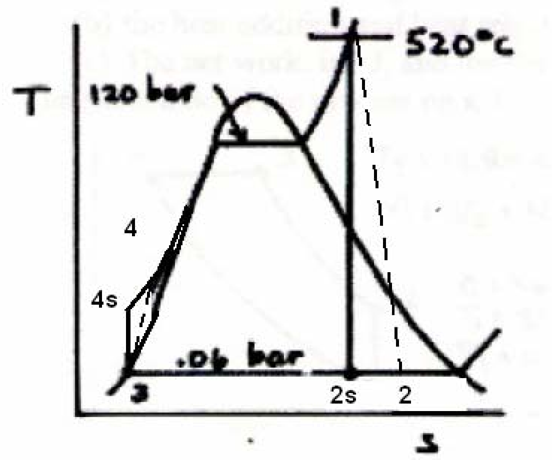
Thus, the mass flow rate is $\frac{100MW}{1122.74kJ/kg} * \frac{10^3\, kJ/s}{1MW}$ **= 89.06 kg/s = 3.206*10^5 kg/hr**

<u>Problem 4</u> (20 points):

Superheated steam at 120 bar and 520°C leaves the steam generator of a vapor power plant and enters a turbine with an efficiency of 90%. The pressure of the turbine at the exit is 0.06 bar. Saturated liquid leaves the condenser at 0.06 bar. The efficiency of the pump is 88%. Compressed liquid leaves the pump at 100 bar. The mass flow rate is 10^6 kg/hr.



a) (5 points) Draw a T-S diagram. Be sure to label clearly states 1, 2, 2s, 3, 4, and 4s.
b) (5 points) Using the isentropic efficiency of the turbine, find the specific enthalpy at state 2.
c) (5 points) Using the isentropic efficiency of the pump, find the specific enthalpy at state 4.
d) (5 points) The net power output, in MW



Solution:

a) T-s diagram is shown here.

b) Given p_1 = 120 bar and T_1 = 520 °C, h_1 = 3401.8 kJ/kg, s_1 = 6.5555 kJ/kgK = s_{2s}

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{6.5555 - 0.5210}{8.3304 - 0.5210} = 0.773$$

$$h_{2s} = 151.53 + 0.773(2415.9) = 2018.35 kJ/kg$$

$$h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 3401.8 - 0.9(3401.8 - 2018.35) = 2156.7 kJ/kg$$

c) h_3 = 151.53 kJ/kg, s_3 = 0.5210 kJ/kgK

$$h_{4s} = h_3 + v_3 * (p_4 - p_3) = 151.53 + 1.0064x10^{-3}(120 - 0.06)10^2 = 163.6\ kJ/kg$$

$$h_4 = h_3 + (1/\eta_p) * (h_{4s} - h_3) = 151.53 + (1/0.88) * (163.6 - 151.53) = 165.25\ kJ/kg$$

d) Net Power = $\left(\frac{\dot{W}_t}{\dot{m}} - \frac{\dot{W}_p}{\dot{m}}\right) * \dot{m}$

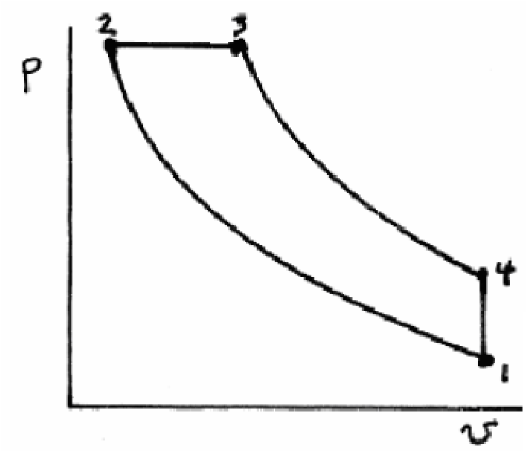
= [(3401.8 – 2156.7) - (165.25-151.53)] *10^6 kg/hr * 1 hr/3600 sec
=342050 kW= 342.05 MW

Problem 5 (15 points)

h_3=1855.5 kJ/kg
u_3=1373.24 kJ/kg
h_4=895.11 kJ/kg
u_4=646.8 kJ/kg
h_1=295.17 kJ/kg
u_1=210.49 kJ/kg
$r_c = V_2/V_3 = 2$
$r = V_1/V_2 = 15.5$

For air: M=28.97 kgr/kmol

R=8.314 kJ/kmol K



In the air-standard Diesel cycle above, the compression ratio is $r = \frac{V_1}{V_2} = 15.5$, the cutoff ratio is $r_c = \frac{V_3}{V_2} = 2$, and the initial (state 1) temperature, pressure and volume are T_1 = 295 K, p_1 = 1 bar and V_1=0.02 m^3. Additional specific enthalpy and specific heat data are provided above (note that not all this data is needed to answer the questions below).

(a) (5 points) Assuming that process 1-2 is isentropic, use the supplied table below to find the temperature at state 2.

(b) (10 points) Compute the heat added to the system in kJ.

(c) (5 points) What is the net work of the cycle?

(d) (5 points) Find the thermal efficiency of the cycle..

Solution:

a) **If the process is isentropic, then** $\frac{v_{r1}}{v_{r2}} = \frac{v_1}{v_2} = \frac{V_1}{V_2} = r$ **. From the supplied table at 295 K,** v_{r1} **= 647.9, so** $v_{r2} = \frac{v_{r1}}{r} = \frac{647.9}{15.5} = 41.8$ **, so from the supplied table,** T_2 **= 840 K.**

TABLE A-22 Ideal Gas Properties of Air

				when $\Delta s = 0$[1]					
T(K), h and u(kJ/kg), s° (kJ/kg · K)									
T	h	u	s°	p_r	v_r	T	h	u	s°
290	290.16	206.91	1.66802	1.2311	676.1	550	554.74	396.86	2.31809
295	295.17	210.49	1.68515	1.3068	647.9	560	565.17	404.42	2.33685
300	300.19	214.07	1.70203	1.3860	621.2	570	575.59	411.97	2.35531
305	305.22	217.67	1.71865	1.4686	596.0	580	586.04	419.55	2.37348
310	310.24	221.25	1.73498	1.5546	572.3	590	596.52	427.15	2.39140
800	821.95	592.30	2.71787	47.75	48.08	1400	1515.42	1113.52	3.36200
820	843.98	608.59	2.74504	52.59	44.84	1420	1539.44	1131.77	3.37901
840	866.08	624.95	2.77170	57.60	41.85	1440	1563.51	1150.13	3.39586
860	888.27	641.40	2.79783	63.09	39.12	1460	1587.63	1168.49	3.41247
880	910.56	657.95	2.82344	68.98	36.61	1480	1611.79	1186.95	3.42892

1. p_r and v_r data for use with Eqs. 6.43 and 6.44, respectively.

b) **In the Diesel cycle, heat is added during the constant pressure process 2-3. From the first law,** $\Delta U = Q - W$ **, but since work is being done at constant pressure,** $W_{23} = \int_2^3 pdV = p(V_3 - V_2) = m \cdot p(v_3 - v_2)$

So $Q_{23} = m(u_3 - u_2) + m \cdot p(v_3 - v_2) = m[(u_3 + pv_3) - (u_2 + pv_2)]$**, but h = u + pv by definition, so**

$$Q_{23} = m(h_3 - h_2)$$

From the ideal gas equation of state,

$$m = \frac{p_1 V_1}{RT} = \frac{\left(1x10^5 \dfrac{N}{m^2}\right)\left(0.02m^3\right)}{\left(\dfrac{8314}{28.97}\dfrac{J}{kg \cdot K}\right)(295K)} = 0.0236kg$$

Therefore $Q_{in} = Q_{23} = 0.0236kg(1855.5 - 866.08)kJ/kg = 23.35kJ$

c) **The net work of any cycle is equal to the net heat transfer, so** $W_{cycle} = Q_{in} - Q_{out} = Q_{23} - Q_{41}$**. From a first law analysis with no work, since process 4-1 is at constant volume,** $Q_{41} = m(u_4 - u_1)$**.**
Therefore $W_{cycle} = 23.35kJ - 0.0236kg(646.48 - 210.49)kJ/kg = 13.06kJ$

d) **The thermal efficiency of the cycle is** $\eta = \dfrac{W_{cycle}}{Q_{in}} = \dfrac{13.06}{23.35} = 0.559 = 55.9\%$

Problem 6 (20 points):

(a) Start with the combined 1st and 2nd laws, dU = T dS – P dV, and consider S(T,V) (i.e. expand dS in terms of dT and dV). Simplify this equation using the Maxwell relation derived from dF=-SdT-PdV and use the definition of C_V to prove the following thermodynamic relation (where familiar notation is used):

$$dU = C_v dT + \left[T\left(\frac{\partial P}{\partial T}\right)_V - P \right] dV \rightarrow \text{(Equation 1)}$$

(b) Applying the concept of exact differential to Equation 1

show that: $\left(\frac{\partial C_v}{\partial v}\right)_T = T\left(\frac{\partial^2 P}{\partial T^2}\right)_V \rightarrow$ (Equation 2)

(c) We have used in many occasions in this course that the specific energy U of an ideal gas is only a function of temperature T. Prove that this is indeed the case starting from Equations (1) and (2) above and using PV=RT for an ideal gas.

Solution:

(Part 1)

$$dU = T\left[\left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV\right] - PdV = T\left(\frac{\partial S}{\partial T}\right)_V dT + \left[T\left(\frac{\partial S}{\partial V}\right)_T - P\right] dV$$

or using the definitions of C_V and the given Maxwell equation:

$$dU = C_v dT + \left[T\left(\frac{\partial P}{\partial T}\right)_V - P \right] dV$$

(Part 2)

Applying the test for exactness of the differential in Equation (1) above, we see that

$$\left(\frac{\partial C_v}{\partial v}\right)_T = \left(\frac{\partial\left(T\left(\frac{\partial P}{\partial T}\right)_V - P\right)}{\partial T}\right)_v \Rightarrow \left(\frac{\partial C_v}{\partial v}\right)_T = T\left(\frac{\partial^2 P}{\partial T^2}\right)_v + \left(\frac{\partial P}{\partial T}\right)_V - \left(\frac{\partial P}{\partial T}\right)_V .$$

or $\left(\dfrac{\partial C_v}{\partial v}\right)_T = T\left(\dfrac{\partial^2 P}{\partial T^2}\right)_v$. Hence proved.

(c) We need to show that $\left[T\left(\dfrac{\partial P}{\partial T}\right)_V - P\right] = 0$ and that C_v is only a function of T.

Using PV=RT, the first term becomes: $T\left(\dfrac{\partial P}{\partial T}\right)_V - P = T\dfrac{R}{V} - P = P - P = 0$ and from

$$\left(\frac{\partial C_v}{\partial v}\right)_T = T\left(\frac{\partial^2 P}{\partial T^2}\right)_v = T\left(\frac{\partial (R/v)}{\partial T}\right)_v = 0$$ which proves the desired result.