

Thursday October 11, 7:30 pm – 9:30 pm

Closed books and notes. Answer all questions. Make sure your answers are legible. Circle your final answer.

The TAs and instructor will not respond to any questions during the exam. If you think that something is wrong with one of the problems below, please state your concern in your exam books.

Problem 1 (10 points) *Multiple choice questions. Answer all questions – explanations are not needed and will not be graded, only provide your answer.*

(1) (2 points) An ideal gas is throttled to a lower pressure. During this process, (select the correct statement)

- (a) the temperature of the gas will increase.
- (b) the temperature of the gas will decrease.
- (c) the temperature of the gas will remain constant.
- (d) the entropy of the gas will decrease.
- (e) the enthalpy of the gas will decrease.

Answer: For a throttling process, h remains constant – so the temperature remains constant (correct answer (c)). Recall for ideal gases $h(T)$.

(2) (3 points) An ideal gas process for which $P v^k = \text{constant}$, where $k=c_p/c_v$ is an (select the correct answer(s))

- (a) incompressible process
- (b) isobaric process
- (c) adiabatic process
- (d) isentropic process
- (e) isothermal process

Answer: The correct answers are (c,d)

(3) (2 points) When a closed system with internal irreversibilities undergoes a process from state 1 to state 2, the entropy change, $S_2 - S_1$, and entropy production, σ , can be

- a) negative and positive, respectively.
- b) negative and zero, respectively.
- c) positive and negative, respectively.
- d) positive and zero, respectively.
- e) all of the above.

Answer: (a) negative and positive respectively; σ is always positive for this process.

(4) (3 points) A refrigeration cycle operating between two reservoirs receives energy Q_C from a cold reservoir at $T_C = 250$ K and rejects energy Q_H to a hot reservoir at $T_H = 300$

K. If $Q_c = 1400$ kJ and $W_{\text{cycle}} = 140$ kJ, does this cycle operate reversibly, irreversibly, or is it impossible?

- Reversibly
- Irreversibly
- It is impossible

Answer: $\beta_{\text{max}} = \frac{T_c}{T_h - T_c} = \frac{250}{300 - 250} = 5$ and $\beta = \frac{Q_c}{W_{\text{cycle}}} = \frac{1400}{140} = 10$, since $\beta > \beta_{\text{max}}$ this is impossible, so the answer is (c).

Problem 2 (5 points) 1.2 kg of liquid water initially at 15°C is to be heated to 95°C in a teapot equipped with a 1200 W electric heating element inside. The specific heat of water can be taken to be 4.18 kJ/kg·°C, and the heat loss from the water during heating can be neglected. The time it takes to heat the water to the desired temperature is approximately: (PROVIDE ONLY YOUR ANSWER – NO PARTIAL CREDIT)

- (a) 4.6 min (b) 5.6 min (c) 6.7 min (d) 9.0 min (e) 11.4 min

Answer (b) 5.6 minutes

(Calculation: $1.2 \times (4.18 \times 10^3) \times (95-15) = 1200 t \rightarrow t=334.4$ seconds $\rightarrow t=5.57$ minutes)

Problem 3 (5 points) Steam expands in an adiabatic turbine from 8 MPa and 450°C to a pressure of 50 kPa at a rate of 1.8 kg/s. The maximum possible power output of the turbine is (PROVIDE ONLY YOUR ANSWER – NO PARTIAL CREDIT)

- (a) 1129 kW (b) 995 kW (c) 2136 kW (d) 718 kW (e) 1791 kW

Answer (e) 1791 kW

Calculation:

From the 1st Law of thermodynamics, for a control volume enclosing the adiabatic turbine, we obtain:

$$\dot{W} = \dot{m}(h_1 - h_2).$$

The maximum power output of the turbine is when the turbine is operating

ISENTROPICALLY. Therefore $h_2 = h_{2s}$.

h_1 = specific enthalpy at 8MPa, 450°C = interpolating from Table A-4 = 3271.68 kJ/kg.

The specific entropy at state 1 = $s_1 = 6.5539$ kJ/kg-K.

Observe that at 50 kPa = 0.5bar, the saturated vapor has specific entropy 6.5539 kJ/kg-K < $s_g = 7.5939$ kJ/kg K (from Table A-3). Therefore, the steam exits the turbine as a saturated mixture.

From Table A-3, calculate the quality for the specific entropy 6.5539 kJ/kg K at pressure $p_2 = 0.5$ bar. The quality is given as

$$x = \frac{6.5539 - 1.091}{7.5939 - 1.091} = 0.84.$$

Using this quality, the specific enthalpy at exit h_{2s} is given as:

$$(1 - 0.84)340.49 + 0.84 \times 2645.9 = 2277.03 \text{ kJ / kg}$$

Therefore, the maximum power = $1.8 * (3271.68 - 2277.03) = 1791$ kW.

Problem 4 (5 points)

Steam is compressed from 1 MPa and 280°C to 8.0 MPa isentropically. Determine the final temperature of the steam.

Answer:

Using Table A4 for superheated steam at $P = 1$ MPa and $T = 280$ °C, $s_1 = 7.0465$ kJ/kg.K. Since $s_2 = s_1$, looking up in the column with $P = 8$ MPa and interpolating for $s_2 = 7.0465$ kJ/kg.K, we obtain $T = 609.6$ °C.

Problem 5 (15 points)

Air (considered ideal gas) is contained in an insulated rigid volume at 20°C and 200 kPa. A paddle wheel inserted in the volume does 720 kJ of work on the air. If the volume is 2m^3 compute the change of entropy of the system ΔS in kJ/°K.

Solution:

Applying the first law of thermodynamics for our isolated system results in the following:

$$-W = \Delta U = m(u_2 - u_1)$$

The mass of the system (considered as ideal gas) is given as:

$$m = \frac{pV}{RT} = \frac{200 \times 2}{0.287 \times 293} = 4.76 \text{ kg}$$

From the Tables of air (Table A-22), we approximate for the given $T_1 = 20$ °C, the following:

$$u_1 = 209.1 \text{ kJ / kg}$$

At this time from the same Table we can also record the value of $s_1^0 = 1.678$

Now solving for u_2 from the 1st law results in:

$$u_2 = -\frac{W}{m} + u_1 = -\frac{-720}{4.76} + 209.1 = 360.4 \text{ kJ / kg}$$

From this value of u_2 , from Table A-22 we can infer that:

$$u_2 = 360.4 \text{ kJ / kg} \Rightarrow T_2 = 501.2 \text{ K}, s_2^0 = 2.22$$

From ideal gas equation: $P_2 = P_1 T_2 / T_1 = 200 \cdot 501.2 / 293 = 342.1 \text{ kPa}$.

Finally, we conclude that:

$$\begin{aligned} \Delta S &= m(s^o(T_2) - s^o(T_1)) - R \ln\left(\frac{P_2}{P_1}\right) \\ &= 4.76(2.22 - 1.678 - 0.287 \ln \frac{342.1}{200}) = 1.856 \text{ kJ / K} \end{aligned}$$

Problem 6 (10 points) Air (ideal gas) enters a compressor operating at steady state at 17 °C, 1 bar and exits at a pressure of 5 bar. KE and PE changes can be ignored. If there are no internal irreversibilities, evaluate the work and heat transfer in kJ/kg for an isothermal compression.

Solution:

An energy rate balance reduces at steady state to give:

$$0 = \frac{\dot{Q}_{cv}}{\dot{m}} - \frac{\dot{W}_{cv}}{\dot{m}} + \dot{m}(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) = \frac{\dot{Q}_{cv}}{\dot{m}} - \frac{\dot{W}_{cv}}{\dot{m}} + \dot{m}(h_1 - h_2)$$

As the specific enthalpy of an ideal gas depends only on temperature and temperature is constant, $h_2 = h_1$. The energy balance then becomes:

$$\dot{Q}_{cv} = \dot{W}_{cv}$$

Since irreversibilities are absent and $\Delta KE = \Delta PE = 0$, the work per unit mass flowing is given by:

$$\frac{\dot{W}_{cv}}{\dot{m}} = -\int_1^2 v dP = -\int_1^2 \frac{RT}{P} dP = -RT \ln \frac{P_2}{P_1} = -\frac{8.314kJ}{28.97kgK} 290K \ln \frac{5}{1} = -133.95kJ / Kg.$$

Problem 7 (10 points)

Two kg of superheated steam at 400°C, 600 kPa is cooled at constant P by transferring heat from a cylinder until the steam is completely condensed. The surroundings are at 25 °C. Determine the net entropy change of the universe.

- Our system is the steam (use Tables A4 and A3):

$$\Delta S_{system} = m(s_2 - s_1) = 2(1.9312 - 7.7086) = -11.55kJ / K$$

Note that entropy is reduced since heat is transferred from the system.

- The heat transfer to the surroundings occurs at constant T (conditions of constant P as well for estimating Q):

$$\Delta S_{surr} = \frac{Q}{T} = \frac{m\Delta h}{T} = \frac{2(3270.2 - 670.6)}{298} = 17.45kJ / K$$

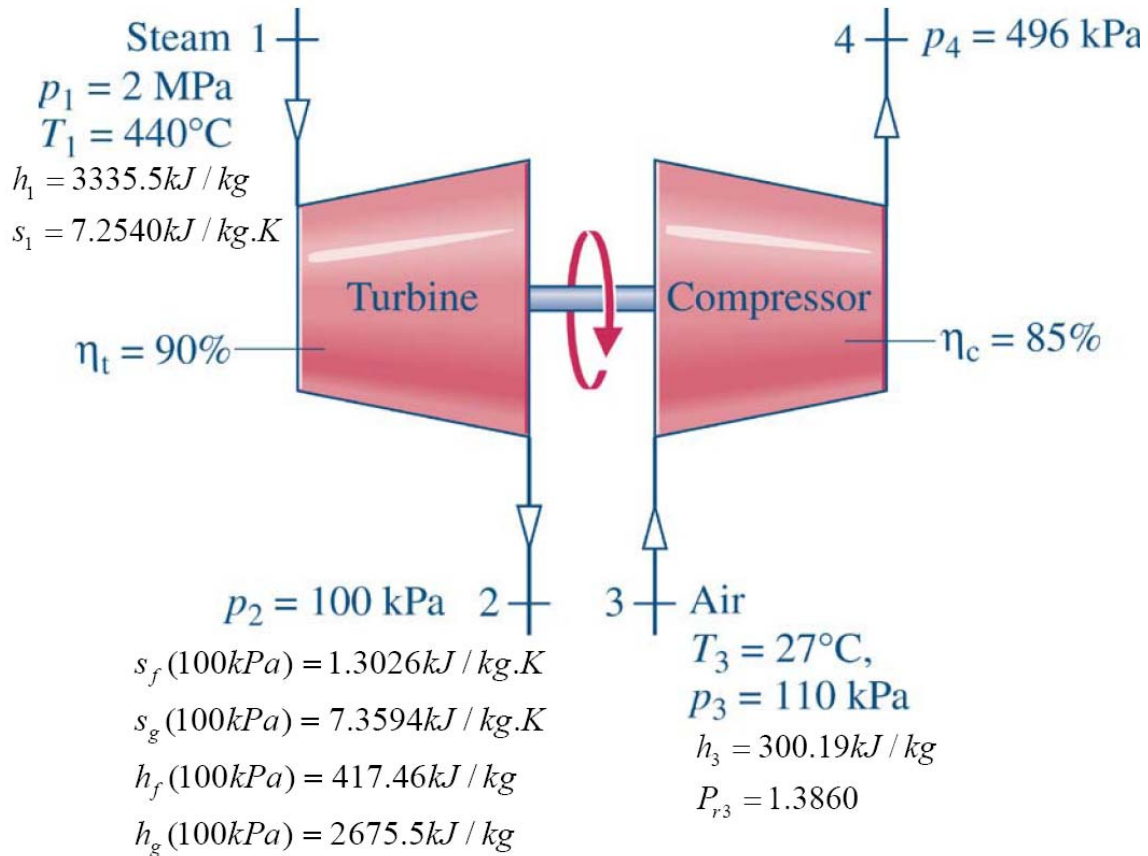
- Finally:

$$\Delta S_{universe} = \Delta S_{system} + \Delta S_{surr} = -11.55 + 17.45 = 5.90kJ / K > 0$$

Problem 8 (20 points)

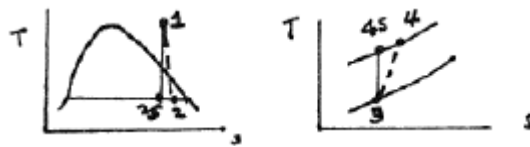
As shown in the Fig., a steam turbine having an isentropic turbine efficiency of 90% drives an air compressor having an isentropic compressor efficiency of 85%. Steady-state operating data are provided on the figure. Determine the mass flow rate of the steam entering the turbine, in kg of steam per kg of air exiting the compressor.

Schematic and Data:



Assumptions:

1. Assume the ideal gas model for air.
2. Ignore stray heat transfer and kinetic and potential energy effects.
3. For control volume, $W_{cv} = 0$.



Analysis:

Mass balances give $\dot{m}_1 = \dot{m}_2$ and $\dot{m}_3 = \dot{m}_4$. An energy balance gives

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1(h_1 - h_2) + \dot{m}_4(h_3 - h_4) \Rightarrow$$

$$0 = 0 - 0 + \dot{m}_1(h_1 - h_2) + \dot{m}_4(h_3 - h_4) \Rightarrow \dot{m}_1 = \dot{m}_4 \left[\frac{h_4 - h_3}{h_1 - h_2} \right] \quad (1)$$

The respective isentropic efficiencies are, $\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}}$, $\eta_c = \frac{h_{4s} - h_3}{h_4 - h_3}$

Substitute into equation 1,

$$\frac{\dot{m}_1}{\dot{m}_4} = \frac{1}{\eta_t \eta_c} \left[\frac{h_{4s} - h_3}{h_1 - h_{2s}} \right] \quad (2)$$

We are given the following data from Table A-4, $h_1 = 3335.5 \text{ kJ/kg}$, $s_1 = 7.2540 \text{ kJ/kg.K}$

Equating $s_{2s} = s_1$, we can obtain the quality of state 2s, with which we can obtain the h_{2s} value. The values s_f and s_g are given (computed from Table A-2 with interpolation) for pressure 100 kPa.

$$x_{2s} = \frac{7.2540 - 1.3026}{7.3594 - 1.3066} = 0.9826$$

We can now compute the enthalpy at state 2s as follows:

$$h_{2s} = (1 - 0.9826)417.46 + 0.9826 \times 2675.5 = 2636.2 \text{ kJ/kg}$$

The process 3→4s is an isentropic process. We are given data extracted From Table A-22. $h_3 = 300.19 \text{ kJ/kg}$, $P_{r3} = 1.3860$. Then we know that for isentropic processes $P_{r4} = P_{r3} (P_4/P_3) = 6.2496$.

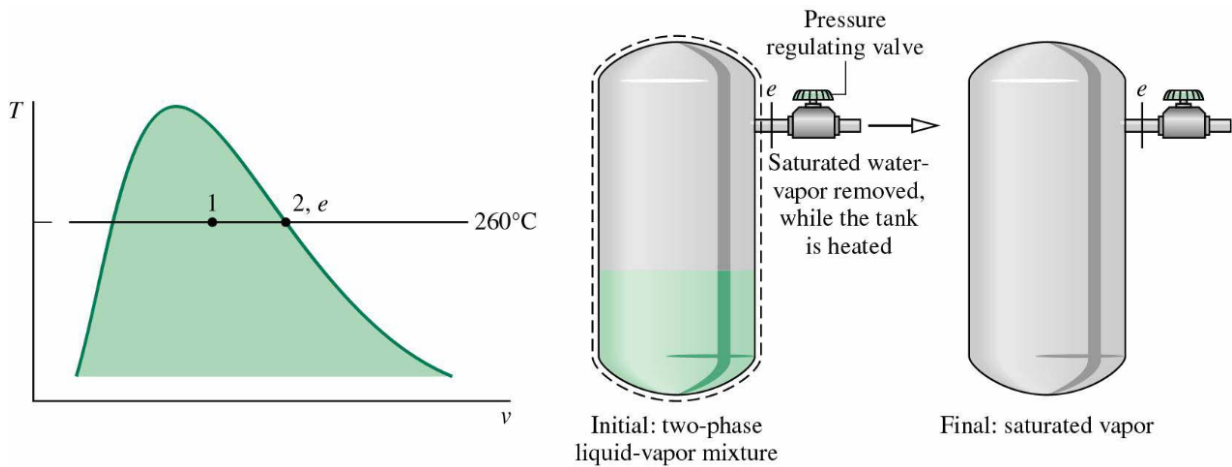
From this value of P_{r4} , we get $h_{4s} = 462.1 \text{ kJ/kg}$.

Thus $\eta_t = 0.9$, $\eta_c = 0.85$

$$\frac{\dot{m}_1}{\dot{m}_4} = \frac{1}{(0.9)(0.85)} \left[\frac{462.1 - 300.19}{3335.5 - 2636.2} \right] = \frac{1}{(0.9)(0.85)} (0.232) = 0.303$$

Problem 9 (20 points)

A tank having a volume of 0.85 m^3 initially contains water as a two-phase liquid-vapor mixture at 260°C and a quality of 0.7. Saturated water vapor at 260°C is slowly withdrawn through a pressure-regulating valve at the top of the tank as energy is transferred by heat to maintain the pressure constant in the tank. This continues until the tank is filled with saturated vapor at 260°C . Determine the amount of heat transfer in kJ. Neglect all kinetic and potential energy effects.



Additional information:

$$u_f(260^\circ C) = 1128.4 \text{ kJ / Kg}, u_g(260^\circ C) = 2599.0 \text{ kJ / Kg}$$

$$v_f(260^\circ C) = 1.2755 \times 10^{-3} \text{ m}^3 / \text{Kg}, v_g(260^\circ C) = 0.04221 \text{ m}^3 / \text{Kg}$$

$$h_g(260^\circ C) = 2796.6 \text{ kJ / Kg}$$

Solution

The mass rate balance takes the form: $\frac{dm_{cv}}{dt} = -\dot{m}_e$

The energy rate balance reduces to: $\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{m}_e h_e$

From the above two equations: $\frac{dU_{cv}}{dt} = \dot{Q}_{cv} + \frac{dm_{cv}}{dt} h_e$

Since the state at the exit is constant, integration of the last equation gives:

$$\Delta U_{cv} = Q_{cv} + \Delta m_{cv} h_e \quad \text{or} \quad Q_{cv} = \Delta U_{cv} - \Delta m_{cv} h_e \quad \text{from which we can}$$

derive:

$$Q_{cv} = (m_2 u_2 - m_1 u_1) - h_e (m_2 - m_1)$$

Here m_1 and m_2 are the initial and final mass within the tank.

How do you get m_1 and u_1 ? From the given data at 260°C

$$u_1 = u_f + x_1(u_g - u_f) = 1128.4 + 0.7(2599.0 - 1128.4) = 2157.8 \text{ kJ / kg}$$

$$v_1 = v_f + x_1(v_g - v_f) = 1.2755 \times 10^{-3} + 0.7(0.04221 - 1.2755 \times 10^{-3}) = 29.93 \times 10^{-3} \text{ m}^3 / \text{kg}$$

$$m_1 = \frac{V}{v_1} = \frac{0.85 \text{ m}^3}{29.93 \times 10^{-3} \text{ m}^3 / \text{kg}} = 28.4 \text{ Kg}$$

How do you get m_2 , u_2 and h_e ? Using the given data for saturated vapor at 260°C .

$$u_2 = u_g(260^\circ \text{C}) = 2599.0 \text{ kJ / Kg}$$

$$v_2 = v_g(260^\circ \text{C}) = 42.21 \times 10^{-3} \text{ m}^3 / \text{kg}$$

$$m_2 = \frac{V}{v_2} = \frac{0.85 \text{ m}^3}{42.21 \times 10^{-3} \text{ m}^3 / \text{kg}} = 20.14 \text{ Kg}$$

$$h_e = h_g(260^\circ \text{C}) = 2796.6 \text{ kJ / kg}$$

Finally:

$$Q_{cv} = (20.14 \times 2599.0 - 28.4 \times 2157.8) - 2796.6(20.14 - 28.4) = 14,162 \text{ kJ}$$